1. The **slope** of the tangent line to the parametric curve

~#18/10.3

$$x = t - \sin t,\ y = 1 - \cos t$$

at the point corresponding to $t = \dfrac{\pi}{2}$ is equal to

(a) 1 ______________________________ (correct)

(b) $\dfrac{1}{2}$

(c) 0

(d) -1

(e) 2

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{0 + \sin t}{1 - \cos t} = \frac{\sin t}{1 - \cos t}$$

$$\text{Slope} = \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \frac{1}{1-0} = \frac{1}{1} = \underline{1}$$

#25/10.2

2. The parametric curve

$$x = 4 + 2\cos t,\ y = -1 + \sin t$$

represents

(a) an ellipse ______________________________ (correct)

(b) a circle with center $(4, -1)$

(c) a line segment

(d) a parabola

(e) a hyperbola

As $\cos^2 t + \sin^2 t = 1$, then

$$\left(\frac{x-4}{2}\right)^2 + (y+1)^2 = 1$$

$$\frac{(x-4)^2}{4} + (y+1)^2 = 1, \text{ an } \underline{\text{ellipse}}$$

$\sim$#15
11.4

3. A vector that is **orthogonal** to both vectors

$\vec{u} = \langle 2, -1, 3\rangle$ and $\vec{v} = \langle 4, 5, 6\rangle$

is

$$\vec{u}\times\vec{v} = \begin{vmatrix} \overset{\oplus}{i} & \overset{\ominus}{j} & \overset{\oplus}{k} \\ 2 & -1 & 3 \\ 4 & 5 & 6 \end{vmatrix} = i\,(-6-15) - j\,(12-12) + k\,(10+4)$$
$$= -21\,i + 14\,k$$

(a) $\langle 3, 0, -2\rangle$ ——— (correct)

(b) $\langle -3, 1, 2\rangle$

(c) $\langle -21, 24, 14\rangle$

(d) $\langle 21, 0, 8\rangle$

(e) $\langle -7, 0, 2\rangle$

$$= \langle -21, 0, 14\rangle$$
$$= 7\langle -3, 0, 2\rangle$$
$$= -7\langle 3, 0, -2\rangle$$

Possible answers:
$\langle -21, 0, 14\rangle$ or $\langle -3, 0, 2\rangle$, $\langle 3, 0, -2\rangle$ ✓

Note: $\vec{v}$ and $k\vec{v}$ are parallel for any $k \neq 0$.

4. The **projection** of $\vec{u} = \langle 4, 2, 0\rangle$ onto $\vec{v} = \langle 2, 1, -1\rangle$ is equal to

$\sim$#42
11.3

(a) $\left\langle \frac{10}{3}, \frac{5}{3}, -\frac{5}{3} \right\rangle$ ——— (correct)

(b) $\left\langle \frac{5}{3}, \frac{5}{3}, -\frac{5}{3} \right\rangle$

(c) $\langle 2, 1, 0\rangle$

(d) $\left\langle 1, \frac{1}{2}, -\frac{1}{2} \right\rangle$

(e) $\left\langle \frac{2}{3}, -\frac{5}{3}, \frac{4}{3} \right\rangle$

$$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\|v\|^2}\right)\vec{v}$$
$$= \frac{8+2+0}{\left(\sqrt{4+1+1}\right)^2}\vec{v}$$
$$= \frac{10}{6}\vec{v}$$
$$= \frac{5}{3}\langle 2, 1, -1\rangle$$
$$= \left\langle \frac{10}{3}, \frac{5}{3}, -\frac{5}{3} \right\rangle$$

5. The **slope** of the tangent line to the polar curve $r = \sin\theta$ at the point corresponding to $\theta = \dfrac{\pi}{3}$ is equal to

~#64
10.4

$f(\theta) = \sin\theta$

(a) $-\sqrt{3}$ ______________________________ (correct)

(b) $-\dfrac{\sqrt{3}}{2}$

(c) $-2\sqrt{3}$

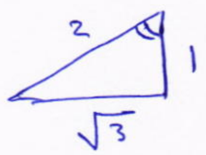
(d) $\dfrac{\sqrt{3}}{3}$

(e) $2\sqrt{3}$

$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$= \frac{\sin\theta\cdot\cos\theta + \cos\theta\cdot\sin\theta}{-\sin\theta\cdot\sin\theta + \cos\theta\cdot\cos\theta} = \frac{2\sin\theta\cos\theta}{-\sin^2\theta + \cos^2\theta}$$

$$\text{Slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}}{-\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$



$$= \frac{\frac{\sqrt{3}}{2}}{-\frac{3}{4}+\frac{1}{4}} = \frac{\frac{\sqrt{3}}{2}}{-\frac{2}{4}} = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= -\sqrt{3}$$

6. A vector **parallel** to the tangent line to the graph of $f(x) = x^2\cos x$ at the point $(\pi, -\pi^2)$ is

~#72
11.1

$f'(x) = -x^2\sin x + \cos x\cdot 2x$

Slope $= f'(\pi) = 0 + (-1)\cdot 2\pi = -2\pi$

(a) $< -1, 2\pi >$ ______________________________ (correct)

(b) $< 1, 2\pi >$

(c) $< \pi, -\pi^2 >$

(d) $< 2, 2\pi >$

(e) $< -\pi, 2\pi >$

a vector parallel to the tangent line is

$\langle 1, -2\pi\rangle$

$\overset{\text{or}}{\Longrightarrow} -\langle 1, -2\pi\rangle = \langle -1, 2\pi\rangle$

Note: $\vec{v}$ and $k\vec{v}$ are parallel vectors for any $k \neq 0$

$\sim$#85
11.2

7. If $\langle a,b,c\rangle$ is the vector of magnitude $\frac{3}{4}$ in the opposite direction of the vector $\langle 2,-2,1\rangle$, then $a+b+c=$

(a) $-\frac{1}{4}$ (correct)

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{5}{4}$

(e) $-\frac{3}{4}$

$\langle a,b,c\rangle = \frac{3}{4}\cdot\frac{-\langle 2,-2,1\rangle}{\|\langle 2,-2,1\rangle\|}$

$= \frac{3}{4}\cdot -\frac{\langle 2,-2,1\rangle}{\sqrt{4+4+1}}$

$= -\frac{3}{4}\cdot\frac{\langle 2,-2,1\rangle}{3}$

$= -\frac{1}{4}\langle 2,-2,1\rangle$

$= \langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{4}\rangle$

$a+b+c = -\frac{1}{2}+\frac{1}{2}-\frac{1}{4} = -\frac{1}{4}$

#39
11.2

8. An equation for the sphere having the points $(2,1,3)$ and $(1,3,-1)$ as endpoints of a diameter is given by

$d = \sqrt{(2-1)^2+(1-3)^2+(3+1)^2} = \sqrt{1+4+16} = \sqrt{21}$

$r = \frac{d}{2} = \frac{1}{2}\sqrt{21}$; Center $= \left(\frac{2+1}{2}, \frac{1+3}{2}, \frac{3-1}{2}\right) = \left(\frac{3}{2}, 2, 1\right)$

(a) $4x^2+4y^2+4z^2-12x-16y-8z+8=0$ (correct)

(b) $4x^2+4y^2+4z^2+12x+16y+8z-8=0$

(c) $4x^2+4y^2+4z^2-6x-8y-4z+8=0$

(d) $4x^2+4y^2+4z^2-6x+16y+8z-16=0$

(e) $4x^2+4y^2+4z^2+12x-16y-4z-8=0$

Eq: $(x-\frac{3}{2})^2+(y-2)^2+(z-1)^2 = \left(\frac{\sqrt{21}}{2}\right)^2$

$x^2-3x+\frac{9}{4}+y^2-4y+4+z^2-2z+1 = \frac{21}{4}$

$\Rightarrow 4x^2-12x+\underline{9}+4y^2-16y+\underline{\underline{16}}+4z^2-8z+\underline{\underline{4}} = 21$

$\Rightarrow 4x^2+4y^2+4z^2-12x-16y-8z+8=0$

~ #52
10.3

9. The **length** of the parametric curve

$$x = \sin^{-1} t,\ y = \ln\sqrt{1-t^2},\ 0 \le t \le \frac{1}{4}$$

$= \frac{1}{2}\ln(1-t^2)$

is equal to

$\cdot\ \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}\ ;\ \frac{dy}{dt} = \frac{1}{2}\cdot\frac{-2t}{1-t^2} = \frac{-t}{1-t^2}$

(a) $\frac{1}{2}\ln\left(\frac{5}{3}\right)$ ——————————(correct)

(b) $2\ln\left(\frac{5}{3}\right)$

(c) $\frac{1}{2}\ln 2$

(d) $\ln\left(\frac{5}{4}\right)$

(e) $\frac{1}{2}\ln\left(\frac{4}{3}\right)$

$\cdot\ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{1}{1-t^2} + \frac{t^2}{(1-t^2)^2}$

$= \frac{1-t^2+t^2}{(1-t^2)^2} = \frac{1}{(1-t^2)^2}$

$\cdot\ \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \frac{1}{1-t^2}$

$L = \int_0^{1/4}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^{1/4}\frac{1}{1-t^2}\,dt = -\int_0^{1/4}\frac{1}{t^2-1}\,dt$

$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Big]_0^{1/4} = -\frac{1}{2}\ln\left|\frac{-\frac{3}{4}}{\frac{5}{4}}\right| - 0$

$= -\frac{1}{2}\ln\left(\frac{3}{5}\right) = \frac{1}{2}\ln\left(\frac{5}{3}\right)$

#52
10.5

10. The polar equation $r = \sec\theta - 2\cos\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, can be converted to the rectangular equation

$\xRightarrow{r} r^2 = r\sec\theta - 2r\cos\theta$

$= \frac{r}{\cos\theta} - 2r\cos\theta$

(a) $y^2 = \frac{x^2(1+x)}{1-x}$ ——————————(correct)

(b) $y^2 = \frac{x^2(1-x)}{1+x}$

(c) $y^2 = \frac{x(1+x^2)}{1-x}$

(d) $y^2 = x^2$

(e) $y^2 = x^2 - x^3$

$r^2 = \frac{r^2}{r\cos\theta} - 2r\cos\theta$

$x^2+y^2 = \frac{x^2+y^2}{x} - 2x$

$\Rightarrow x^3 + xy^2 = x^2 + y^2 - 2x^2$

$\Rightarrow x^3 + xy^2 = y^2 - x^2$

$\Rightarrow x^3 + x^2 = y^2 - xy^2$

$\Rightarrow x^2(1+x) = y^2(1-x)$

$\Rightarrow y^2 = \frac{x^2(1+x)}{1-x}$

11. Which one of the following sets is a **parametrization** of the curve $y = 2x-3$, $x \geq 0$.

$\sim$ #55, 56
10.2

(a) $x = t+1$, $y = 2t-1$, $t \geq -1$ $x \geq 0$; $y = 2(x-1)-1 = 2x-3$ (correct)

(b) $x = t+1$, $y = 2t-1$, $t \in (-\infty, \infty)$ $\Rightarrow x \in (-\infty, \infty)$

(c) $x = t$, $y = 2t-3$, $t \geq 1$ $\Rightarrow x \geq 1$

(d) $x = \dfrac{t+3}{2}$, $y = t$, $t \geq 0$ $\Rightarrow x \geq \frac{3}{2}$

(e) $x = \sin t$, $y = 2\sin t - 3$, $t \in (-\infty, \infty)$ $\Rightarrow -1 \leq x \leq 1$

all give $y = 2x-3$

$\sim$ #38
10.5

12. The **area** of the region in the **common interior** of the polar curves $r = 1+\cos\theta$ and $r = 1-\cos\theta$ is equal to

$\cdot\ 1+\cos\theta = 1-\cos\theta \Rightarrow 2\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} \in QI$

(a) $\dfrac{3\pi}{2} - 4$ (correct)

(b) $3\pi - 4$

(c) $\dfrac{3\pi}{2} - 2$

(d) $\dfrac{\pi}{2} + 2$

(e) $2\pi + 4$

A = 4 · shaded area in QI

$= 4 \int_0^{\pi/2} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$

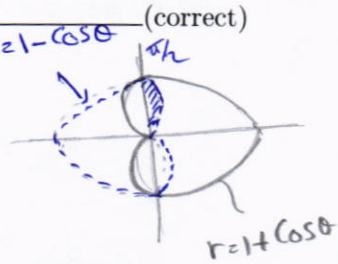
$= 2 \int_0^{\pi/2} 1 - 2\cos\theta + \frac{1+\cos(2\theta)}{2}$

$= 2 \int_0^{\pi/2} \frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos(2\theta) \, d\theta$

$= 2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin(2\theta) \right]_0^{\pi/2}$

$= 2 \left[\frac{3\pi}{4} - 2 \right]$

$= \frac{3\pi}{2} - 4$



13. If the **area of the triangle** with the vertices $A(1,2,0)$, $B(0,0,x)$, and $C(0,-1,0)$ is 3, then $2x^2 =$

$\sim \#26$

$\overline{11.4}$

$\vec{AB} = \langle -1, -2, x\rangle$, $\vec{AC} = \langle -1, -3, 0\rangle$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} i & j & k \\ -1 & -2 & x \\ -1 & -3 & 0 \end{vmatrix}$$

(a) 7 ____ (correct)

(b) 9

(c) 5

(d) 3

(e) 11

$= \langle 3x, -x, 1\rangle$

$\|\vec{AB}\times\vec{AC}\| = \sqrt{9x^2+x^2+1} = \sqrt{10x^2+1}$

area of the triangle $= \frac{1}{2}\|\vec{AB}\times\vec{AC}\|$

$3 = \frac{1}{2}\cdot\sqrt{10x^2+1}$

$6 = \sqrt{10x^2+1}$

$\Rightarrow 36 = 10x^2+1$

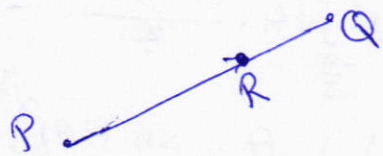
$\Rightarrow 10x^2 = 35$

$\overset{\div 5}{\Rightarrow} 2x^2 = 7$

14. If the point $R\left(\frac{12}{5}, 2, \frac{11}{5}\right)$ lies **three-fifths** of the way from the point $P(0,5,1)$ to the point $Q(a,b,c)$, then $ac+b =$

$\sim \#89$

$\overline{11.2}$



(a) 12 ____ (correct)

(b) -4

(c) 0

(d) 15

(e) $\frac{1}{25}$

$\vec{PR} = \frac{3}{5}\vec{PQ}$

$\cancel{\langle 0-\frac{12}{5}, 5-2, 1-\frac{11}{5}\rangle}$

$\langle \frac{12}{5}-0, 2-5, \frac{11}{5}-1\rangle = \frac{3}{5}\langle a-0, b-5, c-1\rangle$

$\langle \frac{12}{5}, -3, \frac{6}{5}\rangle = \langle \frac{3a}{5}, \frac{3}{5}(b-5), \frac{3}{5}(c-1)\rangle$

$\Rightarrow \frac{12}{5} = \frac{3a}{5}$, $-3 = \frac{3}{5}(b-5)$, $\frac{6}{5} = \frac{3}{5}(c-1)$

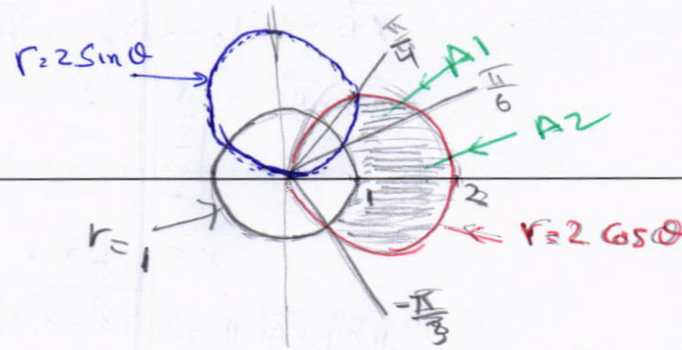
$\Rightarrow 12 = 3a$, $-5 = b-5$, $2 = c-1$

$\Rightarrow a = 4$, $b = 0$, $c = 3$

$\Rightarrow ac+b = (4)(3)+0 = 12$

15. The **area** of the region inside the circle $r = 2\cos\theta$ and outside both circles $r = 2\sin\theta$ and $r = 1$ is equal to

~ #50
10.5



(a) $\dfrac{\pi}{4} + 1$ ____________________ (correct)

(b) $\pi + 4$

(c) $\dfrac{\pi}{2} + 1$

(d) $2\pi - \dfrac{1}{2}$

(e) $\dfrac{3\pi}{2} - 1$

- $2\cos\theta = 2\sin\theta \Rightarrow \cos\theta = \sin\theta \Rightarrow \frac{\pi}{4} \in QI$
- $2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{3} \in QIV$
- $2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \in QI$

$$A_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{2}\left[(2\cos\theta)^2 - (2\sin\theta)^2\right] d\theta$$

$$= \int_{\frac{\pi}{6}}^{\pi/4} 2\cos(2\theta)\, d\theta = \sin(2\theta)\Big]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \sin\frac{\pi}{2} - \sin\frac{\pi}{3} = 1 - \frac{\sqrt{3}}{2}$$

$$A_2 = \int_{-\frac{\pi}{3}}^{\pi/6} \frac{1}{2}\left[(2\cos\theta)^2 - 1^2\right] d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{3}}^{\pi/6} \left[4 \cdot \frac{1+\cos(2\theta)}{2} - 1\right] d\theta = \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} 1 + 2\cos(2\theta)\; d\theta$$

$$= \frac{1}{2}\left[\theta + \sin(2\theta)\right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}} = \frac{1}{2}\left[\left(\frac{\pi}{6} + \frac{\pi}{3}\right) + \left(\sin\frac{\pi}{3} - \sin\left(-\frac{2\pi}{3}\right)\right)\right]$$

$$= \frac{1}{2}\left[\frac{3\pi}{6} + \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{4} + \frac{\sqrt{3}}{2}$$

$$A = A_1 + A_2 = 1 - \frac{\sqrt{3}}{2} + \frac{\pi}{4} + \frac{\sqrt{3}}{2}$$

$$= 1 + \frac{\pi}{4}$$