Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS481: Actuarial Contingencies 2
Dr. Ridwan A. Sanusi
Major Exam II, Term 252
Monday, April 6, 2026
2:00 PM – 4:00 PM

Name…………………………………………………. ID#:_________ Serial #:___

Instructions.

1. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so before the exam starts. Nobody will be allowed to leave the room once the exam starts.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. ***Only answers supported by work will be considered. Unsupported guesses will not be graded.***
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***

The test is 120 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1-5	5		
6-7	1		
PART B			
8	3		
9	3		
10	3		
11	3		
Bonus	3		
Total	18		

Extra blank page

PART A

Show workings for all questions except questions 6 - 7.

Use the following information to answer questions 1-2.

Given the following life table. Assume a 2-state model; State 1: Alive and State 2: Dead.

x	0	1	2	3	4	…	109	110
l_x	100,000	97,408	97,259	97,160	97,082	…	1	0

1. Calculate (p_2, q_3)

A. (0.9741, 0.0008).

B. (0.9990, 0.0008).

C. (0.9990, 0.0010).

D. (0.9741, 0.0259).

E. (0.9992, 0.0008).

2. Calculate π_2

A. (0.9741, 0.0259).

B. (0.9726, 0.0274).

C. (0.9716, 0.0284).

D. (0.9708, 0.0292).

E. (1, 0).

3.

Let T_x and T_y be independent future lifetime random variables. Given $q_x = .080$, $q_y = .004$, ${}_tp_x = 1 - t^2 \cdot q_x$, and ${}_tp_y = 1 - t^2 \cdot q_y$, both for $0 \le t \le 1$, evaluate the PDF of $T_{xy} = 0.55$.

A. 0.0838
B. 0.0837
C. 0.0920
D. 0.8384
E. 0.9368

4.

Two microwave models, denoted Type I and Type II, follow survival models defined by $\mu_x^{I} = \ln 1.25$, for $x > 0$, and $\mu_x^{II} = \frac{1}{9-x}$, for $0 < x < 9$, respectively. Given that both models are currently two years old, and that they have independent lifetimes, find the probability that the first failure will occur between ages 3 and 6.

A. 0.5102
B. 0.4091
C. 0.3980
D. 0.2879
E. 0.1768

5.

For independent lives (x) and (y), the force of failure is constant over each year of age. Find the value of ${}_{.75}p_{\overline{x+.25:y+.25}}$, given that $q_x = .08$ and $q_y = .06$.

A. 0.9973
B. 0.0027
C. 0.9394
D. 0.9547
E. 0.0453

Use the following information to answer questions 6-7.

Consider a tabular survival model for an inanimate object (such as a light bulb) for which the probability of continued survival over each successive discrete time interval, say, p, is the same regardless of the attained age of the object, so that the survival model can be represented by a homogeneous discrete-time Markov model with $X_0 = 0$. (Assume that the object, once failed, remains failed forever.) Define each of the following mulit-state model concepts:

6. $\Pr[X_{10} = 1 | X_9 = 0]$
A. p_9
B. p^9
C. $1 - p^9$
D. $1 - p$
E. q^9

7. π_{90}
A. p_9
B. p^9
C. $1 - p^9$
D. $1 - p$
E. q^9

PART B

8.

Let Y denote the present value random variable for a contingent annuity-due with unit payment made during the first 15 years if at least one of (x) and (y) survive, but made after the first 15 years only if exactly one of (x) and (y) survive. Find the value of $E[Y]$, given the following values:

$$\ddot{a}_x = 9.80 \qquad \ddot{a}_y = 11.60 \qquad \ddot{a}_{xy} = 7.60 \qquad {}_{15|}\ddot{a}_{xy} = 3.70$$

9.

A health insurer divides its insured population into the three risk classes of low, moderate, and high. The following table shows the percentage of insureds in each risk class in Year Z, and the reallocation of them among the three risk classes in Year $Z+1$.

Risk Class	**Year Z Distribution**	**Year $Z+1$ Distribution**		
		Low	**Moderate**	**High**
Low	69.5%	57.4%	11.7%	0.4%
Moderate	28.7%	9.9%	17.7%	1.1%
High	1.8%	0.2%	0.9%	0.7%
Totals	100%	67.5%	30.3%	2.2%

(a) From the data in the table, develop a matrix of transition probabilities for transition among risk classes.

(b) The average claim costs in Year Z for the three risk classes are 500 for low, 10,000 for moderate, and 50,000 for high. Find the average claim cost in Year Z for the entire insured population.

(c) Assuming a homogeneous discrete time Markov process, find the percentage distribution of the insured in Year Z + 2.

10.

Use the nominal annual coupon yields in the table below to calculate the corresponding zero-coupon yields of the same maturities. (In both cases the nominal annual yield rates are convertible semiannually.)

Maturity (in years)	Nominal Annual Yield for Coupon-bearing Bonds	Nominal Annual Yield for Zero-coupon Bonds
0.5	2.0%	
1.0	4.0	
1.5	6.0	
2.0	8.0	

11.

Using the n-year forward one-year rates in the following table, find all determinable spot rates.

n	$f_{n,1}$
0	4.0%
1	5.0
2	6.0
3	7.0
4	8.0

BONUS

12. Using the n-year forward one-year rates from question 11, find all available forward rates.