~ #23→30
§.10.2

1. The parametric curve

$$x = 2\cos^2 t,\ y = 1 - \sin t$$

is represented by the rectangular equation

(a) $x = 4y - 2y^2,\ 0 \le x \le 2$ ______ (correct)

(b) $x = 2y - y^2,\ 0 \le x \le 2$

(c) $x = 1 - 4y + 2y^2,\ 0 \le x \le 2$

(d) $2x = y - y^2,\ x \ge 0$

(e) $2x = 2y - y^2,\ x \ge 2$

- As $\cos^2 t + \sin^2 t = 1$,
  then $\frac{x}{2} + (1-y)^2 = 1$
  $\Rightarrow \frac{x}{2} + 1 - 2y + y^2 = 1$
  $\Rightarrow \frac{x}{2} - 2y + y^2 = 0$
  $\Rightarrow \frac{x}{2} = 2y - y^2$
  $\Rightarrow x = 4y - 2y^2$
- since $0 \le \cos^2 t \le 1$
  then $0 \le 2\cos^2 t \le 2$
  or $0 \le x \le 2$.

a special polar graph
p. 725 / §10.4

2. The graph of the polar equation $r = 3\,\cos(6\theta)$ is

(a) a rose with 12 leaves (petals) ______ (correct)

(b) a rose with 6 leaves (petals)

(c) a rose with 3 leaves (petals)

(d) a circle

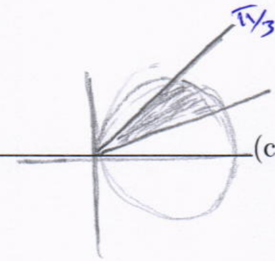
(e) a cardioid

$n = 6$, even
$\Rightarrow 2n = 2\cdot 6 = 12$ petals

~#3
§10.5

3. The **area** of the region bounded by the polar curve $r = \cos\theta$ for $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$ is equal to

$$A = \int_{\pi/6}^{\pi/3} \frac{1}{2} r^2 \, d\theta$$




(a) $\frac{\pi}{24}$ ______ (correct)

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{12}$

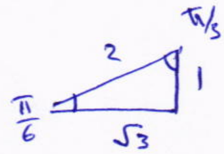
(d) $\frac{\pi}{3}$

(e) $\frac{\pi}{8}$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} \cos^2\theta \, d\theta$$

$$= \frac{1}{4} \int_{\pi/6}^{\pi/3} [1 + \cos(2\theta)] \, d\theta$$

$$= \frac{1}{4} \left[ \theta + \frac{1}{2} \sin(2\theta) \right]_{\pi/6}^{\pi/3}$$




$$= \frac{1}{4} \left[ \theta + \sin\theta\cos\theta \right]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{4} \left[ \left( \frac{\pi}{3} - \frac{\pi}{6} \right) + \underbrace{\left( \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \right)}_{=0} \right]$$

$$= \frac{1}{4} \cdot \frac{\pi}{6}$$

$$= \frac{\pi}{24}$$

~# 11→18
§11.3

4. If the angle between the two vectors

$$\vec{u} = \langle 1, 1, -1 \rangle \text{ and } \vec{v} = \langle 2, x, 0 \rangle$$

is $\frac{\pi}{3}$, then $x^2 + 16x =$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\frac{\pi}{3}$$

(a) $-4$ ______ (correct)

(b) 6

(c) 8

(d) $-2$

(e) 0

$$2 + x + 0 = \sqrt{1+1+1} \cdot \sqrt{4 + x^2 + 0} \cdot \frac{1}{2}$$

$$4 + 2x = \sqrt{3} \cdot \sqrt{4 + x^2}$$

$$16 + 16x + 4x^2 = 3(4 + x^2)$$
$$= 12 + 3x^2$$

$$\Longrightarrow \quad 16x + x^2 = 12 - 16$$

$$\Longrightarrow \quad x^2 + 16x = -4$$

5. The **volume** of the **parallelepiped** with adjacent edges

~#36
§11.4

$$\vec{a} = \langle 1,2,3\rangle,\ \vec{b} = \langle -1,1,2\rangle,\ \vec{c} = \langle 1,-1,1\rangle$$

is equal to

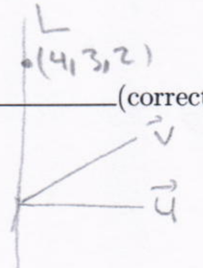
(a) 9 ______________________________ (correct)
(b) 12
(c) 5
(d) 10
(e) 8

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{array}{ccc} + & - & + \end{array}$$
$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= 1(1+2) - 2(-1-2) + 3(1-1)$$
$$= 3 - 2(-3) + 0$$
$$= 3 + 6 = 9$$

$$\text{Vol} = |\vec{a}\cdot(\vec{b}\times\vec{c})| = |9| = 9$$

6. A set of **parametric equations** for the line that passes through the point $(4,3,2)$ and is perpendicular to both vectors $\vec{u} = \langle 2,1,1\rangle$ and $\vec{v} = \langle 1,2,1\rangle$ is given by

Basic
§11.5




(a) $x = 4 - t,\ y = 3 - t,\ z = 2 + 3t,\ t \in (-\infty,\infty)$ ______________ (correct)
(b) $x = 4 + t,\ y = 3 + 2t,\ z = 2 - 2t,\ t \in (-\infty,\infty)$
(c) $x = 4 - 2t,\ y = 3 - 2t,\ z = 2 + 2t,\ t \in (-\infty,\infty)$
(d) $x = 4 + t,\ y = 3 + t,\ z = 2 + 3t,\ t \in (-\infty,\infty)$
(e) $x = 4t,\ y = 3t,\ z = 2t,\ t \in (-\infty,\infty)$

- a vector parallel to the line is

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \langle -1,-1,3\rangle$$

- para. eq: $x = 4 - t$
  $y = 3 - t$ , $t \in (-\infty,\infty)$
  $z = 2 + 3t$

7. The set of all points $(x, y, z)$ that are **equidistant** from the points $(1, 0, 2)$ and $(2, 0, 1)$ represents

#58
§11.5

$d((x,y,z),(1,0,2)) = d((x,y,z),(2,0,1))$

$\sqrt{(x-1)^2+(y-0)^2+(z-2)^2} = \sqrt{(x-2)^2+(y-0)^2+(z-1)^2}$

(a) a plane ______ (correct)

(b) a line

(c) a sphere

(d) an elliptic paraboloid

(e) an elliptic hyperboloid

$\Rightarrow (x-1)^2+y^2+(z-2)^2 = (x-2)^2+y^2+(z-1)^2$

$\Rightarrow x^2-2x+1+y^2+z^2-4z+4 = x^2-4x+4+y^2+z^2-2z+1$

$\Rightarrow -2x+1-4z+4 = -4x+4-2z+1$

$\Rightarrow -2x-4z = -4x-2z$

$\Rightarrow 2x-2z = 0$

$\Rightarrow x-z=0$, a plane

8. The limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+2y^2}$

~#44
§13.2

(a) does not exist ______ (correct)

(b) is equal to 0

(c) is equal to 1

(d) is equal to $\frac{3}{2}$

(e) is equal to 3

along $y=0$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{3x^2y}{x^4+2y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{x^4+0} = \lim_{(x,y)\to(0,0)} 0 = 0$

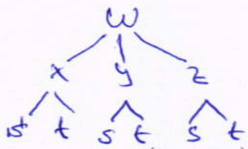
along $y=x^2$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=x^2}} \frac{3x^2y}{x^4+2y^2} = \lim_{(x,y)\to(0,0)} \frac{3x^2 \cdot x^2}{x^4+2x^4} = \lim_{(x,y)\to(0,0)} 1 = 1$

Since the limints along the two paths are not equal, then the given limit dos not exist.

9. Let $w = xyz$, $x = s^2 + t^2$, $y = s^2 - t^2$, $z = st$. The value of $\dfrac{\partial w}{\partial t}$ when $s = 2$ and $t = 1$ is equal to

~#19
§13.5

- $s = 2,\ t = 1 \implies x = 5,\ y = 3,\ z = 2$
- $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial t} + \dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial t}$




(a) 22 ______________________________ (correct)

(b) 80

(c) −72

(d) 63

(e) −42

$= (yz)\cdot(2t) + (xz)\cdot(-2t) + (xy)\cdot(s)$

$= 6\cdot 2 + 10\cdot(-2) + 15\cdot 2$

$= 12 - 20 + 30$

$= 12 + 10$

$= 22$

10. Let $f(x, y, z) = \ln(x + 2y + 3z)$, $P(1, 1, 1)$, $Q(4, 3, 2)$. The **directional derivative** of $f$ at $P$ in the direction of the vector $\overrightarrow{PQ}$ is equal to

~#28
§13.6

- $\overrightarrow{PQ} = \langle 4-1, 3-1, 2-1\rangle = \langle 3, 2, 1\rangle$
- $\vec{u} = \dfrac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \dfrac{1}{\sqrt{9+4+1}}\langle 3, 2, 1\rangle = \dfrac{1}{\sqrt{14}}\langle 3, 2, 1\rangle$

(a) $\dfrac{5}{3\sqrt{14}}$ ______________________________ (correct)

(b) $\dfrac{2}{\sqrt{14}}$

(c) $\dfrac{7}{6\sqrt{14}}$

(d) $\dfrac{1}{3\sqrt{14}}$

(e) $\dfrac{13}{4\sqrt{14}}$

- $\nabla f(x, y, z) = \langle f_x, f_y, f_z\rangle$

$= \left\langle \dfrac{1}{x+2y+3z}, \dfrac{2}{x+2y+3z}, \dfrac{3}{x+2y+3z} \right\rangle$

- $\nabla f(1,1,1) = \left\langle \dfrac{1}{6}, \dfrac{2}{6}, \dfrac{3}{6} \right\rangle$

- $D_{\vec{u}} f(1,1,1) = \nabla f(1,1,1)\cdot \vec{u}$

$= \left\langle \dfrac{1}{6}, \dfrac{2}{6}, \dfrac{3}{6} \right\rangle \cdot \left\langle \dfrac{3}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}} \right\rangle$

$= \dfrac{3}{6\sqrt{14}} + \dfrac{4}{6\sqrt{14}} + \dfrac{3}{6\sqrt{14}}$

$= \dfrac{10}{6\sqrt{14}} = \dfrac{5}{3\sqrt{14}}$

11. If $ax + by + cz + 1 = 0$ is an **equation for the tangent plane** to the surface $z = (x - y)^2$ at the point $(1, 2, 1)$, then $a + b + c =$

\#12

§13.7

$f(x,y) = (x-y)^2$

(a) 1 ________________________________________________(correct)

(b) 0

(c) $-2$

(d) 3

(e) $-4$

$z = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$\cdot\ f_x(x,y) = 2(x-y) \Rightarrow f_x(1,2) = 2(1-2) = -2$

$\cdot\ f_y(x,y) = 2(x-y)(-1) \Rightarrow f_y(1,2) = 2(1-2)(-1) = 2$

$z = 1 - 2(x-1) + 2(y-2)$

$= 1 - 2x + 2 + 2y - 4$

$= -2x + 2y - 1$

$\Rightarrow 2x - 2y + z + 1 = 0$

$\Rightarrow a = 2,\ b = -2,\ c = 1$

$\Rightarrow a + b + c = 2 - 2 + 1 = \underline{1}$

12. The graph of the function $f(x, y) = 4 + x - y - x^2y + xy^2$ has

~ #21

§13.8

(a) two saddle points ________________________________________(correct)

(b) one saddle point and one local minimum point

(c) one local maximum point and one local minimum point

(d) two local maximum points

(e) one saddle point and one local maximum point

$f_x = 1 - 2xy + y^2 = 0 \quad \sim (1)$

$f_y = -1 - x^2 + 2xy = 0 \quad \sim (2)$

; $(1)+(2) \Rightarrow y^2 - x^2 = 0 \Rightarrow y = x$ or $y = -x$

$\cdot\ y = x \overset{(1)}{\Longrightarrow} 1 - 2x^2 + x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\Rightarrow \boxed{(1,1),\ (-1,-1)}$

$\cdot\ y = -x \overset{(1)}{\Longrightarrow} 1 + 2x^2 + x^2 = 0 \Rightarrow 1 + 3x^2 = 0$, no real sol.

$\Rightarrow$ no pts.

$f_{xx} = -2y$ ; $f_{xy} = -2x + 2y$

$f_{yy} = 2x$

$D(x,y) = -4xy - (-2x+2y)^2$

$D(1,1) = -4 - 0 = -4 < 0$

$D(-1,-1) = -4 - 0 = -4 < 0$

$\Rightarrow$ <u>Saddle points</u> at $(-1,-1)$ & $(1,1)$

13. If $M$ and $m$ are respectively the **maximum** and **minimum** values of $f(x, y) = xy + x + y + 1$ subject to the constraint $xy = 4$, then $M + m =$

$\sim$ #8

§ 13.10

$g(x,y) = xy - 4$

note that $y \neq 0, x \neq 0$

(a) 10 ______ (correct)

(b) 12

(c) 8

(d) 14

(e) 6

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g = 0 \end{cases} \Rightarrow \begin{cases} y + 1 = \lambda \cdot y & \text{--- (1)} \\ x + 1 = \lambda \cdot x & \text{--- (2)} \\ xy - 4 = 0 & \text{--- (3)} \end{cases}$$

$(1) \Rightarrow \lambda = \frac{y+1}{y}$

$(2) \Rightarrow \lambda = \frac{x+1}{x}$

$\Rightarrow \frac{y+1}{y} = \frac{x+1}{x} \Rightarrow xy + x = xy + y \Rightarrow \boxed{y = x}$

$\xrightarrow{(3)} x \cdot x - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$\cdot\ x = 2 \Rightarrow y = 2 \Rightarrow (2,2) \xrightarrow{f} f(2,2) = 9 = M$

$\cdot\ x = -2 \Rightarrow y = -2 \Rightarrow (-2,-2) \xrightarrow{f} f(-2,-2) = 1 = m$

$M + m = 9 + 1 = 10.$

14. $\underbrace{\int_0^1 \int_0^2 (2x - y^2)\, dy}\, dx =$

#11

§ 14.1

$2xy - \frac{y^3}{3} \Big]_{y=0}^{y=2} = 4x - \frac{8}{3}$

(a) $-\frac{2}{3}$ ______ (correct)

(b) $\frac{4}{3}$

(c) $\frac{8}{3}$

(d) $-\frac{1}{3}$

(e) 2

$\int_0^1 (4x - \frac{8}{3})\, dx$

$= 2x^2 - \frac{8}{3}x \Big]_{x=0}^{x=1}$

$= 2 - \frac{8}{3}$

$= \frac{6}{3} - \frac{8}{3} = -\frac{2}{3}$

15. $\int_0^2 \int_{y^2}^4 f(x,y)\,dx\,dy =$

$\sim \#43 \to 50$
§ 14.1

$R: y^2 \le x \le 4 \quad , \quad 0 \le y \le 2$

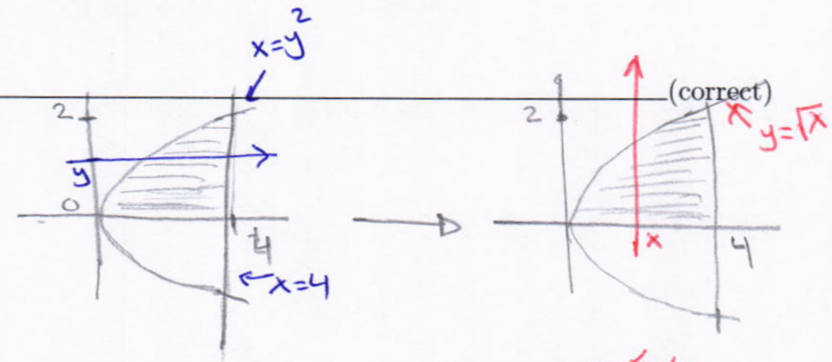
(a) $\int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$ ——— (correct)

(b) $\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$

(c) $\int_0^4 \int_0^{x} f(x,y)\,dy\,dx$

(d) $\int_0^2 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

(e) $\int_{y^2}^4 \int_0^{2} f(x,y)\,dy\,dx$




$R: 0 \le x \le 4$
$0 \le y \le \sqrt{x}$

$$= \int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$$

$\sim \#55/14.2$
$\#63/14.1$

16. The **average value** of $f(x,y) = 4e^{y^2}$ over the plane region $R$ bounded by the curves $y = 2x$, $y = 4$, and $x = 0$ is equal to

(a) $\frac{e^{16}-1}{4}$ ——— (correct)

(b) $\frac{e^{16}-1}{8}$

(c) $e^{16}-1$

(d) $\frac{e^{16}-1}{2}$

(e) $\frac{e^{16}-1}{16}$

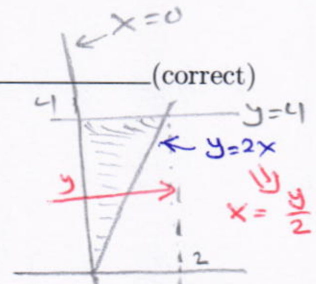
$$f_{ave} = \frac{1}{\text{area}(R)} \iint_R f(x,y)\,dA$$

$$= \frac{1}{4} \int_0^4 \int_0^{y/2} 4e^{y^2}\,dx\,dy$$

$$= \int_0^4 \underbrace{\int_0^{y/2} e^{y^2}\,dx}_{e^{y^2}\cdot x\Big]_{x=0}^{x=y/2}}\,dy$$

$$= \int_0^4 \frac{y}{2} e^{y^2}\,dy$$

$$= \frac{1}{4} e^{y^2}\Big]_{y=0}^{y=4} = \frac{e^{16}-1}{4}$$




$R: 0 \le x \le \frac{y}{2}$
$0 \le y \le 4$
$\text{area}(R) = \frac{1}{2}\cdot 2\cdot 4$
$= 4$

17. If $R = \{(x,y) : x^2+y^2 \le 4,\ x \le 0\}$ then $\displaystyle\iint_R e^{-(x^2+y^2)/2}\, dA =$

~ #30
§14.3

(a) $(1-e^{-2})\,\pi$ (correct)

(b) $(1-e^{-2})\,\frac{\pi}{2}$

(c) $(e^{-2}-1)\,\frac{\pi}{2}$

(d) $(2-e^{-2})\,\pi$

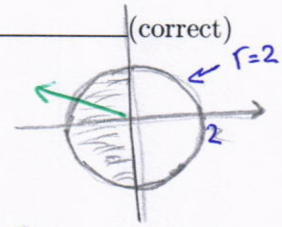
(e) $(1-e^{-2})\,\frac{\pi}{4}$

$$= \int_{\pi/2}^{3\pi/2}\underbrace{\int_0^2 e^{-r^2/2}\cdot r\, dr}_{-e^{-r^2/2}\Big]_{r=0}^{r=2} \;=\; -(e^{-2}-1)}\, d\theta$$

$$= (1-e^{-2})\int_{\pi/2}^{3\pi/2} d\theta$$

$$= (1-e^{-2})\cdot \theta\Big]_{\pi/2}^{3\pi/2}$$

$$= (1-e^{-2})\,\pi$$




R: $0 \le r \le 2$
$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

~ #13, 17
§14.6

18. The **volume** of the solid in the **first octant** bounded by the planes $x=0,\ y=0,$ $z=1$ and $3x+3y+z=4$ is equal to

Intersection of the planes:
$\begin{cases} z=1 \\ 3x+3y+z=4 \end{cases}$

(a) $\dfrac{1}{2}$ (correct)

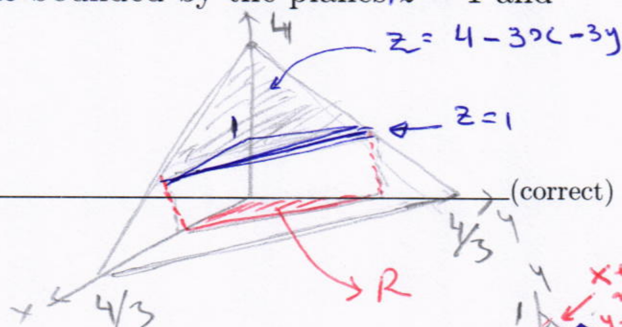
(b) $\dfrac{1}{3}$

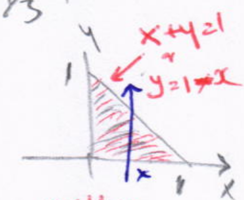
(c) $\dfrac{2}{3}$

(d) $\dfrac{3}{4}$

(e) $1$

$\Rightarrow 3x+3y+1=4$
$\Rightarrow 3x+3y=3$
$\Rightarrow x+y=1$




R: $0 \le y \le 1-x$
$0 \le x \le 1$

$$V = \int_0^1\int_0^{1-x}\underbrace{\int_1^{4-3x-3y} 1\, dz}\, dy\, dx$$

$$= \int_0^1\int_0^{1-x}(3-3x-3y)\, dy\, dx$$

$$= 3\int_0^1\underbrace{\int_0^{1-x}(1-x-y)\, dy}_{(1-x)y-\frac{1}{2}y^2\Big]_{y=0}^{y=1-x} \;=\; (1-x)^2-\frac{1}{2}(1-x)^2 = \frac{1}{2}(1-x)^2}\, dx$$

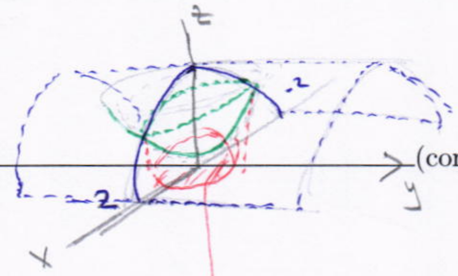
$$= \frac{3}{2}\int_0^1 (1-x)^2\, dx = \frac{3}{2}\cdot -\frac{(1-x)^3}{3}\Big]_{x=0}^{x=1}$$

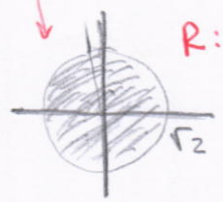
$$= \frac{3}{2}\left(0-\left(-\frac{1}{3}\right)\right) = \frac{3}{2}\cdot\frac{1}{3} = \frac{1}{2}$$

19. The **volume** of the solid bounded above by the parabolic cylinder $z = 4 - x^2$ and below by the paraboloid $z = x^2 + 2y^2$ is equal to

$\sim$ #18
§ 14.6

R: $z = z$
$4 - x^2 = x^2 + 2y^2$
$4 = 2x^2 + 2y^2$
$x^2 + y^2 = 2$
$x^2 + 2y^2 \le z \le 4 - x^2$




R: $0 \le r \le \sqrt{2}$
$0 \le \theta \le 2\pi$

(a) $4\pi$ ______________________________ (correct)

(b) $\dfrac{3\pi}{2}$

(c) $2\sqrt{2}\,\pi$

(d) $\dfrac{\sqrt{2}\pi}{4}$

(e) $2\pi$

$$V = \iint_R \left[\int_{x^2+2y^2}^{4-x^2} dz\right] dA = \iint_R (4 - 2x^2 - 2y^2)\, dA = \int_0^{2\pi}\int_0^{\sqrt{2}} (4 - 2r^2)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \underbrace{\int_0^{\sqrt{2}} (4r - 2r^3)\, dr}_{2r^2 - \frac{1}{2}r^4 \Big]_{r=0}^{r=\sqrt{2}} = 4 - \frac{1}{2}\cdot 4 = 4 - 2 = 2} d\theta$$

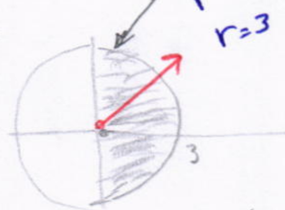
$$= \int_0^{2\pi} 2\, d\theta = 2\theta\Big]_0^{2\pi} = 4\pi$$

20. In **cylindrical coordinates**, $\underbrace{\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}}\int_{x^2+y^2}^{9} y\, dz\, dy\, dx =$

$\sim$ #41
§ 14.7

R: $0 \le x \le 3$
$-\sqrt{9-x^2} \le y \le \sqrt{9-x^2}$

$x^2 + y^2 = 9$
$r^2 = 9$
$r = 3$




(a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^3 \int_{r^2}^9 (r^2 \sin\theta)\, dz\, dr\, d\theta$ ______________________________ (correct)

R: $0 \le r \le 3$
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

(b) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^3 \int_{r^2}^9 (r^2 \sin\theta)\, dz\, dr\, d\theta$

(c) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-3}^3 \int_{r^2}^9 (r^2 \sin\theta)\, dz\, dr\, d\theta$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^3 \int_{r^2}^9 (r \sin\theta)\, dz\, dr\, d\theta$

(e) $\displaystyle\int_0^{\pi} \int_0^3 \int_r^3 (r \sin\theta)\, dz\, dr\, d\theta$

$$\Rightarrow = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^3 \int_{r^2}^9 (r \sin\theta)\, dz \cdot r\, dr\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^3 \int_{r^2}^9 (r^2 \sin\theta)\, dz\, dr\, d\theta$$

21. $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} (8x^2+8y^2+8z^2)^{2/3}\,dz\,dy\,dx =$

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§14.7

(a) $\dfrac{12\pi}{13}$ ________________________________________(correct)

(b) $\dfrac{5\pi}{13}$

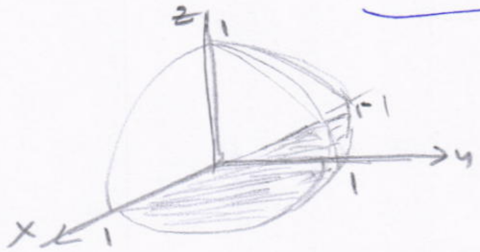
(c) $\dfrac{\pi}{6}$

(d) $\dfrac{3\pi}{8}$

(e) $\dfrac{\pi}{13}$

$0 \le z \le \sqrt{1-x^2-y^2}, \quad \underbrace{0 \le y \le \sqrt{1-x^2}, \ -1 \le x \le 1}$

shaded region in the xy-plane




$0 \le \rho \le 1, \quad 0 \le \phi \le \frac{\pi}{2}, \quad 0 \le \theta \le \pi$

$$= \int_0^{\pi}\int_0^{\pi/2}\int_0^1 \underbrace{(8\rho^2)^{2/3}}_{=4\rho^{4/3}} \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 4\rho^{4/3}\cdot\rho^2 = 4\rho^{10/3}$$

$$= 4\cdot\frac{3}{13}\rho^{13/3}\Big]_{\rho=0}^{\rho=1} = \frac{12}{13}(1-0) = \frac{12}{13}$$

$$= \int_0^{\pi}\underbrace{\int_0^{\pi/2} \frac{12}{13}\sin\phi \, d\phi}\, d\theta$$

$$\frac{12}{13}\cdot(-\cos\phi)\Big]_{\phi=0}^{\phi=\pi/2} = \frac{12}{13}\big(0-(-1)\big) = \frac{12}{13}$$

$$= \int_0^{\pi}\frac{12}{13}\,d\theta = \frac{12}{13}\theta\Big]_0^{\pi} = \frac{12}{13}\pi$$

**Answer KEY**

| Q | MASTER | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | A | B $_{4}$ | D $_{4}$ | B $_{1}$ | A $_{6}$ |
| 2 | A | E $_{2}$ | D $_{1}$ | D $_{5}$ | B $_{4}$ |
| 3 | A | E $_{1}$ | E $_{3}$ | E $_{4}$ | B $_{1}$ |
| 4 | A | A $_{3}$ | B $_{5}$ | D $_{2}$ | A $_{3}$ |
| 5 | A | E $_{6}$ | C $_{6}$ | D $_{3}$ | D $_{2}$ |
| 6 | A | D $_{5}$ | E $_{2}$ | E $_{6}$ | D $_{5}$ |
| 7 | A | E $_{12}$ | C $_{12}$ | E $_{8}$ | A $_{9}$ |
| 8 | A | E $_{10}$ | D $_{11}$ | C $_{13}$ | C $_{11}$ |
| 9 | A | A $_{7}$ | E $_{10}$ | E $_{9}$ | C $_{13}$ |
| 10 | A | D $_{13}$ | C $_{7}$ | D $_{12}$ | E $_{12}$ |
| 11 | A | B $_{8}$ | B $_{8}$ | C $_{10}$ | E $_{10}$ |
| 12 | A | D $_{11}$ | B $_{9}$ | E $_{11}$ | E $_{8}$ |
| 13 | A | E $_{9}$ | B $_{13}$ | D $_{7}$ | B $_{7}$ |
| 14 | A | B $_{16}$ | A $_{19}$ | B $_{21}$ | D $_{20}$ |
| 15 | A | A $_{19}$ | A $_{16}$ | C $_{16}$ | A $_{14}$ |
| 16 | A | E $_{17}$ | C $_{14}$ | E $_{14}$ | D $_{16}$ |
| 17 | A | A $_{21}$ | B $_{18}$ | A $_{19}$ | D $_{15}$ |
| 18 | A | C $_{15}$ | A $_{15}$ | E $_{15}$ | E $_{17}$ |
| 19 | A | A $_{14}$ | D $_{21}$ | B $_{20}$ | E $_{19}$ |
| 20 | A | C $_{20}$ | D $_{20}$ | D $_{18}$ | A $_{18}$ |
| 21 | A | C $_{18}$ | D $_{17}$ | D $_{17}$ | A $_{21}$ |