King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 201

Major Exam II

252

14 April 2026

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If the parametric equations of the line passing through the point $(2,3,4)$ and perpendicular to the plane given by $3x+2y-z=6$ are

$$x = 2 + at, \quad y = 3 + bt, \quad z = 4 - t,$$

then $a+b=$

(a) 5 ________________ (correct)

(b) 3

(c) 1

(d) -5

(e) -3

2. The graph of the equation $x^2 - 2y^2 + z^2 - 4x + 4y - 6z = k$, represents

(a) an elliptic cone if $k = -11$. ________________ (correct)

(b) a hyperboloid with two sheets if $k = 0$.

(c) a hyperboloid with one sheet if $k = -20$.

(d) an elliptic cone if $k = 20$.

(e) an ellipsoid if $k = 1$.

3. If z is defined **implicitly** as a differentiable function of x and y by the equation

$$3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0,$$

then $\dfrac{\partial z}{\partial x}$ is equal to

(a) $\dfrac{2x(y^2 - 3z)}{3(x^2 + 2z^2 + y)}$ ________________ (correct)

(b) $\dfrac{2xy^2}{3(x^2 + 2z^2 + y)}$

(c) $\dfrac{-2x(y^2 - 3z)}{3(x^2 + 2z^2 + y)}$

(d) $\dfrac{2(xy^2 - 3xz - 3z^2)}{3(x^2 + y)}$

(e) $\dfrac{2x^2y - 3z}{3(x^2 + 2z^2 + y)}$

4. The **range** of the function $f(x, y) = \ln(25 - x^2 - y^2)$ is

(a) $(-\infty, 2\ln 5]$ ________________ (correct)

(b) $(-\infty, 5\ln 2]$

(c) $[2\ln 5, \infty)$

(d) $(0, 5\ln 5]$

(e) $(-\infty, \infty)$

5. If the symmetric equations of the **normal** line to the surface $z = ye^{2xy}$ at the point $(0, 2, 2)$ are given by

$$\frac{x}{8} = \frac{y-2}{b} = \frac{z-2}{c},$$

then $b + c =$

(a) 0 ________________ (correct)
(b) 1
(c) 2
(d) -1
(e) -2

6. Consider the function $f(x, y) = (x^2 + y^2)^{\frac{2}{3}}$, then

(a) $f_x(0, 0) = 0$. ________________ (correct)
(b) $f_x(0, 0)$ and $f_y(0, 0)$ do not exist.
(c) $f_x(2, 2) = \frac{3}{4}$.
(d) $f_y(1, 1) = \frac{4}{3}$.
(e) $f_x(1, 1) \neq f_x(1, -1)$.

7. The total resistance R (in ohms) of two resistors connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

By using the **differentials** dR, the change in R as R_1 is increased from 10 ohms to 10.5 ohms and R_2 is decreased from 15 ohms to 13 ohms, is approximately equal to

(a) -0.14 ______________________________ (correct)

(b) 0.14

(c) -0.39

(d) 0.39

(e) -1.40

8. $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x}-\sqrt{y}} =$

see the last

(a) does not exist ______________________________ (correct)

(b) 0

(c) 1

(d) 2

(e) $\sqrt{2}$

9. Let $f(x,y,z) = ye^x + x\ln z$, then $f_{xzz}(1,1,1) + f_{zxz}(1,1,1) + f_{zzx}(1,1,1) =$

   (a) -3 ______ (correct)

   (b) -1

   (c) 1

   (d) -27

   (e) -8

10. The **maximum value** of the directional derivative of $f(x,y) = y^2 - x\sqrt{y}$ at the point $(0,3)$ is equal to

   (a) $\sqrt{39}$ ______ (correct)

   (b) $\sqrt{33}$

   (c) 33

   (d) $\sqrt{45}$

   (e) 39

11. If the equation of the tangent plane to the surface $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$ is given by $x + ay + bz + c = 0$, then $a + b + c =$

(a) 3 ____ (correct)

(b) 4

(c) -2

(d) 2

(e) 5

12. The distance between the point $(2, 8, 4)$ and the plane $2x + y + z = 5$, is equal to

(a) $\dfrac{11}{\sqrt{6}}$ ____ (correct)

(b) $\dfrac{16}{\sqrt{6}}$

(c) $\dfrac{21}{\sqrt{6}}$

(d) $\dfrac{11}{2}$

(e) $\dfrac{11}{2\sqrt{21}}$

13. The directional derivative of $f(x,y) = x^2 \sin 2y$ at the point $\left(1, \frac{\pi}{2}\right)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$, is equal to

(a) $\frac{8}{5}$ ________________ (correct)

(b) $\frac{6}{5}$

(c) $-\frac{8}{5}$

(d) 8

(e) -8

14. If $w = xy + yz + xz$, where $x = s\cos t$, $y = s\sin t$, and $z = t$, then $\frac{\partial w}{\partial s}$ when $s = 1$ and $t = 2\pi$ is equal to

(a) 2π ________________ (correct)

(b) $1 + 2\pi$

(c) 0

(d) 4π

(e) 2

15. If θ is the angle of inclination of the tangent plane to the surface $3x^2 + 2y^2 - z = 4$ at the point $(1, 1, 1)$, then $\cos\theta =$

(a) $\dfrac{1}{\sqrt{53}}$ ____(correct)

(b) $\dfrac{1}{2\sqrt{13}}$

(c) $\dfrac{6}{\sqrt{53}}$

(d) $\dfrac{1}{\sqrt{11}}$

(e) $\dfrac{4}{\sqrt{53}}$

8. $\lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x}-\sqrt{y}} =$

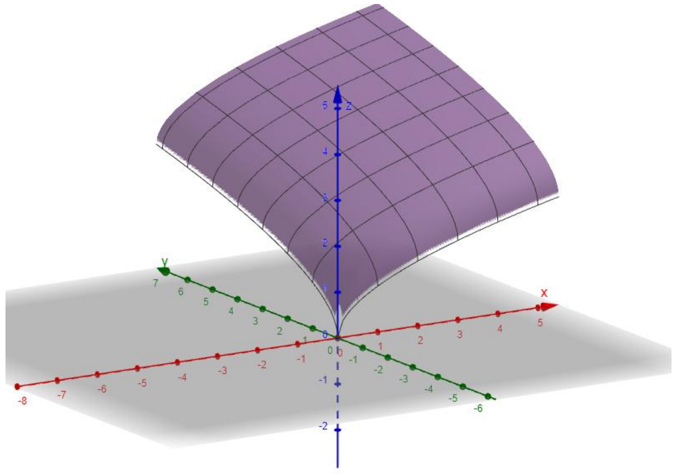
(a) does not exist _(correct)

(b) 0

(c) 1

(d) 2

(e) $\sqrt{2}$



The domain of the function $f(x,y)=\frac{x-y}{\sqrt{x}-\sqrt{y}}$ is $D=\{(x,y): x>0 \text{ and } y>0\}$. No matter which curve is selected in the first quadrant, the limit approaches zero.

According to the definition of the limit stated in our textbook (page 885, Section 13.2), the *Limit of a Function of Two Variables* is defined as follows:

Definition of the Limit of a Function of Two Variables

Let f be a function of two variables defined, except possibly at (x_0, y_0), on an open disk centered at (x_0, y_0), and let L be a real number. Then

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

if for each $\varepsilon > 0$ there corresponds a $\delta > 0$ such that

$$|f(x,y) - L| < \varepsilon \quad \text{whenever} \quad 0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta.$$

This definition clearly indicates that for a limit to exist, there must be an open disk containing the point (x_0, y_0) in which the function f is defined on this desk (except possibly at (x_0, y_0)). If such a disk cannot be found, then the limit does not exist.

The *Limit of a Function of single Variable* is defined as follows:

Definition of Limit

Let f be a function defined on an open interval containing c (except possibly at c), and let L be a real number. The statement

$$\lim_{x\to c} f(x) = L$$

means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$$0 < |x - c| < \delta$$

then

$$|f(x) - L| < \varepsilon.$$

This definition indicates that for a limit to exist, there must be an open interval containing the point c in which the function f is defined on this open interval (except possibly at c). If such an interval cannot be found, then the limit does not exist. For example $\lim_{x\to 0} \sqrt{x}$ DNE.