

Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS251 – Mathematics of Financial Derivatives
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T252 – Major 2

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Name: _____ ID: _____

Instructions

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two persons can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, ipad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. Write important steps to arrive at the solution of the exam problems.
9. Important: Questions in Part A worth 1 point each, while Questions in Part B worth 2 points each (1 point for correct option, 1 point for correct working).

The test is 120 minutes, GOOD LUCK, and you may begin now.

Part A: Concept Questions (1 point each)

1. An investor has written a covered call. Which of the following represents the investor's position?
 - (A) Short the call and short the stock
 - (B) Short the call and long the stock
 - (C) Short the call and no position on the stock
 - (D) Long the call and short the stock
 - (E) Long the call and long the stock

2. Which of the following statements about futures and forward contracts is **false**?
 - (A) Frequent marking-to-market of futures can lead to pricing differences from forwards
 - (B) Forward contracts can be customized; futures contracts are standardized
 - (C) Users of forward contracts are better able to protect themselves from credit risks than users of futures contracts
 - (D) Forward contracts can be used to synthetically switch a portfolio invested in stocks into bonds
 - (E) The holder of a long futures contract must place a fraction of the cost with an intermediary

3. An airline company faces risk concerning the price of jet fuel. Which hedging strategy best protects the company against an **increase** in the price of jet fuel?
 - (A) Buying calls on jet fuel
 - (B) Buying collars on jet fuel
 - (C) Buying puts on jet fuel
 - (D) Selling puts on jet fuel
 - (E) Selling calls on jet fuel

4. Which of the following statements about options is **true**?
 - (A) Naked writing is the practice of buying options without taking an offsetting position
 - (B) A covered call involves taking a long position in an asset together with a written call on the same asset
 - (C) An American style option can only be exercised during specified periods
 - (D) A Bermudan style option allows exercise at any time during the life of the option
 - (E) An in-the-money option is one which would have a positive profit if exercised immediately

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5. Which of the following statements about early exercise of options is **true**?
- (A) Put/call parity implies puts and calls should trade at the same price when stock price equals exercise price
 - (B) Early exercise of an American call option only makes sense when there is positive cash flow prior to maturity on the underlying asset
 - (C) As price volatility increases, call option prices rise and put option prices fall
 - (D) European options can be exercised at any time
 - (E) Dividends never affect early exercise decisions
6. Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of:
- (A) Speculation on stock price direction
 - (B) Borrowing or lending money with no stock price risk
 - (C) Hedging against volatility increases
 - (D) Creating unlimited profit potential
 - (E) Reducing transaction costs
7. Which of the following strategies has greater payoffs for **lower** stock prices?
- (A) Bull spread using calls
 - (B) Bear spread using puts
 - (C) Box spread
 - (D) Straddle
 - (E) Strangle
8. The practice of selling a call option without owning the underlying asset is called:
- (A) Covered call
 - (B) Protective put
 - (C) Naked writing
 - (D) Married put
 - (E) Collar

9. Put-call parity for a non-dividend paying stock states that:

(A) $C - P = S_0 - Ke^{-rT}$

(B) $C + P = S_0 + Ke^{-rT}$

(C) $C - P = S_0 + Ke^{-rT}$

(D) $C + P = S_0 - Ke^{-rT}$

(E) $C = P$ always

10. Option prices must satisfy convexity. For call options with strikes $K_1 < K_2 < K_3$, the convexity condition is:

(A) $C(K_2) \leq \lambda C(K_1) + (1 - \lambda)C(K_3)$ where $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$

(B) $C(K_2) \geq \lambda C(K_1) + (1 - \lambda)C(K_3)$

(C) $C(K_2) = C(K_1) + C(K_3)$

(D) $C(K_2) \leq C(K_1) \leq C(K_3)$

(E) $C(K_2) \geq \max(C(K_1), C(K_3))$

Part B: Calculation Questions (2 points each)

11. A four-month European call option with a strike price of 60 is selling for 5. The price of the underlying stock is 61, and the annual continuously compounded risk-free rate is 12%. The stock pays no dividends.

Calculate the value of a four-month European put option with a strike price of 60.

(A) 1.25

(B) 1.65

(C) 2.00

(D) 2.35

(E) 2.80

12. For a nondividend paying stock:

- Current price is 30
- A European call option with one year to expiration and strike price 25 costs 8.05
- Continuously compounded risk-free interest rate is 0.05

Determine the premium of a 1-year European put option on the stock with strike 25.

- (A) 0.83
- (B) 1.23
- (C) 1.83
- (D) 2.43
- (E) 3.03

13. Happy Jalapenos will deliver 10,000 jalapenos in one year at market price. Cost to provide = 1,000. They buy 10,000 put options (strike 0.12) for 84.30 and sell 10,000 call options (strike 0.14) for 74.80. Continuously compounded risk-free rate = 6%.

Calculate the range of possible profit one year from now.

- (A) -200 to 100
- (B) -110 to 190
- (C) -100 to 200
- (D) 190 to 390
- (E) 200 to 400

14. Stock price = 35, continuous dividend yield = 4%, risk-free rate = 6%, one-year American put with strike 34. Determine the lowest possible price for this put option.

- (A) 0
- (B) 0.50
- (C) 1.00
- (D) 1.50
- (E) 2.00

15. You want to borrow \$10,000 for one year using a box spread. Option prices:

Strike	Call Premium	Put Premium
30	8.06	1.31
40	3.13	5.80

Risk-free rate = 6% continuously compounded. One unit = long 30-40 bull call spread + long 40-30 bear put spread. Calculate the number of units needed (bought or sold).

- (A) 800 units bought
(B) 900 units sold
(C) 1062 units sold
(D) 1100 units bought
(E) 1200 units sold
16. An investor bought a 70-strike European put option for 1 and wrote an 80-strike European put option for 8 (same expiration, 6 months). Interest rate = 0%. Calculate the index price at expiration that allows the investor to break even.
- (A) 63
(B) 73
(C) 77
(D) 80
(E) 87

17. American call option prices:

- Strike 40: price = 12
- Strike 50: price = 3

Determine the highest possible price for an American call option with strike price 48.

- (A) 2.4
(B) 3.6
(C) 4.8
(D) 6.0
(E) 7.2

18. European put option prices:

- Strike 40: price = 4
- Strike 50: price = 9

Determine the highest possible price for a European put option with strike price 44.

- (A) 4.0
- (B) 5.0
- (C) 6.0
- (D) 7.0
- (E) 8.0

19. A 182-day American call option on a stock has strike price 100. Risk-free rate = 4%. A dividend is payable on day 91.

Determine the minimum dividend for which early exercise may be rational.

- (A) 0.50
- (B) 0.75
- (C) 1.00
- (D) 1.25
- (E) 1.50

20. A 6-month dollar-denominated European call on pounds (strike 1.45) costs \$0.05. The put costs \$0.02. Dollar rate = 4%, pound rate = 6%.

Calculate the 6-month forward exchange rate (\$/£).

- (A) 1.40
- (B) 1.45
- (C) 1.48
- (D) 1.52
- (E) 1.55

Bonus Questions (1 point each)

21. A box spread with strikes $K_1 = 50$ and $K_2 = 60$ costs \$8 today. The guaranteed payoff at expiration (6 months) is \$10. The continuously compounded risk-free rate is 5%.

Is there an arbitrage opportunity? If so, calculate the arbitrage profit per unit (ignore discounting for simplicity).

- (A) No arbitrage
- (B) Yes, profit = \$1
- (C) Yes, profit = \$2
- (D) Yes, profit = \$3
- (E) Yes, profit = \$4

22. You wish to create a synthetic 182-day Treasury bill with maturity value \$10,000 using a stock and 6-month European options with strike price 95. The stock pays quarterly dividends of \$2, with the next dividend in 3 months. The continuously compounded risk-free rate is 4%.

For every share of stock purchased, the maturity value from the synthetic T-bill is:

$$K + (\text{Cumulative value of dividends}) = 95 + 2e^{0.04(0.25)} + 2 = 99.0201$$

Calculate the number of shares of stock you should purchase.

- (A) 95
- (B) 99
- (C) 101
- (D) 105
- (E) 110

23. A butterfly spread on a stock has strikes $K_1 = 90$, $K_2 = 100$, $K_3 = 110$. Option premiums: $C_{90} = 12$, $C_{100} = 7$, $C_{110} = 3$.

Which of the following positions will **NOT** produce the butterfly profit diagram?

- (A) Buy a 90 call, buy a 110 call, sell two 100 calls
- (B) Buy a 90 put, buy a 110 put, sell two 100 puts
- (C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call
- (D) Buy one share of stock, buy a 90 call, buy a 110 put, sell two 100 puts
- (E) Buy one share of stock, buy a 90 put, buy a 110 call, sell two 100 calls

Solutions

1. **B. Short the call and long the stock**
2. **C. Users of forward contracts are better able to protect themselves from credit risks than users of futures contracts** (Futures have margin accounts that reduce credit risk)
3. **A. Buying calls on jet fuel**
4. **B. A covered call involves taking a long position in an asset together with a written call on the same asset**
5. **B. Early exercise of an American call option only makes sense when there is positive cash flow prior to maturity on the underlying asset**
6. **B. Borrowing or lending money with no stock price risk**
7. **B. Bear spread using puts**
8. **C. Naked writing**
9. **A. $C - P = S_0 - Ke^{-rT}$**
10. **A. $C(K_2) \leq \lambda C(K_1) + (1 - \lambda)C(K_3)$ where $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$**
11. **B. 1.65**
By put-call parity: $P = C + Ke^{-rT} - S = 5 + 60e^{-0.04} - 61 = 5 + 57.65 - 61 = 1.65$
12. **C. 1.83**
 $C - P = S - Ke^{-rT} \Rightarrow 8.05 - P = 30 - 25e^{-0.05} = 30 - 23.78 = 6.22 \Rightarrow P = 1.83$
13. **D. 190 to 390**
Lowest sale price = $0.12 \times 10,000 = 1200$, highest = $0.14 \times 10,000 = 1400$
Net option cost = $84.30 - 74.80 = 9.50$, future value = $9.50e^{0.06} = 10.09$
Total cost = $1000 + 10.09 = 1010.09$
Profit range = $1200 - 1010.09 = 190$ to $1400 - 1010.09 = 390$
14. **A. 0**
 $P \geq \max(0, Ke^{-rT} - Se^{-\delta T}) = \max(0, 34e^{-0.06} - 35e^{-0.04}) = \max(0, 32.02 - 33.63) = 0$
15. **C. 1062 units sold**
Bull call spread cost = $8.06 - 3.13 = 4.93$
Bear put spread cost = $5.80 - 1.31 = 4.49$
Total cost per unit = 9.42
To borrow, sell units: $10,000/9.42 = 1061.57 \approx 1062$

16. **B. 73**

$$\text{Net credit} = 8 - 1 = 7$$

$$\text{If } 70 \leq S \leq 80, \text{ payoff} = 80 - S$$

$$\text{Break-even: } 80 - S = 7 \Rightarrow S = 73$$

17. **C. 4.8**

$$\lambda = \frac{50-48}{50-40} = 0.2$$

$$C(48) \leq 0.2(12) + 0.8(3) = 2.4 + 2.4 = 4.8$$

18. **C. 6.0**

$$\lambda = \frac{50-44}{50-40} = 0.6$$

$$P(44) \leq 0.6(4) + 0.4(9) = 2.4 + 3.6 = 6.0$$

19. **C. 1.00**

$$\text{Interest lost} = 100(1 - e^{-0.04 \times 0.25}) = 100(1 - 0.99005) = 0.995 \approx 1.00$$

20. **C. 1.48**

$$C - P = x_0 e^{-r_f T} - K e^{-r_d T} \Rightarrow 0.03 = x_0 e^{-0.03} - 1.45 e^{-0.02} \Rightarrow x_0 = 1.4513 e^{0.03} = 1.495$$

$$\text{Forward rate} = x_0 e^{(r_d - r_f)T} = 1.495 e^{-0.01} = 1.48$$

— End of Major 2 Examination —

Bonus Solutions

21. **C. Yes, profit = \$2**

$$\text{Guaranteed payoff} = K_2 - K_1 = 60 - 50 = 10$$

$$\text{Cost} = \$8$$

$$\text{Arbitrage profit} = 10 - 8 = \$2 \text{ (ignoring discounting)}$$

22. **C. 101**

$$\text{Maturity value per share} = K + \text{Cumulative dividends} = 95 + 2e^{0.04(0.25)} + 2$$

$$= 95 + 2(1.01005) + 2 = 95 + 2.0201 + 2 = 99.0201$$

$$\text{Number of shares} = 10,000/99.0201 = 100.99 \approx \mathbf{101}$$

23. **D. Buy one share of stock, buy a 90 call, buy a 110 put, sell two 100 puts**

This position results in **unbounded profit** if the stock price goes above 110, which does NOT match the limited-profit butterfly diagram. All other options (A, B, C, E) produce the symmetric butterfly payoff.

— End of Bonus Questions —