1. If $y = Ax + B$ is the **level curve** of the function $f(x,y) = \ln(x-y)$ that passes through $(e,0)$, then $AB =$

#58
13.1

(a) $-e$ ______________________________ (correct)

(b) $3e$

(c) $-3e$

(d) e

(e) 0

$f(x,y) = C$

$\ln(x-y) = C$, Find C?

$\ln(e-0) = C$

$1 = C$

$\Rightarrow \ln(x-y) = 1 \Rightarrow x-y = e$

$\Rightarrow y = x-e$

$\Rightarrow A = 1, B = -e$

$\Rightarrow AB = -e$

2. Which one of the following statements is **FALSE** about the graph of the equation $4x^2 - 4y + z^2 = 0$?

#9
11.6

(a) Its trace in the plane $y = -1$ is an ellipse ______________________________ (correct)

$4x^2 - 4(-1) + z^2 = 0 \Rightarrow 4x^2 + z^2 = -4$ not an ellipse

(b) Its trace in the plane $y = 1$ is an ellipse

(c) Its trace in the xy-plane is a parabola

(d) Its trace in the xz-plane is the origin

(e) It is an elliptic paraboloid

#33
11.5

3. If (a, b, c) is the **point of intersection** between the lines

$$L_1 : x = 2 + 4t,\ y = 3,\ z = 1 - t$$

$$L_2 : x = 2 + 2s,\ y = 3 + 2s,\ z = 1 + s$$

then $abc =$

$\cdot\ x = x,\ y = y,\ z = z \Rightarrow 2+4t = 2+2s',\ 3 = 3+2s' \Rightarrow s' = 0,\ 1-t = 1+s \Rightarrow t = 0 \Rightarrow s = 0,\ t = 0$

(a) 6 ______ (correct)

(b) -4

(c) 0

(d) 8

(e) -10

$L_1 \xRightarrow{t=0} (x, y, z) = (2, 3, 1)$

$\Rightarrow a = 2,\ b = 3,\ c = 1$

$\Rightarrow abc = 6.$

~#91
11.5

4. The **distance** between the parallel planes

$$x - 3y + 2z = 1 \leftarrow \text{point: } (1, 0, 0)$$
$$2x - 6y + 4z = 3 \longrightarrow 2x - 6y + 4z = 3$$

is equal to

$$d = \frac{|2(1) - 6(0) + 4(0) - 3|}{\sqrt{4 + 36 + 16}} = \frac{1}{\sqrt{56}} = \frac{1}{2\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{\sqrt{14}}{28}$$

(a) $\frac{\sqrt{14}}{28}$ ______ (correct)

(b) $\frac{\sqrt{14}}{14}$

(c) $\frac{\sqrt{14}}{7}$

(d) $\frac{\sqrt{14}}{4}$

(e) $\frac{\sqrt{14}}{2}$

5. The set of **all** points where $f(x,y,z) = \dfrac{\sin z}{e^x - e^y}$ is **continuous** is given by

~ #63
13.2

(a) $\{(x,y,z) : x \neq y,\ z \in (-\infty, \infty)\}$ ________________(correct)

(b) $\{(x,y,z) : x, y, z \in (-\infty, \infty)\}$

(c) $\{(x,y,z) : x, y \in (-\infty, \infty),\ z \in [-1, 1)\}$

(d) $\{(x,y,z) : x = y\}$

(e) $\{(x,y,z) : x \neq y,\ z \in [-1, 1]\}$

- $e^x - e^y \neq 0 \Rightarrow e^x \neq e^y$
  $\Rightarrow x \neq y$
- $z \in (-\infty, \infty)$.

6. If the equation $z^3 - 2xy + 2yz + y^3 - 2 = 0$ defines z implicitly as a differentiable function of x and y, then the value of $\dfrac{\partial z}{\partial y}$ when $(x,y,z) = (1,1,1)$ is equal to

~ #29
13.5

(a) $-\dfrac{3}{5}$ ________________(correct)

(b) 0

(c) $\dfrac{2}{5}$

(d) $\dfrac{4}{5}$

(e) $-\dfrac{2}{5}$

$F(x,y,z) = z^3 - 2xy + 2yz + y^3 - 2$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{0 - 2x + 2z + 3y^2 - 0}{3z^2 - 0 + 2y + 0 - 0}$$

$$= -\frac{-2x + 2z + 3y^2}{3z^2 + 2y}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(1,1,1)} = -\frac{-2+2+3}{3+2}$$

$$= -\frac{3}{5}$$

7. If $f(x,y,z) = y^2 e^{xz^2}$, then $\dfrac{\partial^3 f}{\partial z \partial y \partial x} = f_{xyz}$

~ # 93
13.3

(a) $4yz(xz^2+1)\,e^{xz^2}$ ________________________________ (correct)

(b) $4xy(z^3+1)\,e^{xz^2}$

(c) $xyz(z^2+1)\,e^{xz^2}$

(d) $2xz(z^4+1)\,e^{xz^2}$

(e) $2yz(xz^2+1)\,e^{xz^2}$

$f_x = y^2 e^{xz^2} \cdot z^2$

$f_{xy} = 2y \cdot e^{xz^2} \cdot z^2$

$f_{xyz} = 2y\left[e^{xz^2} \cdot 2z + z^2 \cdot e^{xz^2} \cdot 2xz\right]$

$= 4yz\left[e^{xz^2} + xz^2 e^{xz^2}\right]$

$= 4yz(1 + xz^2)\,e^{xz^2}$

~ #109
11.5

8. If (a,b,c) is the **point of intersection** of the plane $x + 2y - z = -9$ and the line through $(2,-1,3)$ that is perpendicular to this plane, then $abc =$

• L: $(2,-1,3)$, $\vec{v} = \langle 1,2,-1\rangle = \vec{n}$

$x = 2+t$, $y = -1+2t$, $z = 3-t$, $t \in \mathbb{R}$

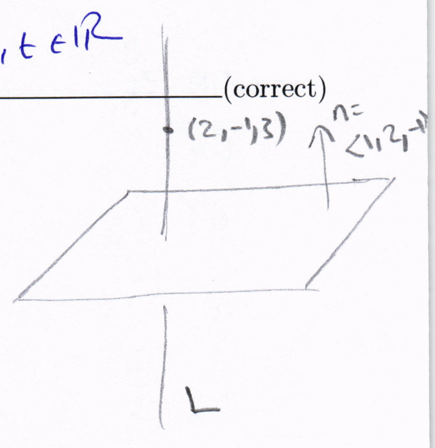
(a) -12 ________________________________ (correct)

(b) -6

(c) 6

(d) -9

(e) 0



• $x + 2y - z = -9$

$(2+t) + 2(-1+2t) - (3-t) = -9$

$2 + t - 2 + 4t - 3 + t = -9$

$6t = -6$

$\Rightarrow t = -1$

$\overset{L}{\Longrightarrow} (x,y,z) = (1,-3,4)$

$\Rightarrow a = 1,\ b = -3,\ c = 4$

$\Rightarrow abc = -12$

9. Let $z = xy + \dfrac{y}{x}$. Using **differentials**, the change in z when (x, y), moves from the point $(2, 1)$ to the point $(2.1,\ 1.05)$ is approximately equal to

~#12
13.4

$f(x,y) = xy + \frac{y}{x}$; $x = 2$, $y = 1$

$dx = \Delta x = 2.1 - 2 = 0.1$

$dy \approx \Delta y = 1.05 - 1 \approx 0.05$

(a) $\dfrac{1}{5}$ ______ (correct)

(b) $\dfrac{3}{40}$

(c) $\dfrac{2}{5}$

(d) $\dfrac{1}{40}$

(e) $\dfrac{3}{5}$

$\Delta z \approx dz$; $dz = f_x\,dx + f_y\,dy$

$$= \left(y - \frac{y}{x^2}\right)dx + \left(x + \frac{1}{x}\right)dy$$

$$= \left(1 - \frac{1}{4}\right)\cdot\frac{1}{10} + \left(2 + \frac{1}{2}\right)\cdot\frac{5}{100}$$

$$= \frac{3}{4}\cdot\frac{1}{10} + \frac{5}{2}\cdot\frac{1}{20}$$

$$= \frac{3}{40} + \frac{5}{40}$$

$$= \frac{8}{40}$$

$$= \frac{1}{5}$$

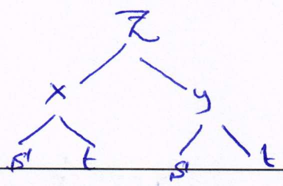
~#18/13.5
#86/13.3

$s = 1$, $t = e$

10. Let $z = \tan^{-1}\left(\dfrac{x}{y}\right)$, $x = te^s$, $y = s\ln t$. The value of $\dfrac{\partial z}{\partial t}$ when $(s, t) = (1, e)$ is equal to

$\Rightarrow$ $x = e\cdot e^1 = e^2$

$y = 1\cdot \ln e = 1$



(a) 0 ______ (correct)

(b) $\dfrac{2e}{e^4+1}$

(c) $\dfrac{e-1}{e^4+1}$

(d) $\dfrac{-e}{e^4+1}$

(e) $\dfrac{e+1}{e^4+1}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{1}{y}\cdot e^{s} + \frac{1}{1+\left(\frac{x}{y}\right)^2}\cdot\frac{-x}{y^2}\cdot\frac{s}{t}$$

$$\left.\frac{\partial z}{\partial t}\right|_{\substack{s=1\\t=e}} = \frac{1}{1+e^4}\cdot\frac{1}{1}\cdot e^1 + \frac{1}{1+e^4}\cdot\frac{-e^2}{1}\cdot\frac{1}{e}$$

$$= \frac{e}{1+e^4} - \frac{e}{1+e^4}$$

$$= 0$$

11. The **range** of the function $f(x, y) = 2e^{xy}$ is

$\sim$#22
13.1

Domain = $\{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$

(a) $(0, \infty)$ ____ (correct)

(b) $[2, \infty)$

(c) $(-\infty, \infty)$

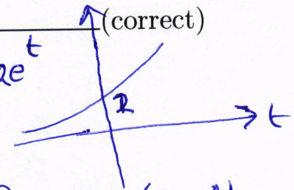
(d) $[1, \infty)$

(e) $(3, \infty)$

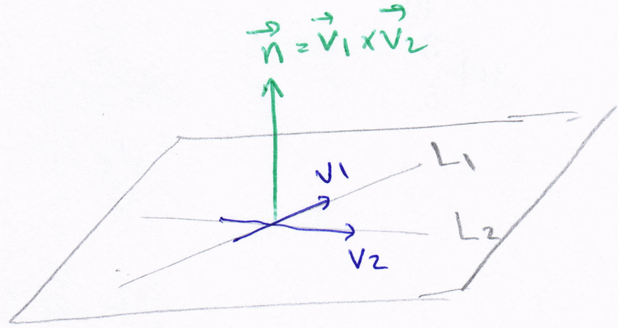
$xy \in \mathbb{R}$

Range : $(0, \infty)$

$f(t) = 2e^t$



Range = $(0, \infty)$



$\vec{v}_1 = \langle -2, 1, 1 \rangle$

$\vec{v}_2 = \langle -3, 4, -1 \rangle$

12. If $x + by + cz = d$ is an **equation of the plane** that contains the lines

#51
11.5

$$\frac{x-1}{-2} = y - 4 = z \text{ and } \frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1},$$

then $b + c + d =$

(a) 7 ____ (correct)

(b) 5

(c) 0

(d) -3

(e) -4

- $\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = \langle -5, -5, -5 \rangle$

- a point in the plane : $(1, 4, 0)$, a point on L_1

Eq : $-5(x-1) - 5(y-4) - 5(z-0) = 0$

$\xRightarrow{\div(-5)}$ $(x-1) + (y-4) + (z-0) = 0$

$x + y + z = 5$

$\Rightarrow b = 1, \; c = 1, \; d = 5$

$\Rightarrow b + c + d = 7$

13. If $f(x,y) = \ln\left(\frac{x+y}{x-y}\right)$, then $xf_y(x,y) + yf_x(x,y) =$

#24
13.3

$= \ln(x+y) - \ln(x-y)$

(a) 2 ——— (correct)

(b) -2

(c) 0

(d) 1

(e) $-\frac{1}{2}$

- $f_x(x,y) = \frac{1}{x+y} - \frac{1}{x-y} = \frac{(x-y)-(x+y)}{(x+y)(x-y)} = \frac{-2y}{x^2-y^2}$
- $f_y(x,y) = \frac{1}{x+y} - \frac{-1}{x-y} = \frac{(x-y)+(x+y)}{(x+y)(x-y)} = \frac{2x}{x^2-y^2}$
- $x f_y(x,y) + y f_x(x,y) = x \cdot \frac{2x}{x^2-y^2} + y \cdot \frac{-2y}{x^2-y^2}$

$= \frac{2x^2}{x^2-y^2} - \frac{2y^2}{x^2-y^2}$

$= \frac{2x^2-2y^2}{x^2-y^2} = \frac{2(x^2-y^2)}{x^2-y^2}$

$= 2$

~ #37→60
13.2

14. $\lim_{(x,y)\to(0,0)} \left[\frac{1}{\ln(1+x^2+y^2)} - \frac{1}{x^2+y^2}\right] =$ $\quad \infty - \infty$

Method I: Polar Coord: $r^2 = x^2+y^2$: $\lim_{r\to 0^+} \left[\frac{1}{\ln(1+r^2)} - \frac{1}{r^2}\right]$ ....

(a) $\frac{1}{2}$ ——— (correct)

(b) 0

(c) 1

(d) 2

(e) The limit does not exist

Method II: Let $w = x^2+y^2$.

$\lim_{w\to 0^+} \left[\frac{1}{\ln(1+w)} - \frac{1}{w}\right] = \lim_{w\to 0^+} \frac{w - \ln(1+w)}{w \cdot \ln(1+w)}$, $\frac{0}{0}$

$\overset{L'R}{=} \lim_{w\to 0^+} \frac{1 - \frac{1}{1+w}}{w \cdot \frac{1}{1+w} + \ln(1+w)}$

$\underline{\underline{\text{Simplify}}}$ $\lim_{w\to 0^+} \frac{1+w-1}{w+(1+w)\ln(1+w)} = \lim_{w\to 0^+} \frac{w}{w+(1+w)\cdot\ln(1+w)}$, $\frac{0}{0}$

$\overset{L'R}{=} \lim_{w\to 0^+} \frac{1}{1+(1+w)\cdot\frac{1}{1+w} + \ln(1+w)\cdot 1} = \lim_{w\to 0^+} \frac{1}{2+\ln(1+w)}$

$= \frac{1}{2+0} = \frac{1}{2}$

11.6

15. The graph of the equation

$$x^2 + 2y^2 - z^2 + 2x + 6z = k^2 + k + 8$$

is

(a) a hyperboloid of two sheets if $-1 < k < 0$ ______________________(correct)

(b) a hyperboloid of one sheet if $-1 < k < 0$

(c) an elliptic cone if $k > 0$

(d) a hyperboloid of one sheet for $k > -1$

(e) an elliptic paraboloid if $k = 0$ and $k = -1$

$x^2 + 2x + 1 + 2y^2 - (z^2 - 6z + 9) = k^2 + k + 8 + 1 - 9$

$(x+1)^2 + 2y^2 - (z-3)^2 = k^2 + k$

$= k(k+1)$ $\rightsquigarrow$ + -1 − 0 +

$\downarrow$ if $-1<k<0$

$-$

$\Rightarrow -(x+1)^2 - 2y^2 + (z-3)^2 = \underbrace{-k(k+1)}_{+}$

a hyperboloid of <u>two</u> sheets.