

Dept. of Mathematics and Statistics  
King Fahd University of Petroleum & Minerals  
AS251: Mathematics of Financial Derivatives  
Instructor: Dr. Ridwan A. Sanusi  
Final Exam Term 252  
Wednesday, May 20, 2026  
7:00 PM - 9:30 PM

**Name:** \_\_\_\_\_ **ID:** \_\_\_\_\_

**Time:** 2.5 Hours

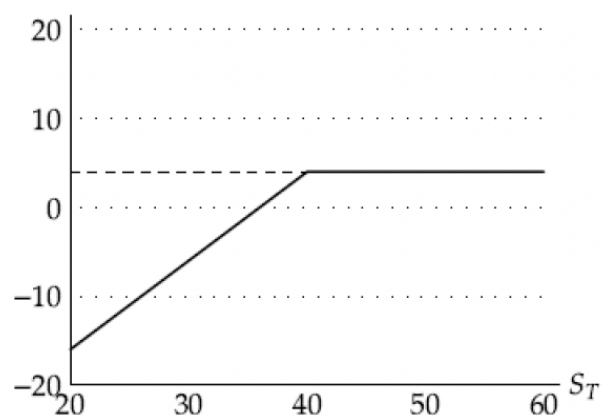
### **Instructions**

1. No phones or any other smart devices. Any student caught during the exam will be considered under the cheating rules of the University.
2. Once the exam starts, nobody will be allowed to leave the exam room until the end of the exam.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
7. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only.
8. Answer all questions.
9. Choose the **best** answer for multiple-choice questions.
10. For Part B, show your work. Write important steps to arrive at the solution of the exam problems. Only answers supported by work will be considered. Unsupported guesses will not be graded.
11. Formula sheet is provided on the last page.
12. If needed, take  $N(-0.03725) = 0.48514$ ,  $N(-0.17867) = 0.42910$ ,  $N(-0.10) = 0.46017$ , and  $N(-0.05) = 0.48006$  from standard Normal Table.

**Part A (1 point each)**

1. A forward contract is best described as:
  - (A) An agreement that gives the buyer the right, but not the obligation, to purchase an asset at a predetermined price on a future date
  - (B) A standardized exchange-traded agreement to buy or sell an asset at a specified future date
  - (C) An over-the-counter agreement to buy or sell an asset at a specified future date at a price agreed upon today
  - (D) A contract that requires daily settlement of gains and losses through a margin account
  - (E) An agreement that can only be settled by physical delivery of the underlying asset
  
2. Sama expects to harvest 10,000 bushels of wheat in 3 months. To lock in a selling price today, Sama should:
  - (A) Buy a forward contract on wheat
  - (B) Sell a forward contract on wheat
  - (C) Buy a put option on wheat
  - (D) Sell a call option on wheat
  - (E) Do nothing because prices cannot be locked in
  
3. Determine which of the following positions has the same cash flow as a long zero-coupon bond position.
  - (A) Long stock and long forward
  - (B) Long stock and short forward
  - (C) Short stock and long forward
  - (D) Short stock and short forward
  - (E) Long forward and short forward
  
4. Stock XYZ has a current price of 100. The forward price for delivery of this stock in 1 year is 110. Unless otherwise indicated, the stock pays no dividends and the annual effective risk-free interest rate is 10%. Determine which of the following statements is FALSE.
  - (A) The time-1 profit diagram and the time-1 payoff diagram for long positions in this forward contract are identical.
  - (B) The time-1 profit for a long position in this forward contract is exactly opposite to the time-1 profit for the corresponding short forward position.
  - (C) There is no comparative advantage to investing in the stock versus investing in the forward contract.
  - (D) If the 10% interest rate was continuously compounded instead of annual effective, then it would be more beneficial to invest in the stock, rather than the forward contract.
  - (E) If there was a dividend of 3.00 paid 6 months from now, then it would be more beneficial to invest in the stock, rather than the forward contract.

5. The figure below shows a payoff/profit diagram. The figure is the:

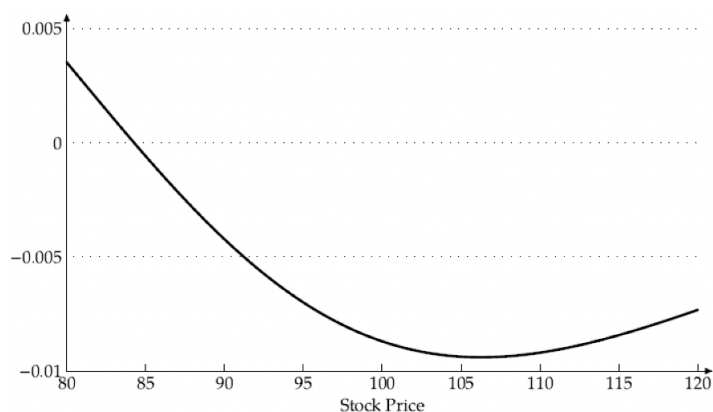


- (A) Payoff on put option with strike price 40
- (B) Profit on call option with strike price 40
- (C) Payoff on short call option with strike price 40
- (D) Profit on short call option with strike price 40
- (E) Profit on short put option with strike price 40

6. An option is said to be *at-the-money* if:

- (A) the strike price equals the current underlying price
- (B) exercising the option would result in a positive cash settlement
- (C) exercising the option would result in a negative cash settlement
- (D) the current price of the underlying is less than the strike price
- (E) None of the above.

7. The figure below is a graph of an option Greek as a function of stock price. Which option Greek is this most likely to be?



- (A)  $\Delta$  (Delta)
- (B)  $\Gamma$  (Gamma)
- (C)  $\theta$  (Theta)
- (D)  $\Psi$  (Psi)
- (E)  $\rho$  (Rho)

8. Regarding option Greeks, which of the following statement(s) is/are true?

- I.  $\theta$  for a call option is negative.
- II.  $\Gamma$  is the same for a call option and a put option with same strike price and time to expiry.
- III. Vega is maximal for at-the-money options.
- IV.  $\rho$  for a call option is positive.
- V.  $\Psi$  increases for a put option as time to expiration increases.

- (A) I, III
- (B) II, IV, V
- (C) I, II, IV, V
- (D) I, II, III, IV, V
- (E) II

9. Which of the following 3-month put options is most valuable?

- (A) European put with strike price 40.
- (B) Asian arithmetic average weekly price put with strike price 40.
- (C) Asian arithmetic average daily price put with strike price 40.
- (D) Asian geometric average weekly price put with strike price 40.
- (E) Asian geometric average daily price put with strike price 40.

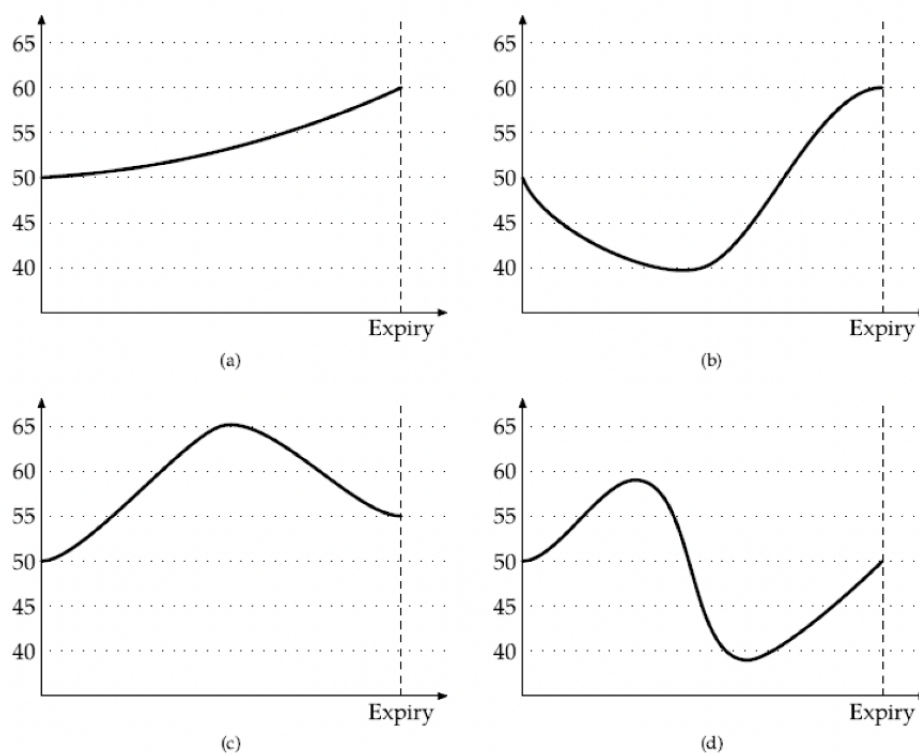
10. Consider the following 6 call options on a stock with price 50.

- I. Standard option, strike 55.
- II. Standard option, strike 60.
- III. Down-and-in option, strike 60, barrier 45.
- IV. Down-and-out option, strike 60, barrier 45.
- V. Up-and-in option, strike 60, barrier 55.
- VI. Up-and-out option, strike 60, barrier 55.

Select the correct inequality relationship between the options.

- (A)  $IV < III < II < I$
- (B)  $VI < IV < I < II$
- (C)  $I < II < IV < VI$
- (D)  $III < VI < V < II$
- (E)  $VI < III < V < I$

11. Consider a knock-out barrier call option with strike price 50 and barrier 65. Which of the figures below will not pay?



- (A) A  
 (B) B  
 (C) C  
 (D) D  
 (E) C and D

**Part B (2 points each. Show your workings.)**

12. A stock's current price is 40. It pays continuous dividends at a rate of 4%. The continuously compounded risk-free rate is 10%. Party A buys 10,000 180-day futures contracts from Party B. Each contract allows purchase of 1 share of stock. After 1 day, the price of the stock increases to 42. Calculate the amount paid by Party B to Party A after 1 day.
- (A) 20,530  
 (B) 412,012  
 (C) 432,542  
 (D) 31,640  
 (E) 420,220

13. A 1-year European put option on a nondividend paying stock has a strike price of 50. The current stock price is 50. At the end of a year, the stock price will be either 40 or 60. The risk-free rate is 5%.

The replicating portfolio consists of  $\Delta$  shares of stock and lending  $B$ . Determine  $B$ .

- (A) 3.5369
- (B) -0.5000
- (C) 28.5369
- (D) 0.6282
- (E) 24.6549

14. The spot exchange rate of dollars for euros is 1.15 \$/€. A 6-month American call option allows purchase of euros at 1.25 \$/€. You are given:

- (i)  $r_s = 0.05$ .
- (ii)  $r_{\text{€}} = 0.04$ .
- (iii) The annual volatility of the exchange rate is 0.1.

A 2-period binomial tree based on forward prices is used to value the option.

Determine the call premium.

- (A) 0.0109
- (B) 0.0074
- (C) 0.0063
- (D) 0.0052
- (E) 0.0041

15. For an American call option on a stock, the stock price is initially 60, the strike price is 60,  $r = 0.05$ , and  $\delta = 0.04$ . There are 3 months to expiry. If the value of a 3-month put option with strike price 60 is 1 and early exercise at this point is optimal, what is the lowest possible current value for the stock?
- (A) 53.3043
  - (B) 74.2001
  - (C) 126.7009
  - (D) 175.4070
  - (E) 204.2322
16.  $S_t$  is the price of a nondividend paying stock at time  $t$ .  $S_t$  follows a lognormal model. You are given:  $S_0 = 45$ , the continuously compounded expected rate of return is 0.15, and the volatility is 0.2. Let  $p_t$  be the probability that  $S_t$  is less than 40. Determine the value of  $t$  for which  $p_t$  is maximized over all positive  $t$ .
- (A) 33.7080
  - (B) 0.5322
  - (C) 1.5444
  - (D) 0.9060
  - (E) 0.3664

17. You are given: The price of a futures contract is 325 and Volatility of the contract is 0.2. A 6-month call option on the futures contract with strike price 330 has price 15.50 based on the Black formula. Determine the continuously compounded risk-free interest rate.

- (A) 0.1053
- (B) 0.0942
- (C) 0.0831
- (D) 0.0720
- (E) 0.0619

18. Assume you have purchased European put options from Fajr for 100,000 shares of a non-dividend paying stock and you are given the following information: Price of stock = \$49.16, Strike price = \$50.00, Continuously compounded risk-free interest rate = 5% per annum, Volatility = 20% per annum, and there are 20 weeks remaining until maturity.

You now have the following information:

$T$ (weeks)	Stock price	$d_1$
1	\$49.33	0.10
2	\$49.09	0.05

You decide to readjust the delta hedging strategy on a weekly basis.

Calculate the cumulative cost, including interest, of the hedge at the end of week 2.

- (A) 2,364,095
- (B) 2,304,821
- (C) 2,360,879
- (D) 2,263,190
- (E) 2,261,016

## Formula Sheet

### Forward Pricing

$$F = S_0 e^{rT} \quad (\text{non-dividend stock})$$
$$F = S_0 e^{(r-\delta)T} \quad (\text{continuous dividends})$$
$$F = x_0 e^{(r_d - r_f)T} \quad (\text{currency})$$

### Put-Call Parity

$$C - P = S_0 e^{-\delta T} - K e^{-rT}$$
$$C - P = S_0 - K e^{-rT} \quad (\text{non-dividend})$$

### Binomial Model

$$u = e^{\sigma\sqrt{h}}, \quad d = e^{-\sigma\sqrt{h}} \quad (\text{or given})$$
$$p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$
$$\Delta = \frac{C_u - C_d}{S_0 u - S_0 d}$$
$$C = e^{-rh} [p^* C_u + (1 - p^*) C_d]$$

### Black-Scholes Formula

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$
$$C = S e^{-\delta T} N(d_1) - K e^{-rT} N(d_2)$$
$$P = K e^{-rT} N(-d_2) - S e^{-\delta T} N(-d_1)$$