King Fahd University of Petroleum and Minerals
Dept. of Mathematics and Statistics
AS481: Actuarial Contingencies 2
FINAL Exam, T252
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12:30 PM – 2:30 PM

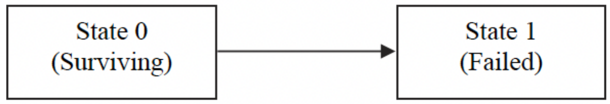
Name: ______________________ ID#: ______________________

Instructions

1. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so before the exam starts. Nobody will be allowed to leave the room once the exam starts.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly the question number.
6. **Only answers supported by work will be considered. Unsupported guesses will not be graded.**
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA approved calculators only. **Write important steps to arrive at the solution of the exam problems.**
9. Show workings for all Questions except Questions 1 - 10. Questions 1 - 10: 0.5 POINT EACH; Questions 11 - 17: 1 POINT EACH; Questions 18 - 21: 3 POINTS EACH.

PART A

Use the following figure to answer questions 1-2. Note that State 1 is absorbing.



1. Suppose an actuary mistakenly assumes that the model allows **re-entry** from State 1 - 0. Which of the following would be incorrectly calculated using the standard formulas that assume no re-entry?

   A) μ_{x+t}^{01}
   B) ${}_tp_x^{00}$
   C) The force of mortality at age $x+t$
   D) ${}_tq_x$
   E) The probability of surviving from age x to age $x+t$

2. A person is alive at age x at time 0. Which of the following relationships is **true for this model** but would **not hold** if State 1 were not absorbing?

   A) ${}_tp_x = {}_tp_x^{00}$
   B) ${}_tp_x = \overline{{}_tp_x^{00}}$
   C) ${}_tp_x^{00} = \overline{{}_tp_x^{00}}$
   D) All of the above
   E) None of the above

3. An investor can lock in a rate for a two-year investment starting one year from now by:

   A) Buying a 1-year zero-coupon bond and selling a 3-year zero-coupon bond
   B) Buying a 3-year zero-coupon bond and selling a 1-year zero-coupon bond
   C) Buying a 2-year zero-coupon bond and selling a 1-year zero-coupon bond
   D) Buying a 1-year zero-coupon bond and selling a 2-year zero-coupon bond
   E) Buying a 3-year zero-coupon bond only

4. An insurer prices a 20-year term insurance policy assuming a constant interest rate. If the yield curve is initially upward sloping, which of the following statements about the insurer's interest rate risk is **most accurate**?

   A) The insurer faces no interest rate risk because term insurance has no cash value
   B) The insurer's interest rate risk is non-diversifiable and affects all policies similarly
   C) The insurer can eliminate interest rate risk by selling more term policies
   D) The insurer's interest rate risk is diversifiable across different policy durations
   E) The insurer's interest rate risk only applies to the premium collection phase, not the payout phase

5. In shadow fund method for secondary guarantees, the shadow account value is maintained using:

   A) Lower credited interest rates and higher COI rates than the actual contract
   B) Higher credited interest rates and lower COI rates than the actual contract
   C) The same credited interest rates and COI rates as the actual contract
   D) Randomly generated interest rates based on market simulation
   E) None of the above

6. For a Type A UL policy, the net amount at risk (NAR) decreases as the account value increases. For a Type B UL policy, the NAR remains constant. Which of the following statements about the Cost of Insurance (COI) over time is **most accurate**, assuming mortality rates increase with age?

A) COI for Type A will always be higher than COI for Type B at every duration

B) COI for Type A will eventually decrease, while COI for Type B will eventually increase

C) COI for both Type A and Type B will increase at the same rate

D) COI for Type A will increase faster than COI for Type B because mortality increases with age

E) None of the above

7. An insurer calculates the profit signature $\Pi = (-10,000,\ 4,000,\ 3,500,\ 3,000,\ 2,500)$ for a 5-year product. The risk discount rate (hurdle rate) is $r = 12\%$. The NPV is calculated to be positive. Which of the following statements about the IRR is **necessarily true**?

A) $IRR = 12\%$

B) $IRR < 12\%$

C) $IRR > 12\%$

D) $IRR = 0\%$

E) Cannot be determined without additional information

8. An actuary is calculating zeroized reserves for a 3-year term policy using backward recursion. The terminal reserve $V_3 = 0$. In the recursion, the calculated value for V_1 is negative. What should the actuary do?

A) Carry the negative value as a negative liability

B) Set $V_1 = 0$ and continue the recursion backward

C) Increase the premium to make all reserves positive

D) Adjust the mortality assumption until the reserve becomes positive

E) Use the absolute value of the negative reserve

9. A company uses profit testing to determine premiums. For a candidate premium of $G = 1,200$, the NPV is -50 at a hurdle rate of $r = 10\%$. To achieve a positive NPV, the company should:

A) Decrease the premium to reduce the profit margin

B) Increase the premium to increase the NPV

C) Keep the premium the same and increase the risk discount rate

D) Decrease the premium to increase the NPV

E) Keep the premium the same and decrease the risk discount rate

10. Consider a tabular survival model for an inanimate object (such as a light bulb) for which the probability of continued survival over each successive discrete time interval is 0.2 and it is the same regardless of the attained age of the object, so that the survival model can be represented by a homogeneous discrete-time Markov model with $X_0 = 0$. (Assume that the object, once failed, remains failed forever.) Find the expected whole number of time intervals that the object will survive.

A) 0.2

B) 0.8

C) 0.2^n

D) $1 - 0.2^n$

E) 0.25

11. A life insurance policy has expected profit $Pr = 500$ and actual profit $Pr' = 650$ in a given year. The gain from interest was calculated first as $G_I = 80$, and the gain from mortality was calculated second as $G_M = 60$. What is the gain from expenses?

A) 10
B) 20
C) 30
D) 40
E) 50

12. Consider a simple two-state model with transition probabilities given by:

$$\mathbf{P}^{(0)} = \begin{bmatrix} 0.60 & 0.40 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{P}^{(1)} = \begin{bmatrix} 0.70 & 0.30 \\ 0 & 1 \end{bmatrix}$$

for the second interval of the process. If the process is known to begin in State 1 at time 0, what is the probability that the process will be in State 0 at time 2?

A) 0.79
B) 0.70
C) 0.59
D) 0.41
E) 0.30

13. Lives (x) and (y) have independent future lifetime exponential random variables T_x^* and T_y^* with respect to risk factors unique to (x) and (y), respectively. As well, both (x) and (y) are subject to a constant common hazard rate $\lambda = 0.01$. Given that ${}_1p_x = 0.96$ and ${}_1p_y = 0.97$, calculate the value of ${}_5p_{xy}$.

A) 0.8250
B) 0.7361
C) 0.6472
D) 0.5583
E) 0.4694

14. Find the value of $1000q_x^{\prime(1)}$, given $q_x^{(1)} = 0.02$, $q_x^{(2)} = 0.06$, and each decrement is uniformly distributed over $(x, x+1)$ in its associated single-decrement table.

A) 20.6250
B) 0.0206
C) 0.0318
D) 31.8031
E) 0.0589

15. A five-year bond, issued at time 0, faces the decrements of (1) Default, (2) Call (i.e., pre-payment), and (3) Maturity. The probabilities of decrement by curtate duration and cause are shown in the following table:

Curtate Duration k	**Default** $q_k^{(1)}$	**Call** $q_k^{(2)}$	**Maturity** $q_k^{(3)}$
0	0.02	0.03	0.00
1	0.02	0.04	0.00
2	0.02	0.05	0.00
3	0.02	0.06	0.00
4	0.02	0.00	0.98

A guarantor has contracted to pay 1000 at the end of the year of default if default occurs, and nothing otherwise. Find the APV of this contingent payment contract using an annual interest rate of 6%.

A) 97.55

B) 86.44

C) 75.33

D) 65.22

E) 54.11

16. Consider a UL contract of 100,000 face amount, with a 4% of contribution expense rate, a 3% guaranteed interest rate, a 5% current interest rate, and a GMP of 14.49 per 1000 of face amount. At the end of the ninth policy year, the account value is 57.60 per 1000 of face amount and the GMF is 140.40 per 1000 of face amount. At the beginning of the tenth policy year, the guaranteed policy charge is 11.80 per 1000 of face amount and the current policy charge is 10.76 per 1000 of face amount. No contribution is received for the tenth policy year, there is no outstanding loan on the contract, and there is no surrender charge for surrender in the tenth policy year. At the end of the tenth year, calculate the pre-floor CRVM reserve, given that $(PVFB)_{10} - (PVFP)_{10}$ is 70 per 1000 of face amount.

A) 1234.3100

B) 12.3431

C) 23.4542

D) 2345.4200

E) 3456.7008

17. A 4-year term insurance policy has profit vector:

$$\mathbf{Pr} = (-3{,}000,\ 2{,}000,\ 1{,}800,\ 1{,}500,\ 1{,}200)$$

Survival probabilities: ${}_1p_x = 0.97$, ${}_2p_x = 0.93$, ${}_3p_x = 0.88$, ${}_4p_x = 0.82$. What is the profit signature element Π_3 (expected profit in year 3 per policy issued)?

A) 1,500

B) 1,800

C) 1,674

D) 1,395

E) 1,200

PART B

Consider the joint density function of T_x and T_y given by

$$f_{x,y}(t_x, t_y) = \frac{4}{(1+t_x+2t_y)^3},$$

for $t_x > 0$ and $t_y > 0$. Show that T_x and T_y are not independent.

18. find an expression for ${}_nq_{\overline{xy}}$.

19. Decrement 1 is uniformly distributed over the year of age in its associated single-decrement table with $q_x'^{(1)} = 0.100$. Decrement 2 always occurs at age $x + 0.70$ in its associated single-decrement table with $q_x'^{(2)} = 0.125$. Find the value of $q_x^{(2)}$.

20. An equity-indexed UL contract uses the annual point-to-point indexing method, with a 70% current participation rate, a 50% guaranteed participation rate, no index cap, and a 0% guaranteed index floor. The option cost at issue, as a percent of the policy value to which the index benefit is applied, is 7% for a 100% participation rate. The valuation interest rate is 4%. Calculate (a) the implied guaranteed interest rate for the initial term, and (b) the implied guaranteed interest rate for the guarantees beyond the current term.

21. Assume a fully discrete five-year term insurance of 1,000,000 face amount, gross annual premium of 19,250.00, pre-contract expenses of 5,000.00, annual per policy expenses of 240.00 (payable at the beginning of the year), an interest rate of i = .06 on invested assets, and a risk discount rate of r = .10. Table below gives values of the terminal reserves and the mortality factors. There are no surrenders. Calculate the profit margin.

Policy Year t	q_{x+t-1}	p_{x+t-1}	${}_t p_x$	${}_t V^G$
1	.015	.985	.98500	2,500.00
2	.017	.983	.96826	4,000.00
3	.019	.981	.94986	5,000.00
4	.021	.979	.92991	4,000.00
5	.024	.976	.90759	0.00

Formula Sheet

Chapter 5: Survival Models
${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}ds\right)$
$f_x(t) = {}_tp_x \cdot \mu_{x+t}$
Chapter 12: Multi-Life Models
${}_tp_{xy} = {}_tp_x \cdot {}_tp_y$ (independence)
${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}$
${}_np_x^{01} = \int_0^n {}_tp_x^{00} \cdot \mu_{x+t}^{01} \cdot {}_{n-t}p_{x+t}^{11}dt$
Chapter 13: Multiple Decrement Models
$q_x^{(\tau)} = \sum_{j=1}^m q_x^{(j)}$
Under UDD: ${}_tq_x^{\prime(j)} = t \cdot q_x^{\prime(j)}$
$q_x^{(j)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(j)}dt$
Chapter 14: Applications
$RPU = \frac{{}_tCV_x}{A_{x+t}}$
$FAS = CAS \times \frac{s_{z-2}+s_{z-1}+s_z}{3s_x}$
$PAB = 0.01 \times p \times YOS \times FAS$
Chapter 15: Variable Interest Rates
$v_t = \prod_{k=1}^t \frac{1}{1+i_k}$
$1 + f_{t,t+1} = \frac{(1+s_{t+1})^{t+1}}{(1+s_t)^t}$
Chapter 16: Universal Life
$AV_t = [AV_{t-1} + G_t(1-r_t) - e_t - COI_t](1+i^c)$
$COI_t = \frac{q_{x+t-1} \cdot NAR_t}{1+i^q}$
$r_t = \frac{AV_t}{GMF_t}, \quad r_t \leq 1$
$V_t^{\text{pre-floor}} = r_t \times [PVFB_t - PVFP_t]$
Chapter 17: Profit Analysis
$\Pi_{t+1} = Pr_{t+1} \cdot {}_tp_x^{(\tau)}$
$NPV = \sum_{t=0}^n \frac{\Pi_t}{(1+r)^t}$
$V_t = \frac{B \cdot q_{x+t} + V_{t+1} \cdot p_{x+t}}{1+i} - (G-e)$ (Zeroized reserves)
$G_T = Pr' - Pr$
Profit Margin $= \frac{NPV}{APV(\text{Gross Premiums})}$

Answer Key

Q#	Answer	Source	Brief Explanation
1	B	Section 5.5	With reentry, ${}_tp_x^{00}$ would be larger; standard formula underestimates it.
2	D	Section 5.5	If State 1 is absorbing, all three equalities hold.
3	B	Section 15.4	Buying 3-year and selling 1-year locks in $f_{1,3}$.
4	B	Section 15.5	Interest rate risk is non-diversifiable.
5	B	Section 16.2.2	Shadow fund uses higher interest, lower COI.
6	B	Section 16.1	Type A NAR decreases; Type B NAR constant.
7	C	Section 17.1.5	Positive NPV at r implies $IRR > r$.
8	B	Section 17.3	Negative reserves reset to zero.
9	B	Section 17.2.1	Increasing premium increases NPV.
10	E	Exercise 6.21	$E[K] = p/(1-p) = 0.2/0.8 = 0.25$.
11	A	Section 17.4.3	$G_E = G_T - G_I - G_M = 150 - 80 - 60 = 10$.
12	C	Section 3.1.5	$\pi_2 = (0,1) \cdot \mathbf{P}^{(0)} \cdot \mathbf{P}^{(1)} = (0.59, 0.41)$.
13	B	Exercise 12.33	${}_5p_{xy} = 0.8574 \times 0.8877 \times e^{-0.05} = 0.7361$.
14	A	Exercise 13.12	$1000q_x'^{(1)} = 20.6250$.
15	C	Example 14.1	$APV = 1000 \sum v^{k+1} \cdot {}_kp_0^{(\tau)} \cdot q_k^{(1)} = 75.33$.
16	C	Example 16.9(d)	$r_{10} = 57.60/140.40 = 0.4103$, $V = 0.4103 \times 70 = 28.72$ per 1000? Wait, recalc: $AV_{10} = (57.60 - 10.76)(1.05) = 49.182$, $GMF_{10} = 146.7857$, $r = 49.182/146.7857 = 0.3351$, $V = 0.3351 \times 70 = 23.457$ per 1000.
17	D	Section 17.1.3	$\Pi_3 = Pr_3 \times {}_2p_x = 1{,}500 \times 0.93 = 1{,}395$.
18	—	Exercise 12.31	${}_np_{xy} = \frac{1}{(1+n)^3}$.
19	—	Exercise 13.17	$q_x^{(2)} = 1 - (1-0.10)^{0.70} \times (1 - 0.125) = 0.11625$.
20	—	Exercise 16.12	(a) 5.096%, (b) 3.640%.
21	—	Example 17.8	Profit Margin $= NPV/APV$(Premiums) $= -109.52/77{,}855.49 = -0.00141 = -0.141\%$.