1. The sum of all real values of m that makes the equation

$$(mx^2y + 2xy - y^3)\,dx + (x^2 - 3xy^2 + \frac{1}{3}m^3x^3)\,dy = 0$$

an Exact differential equation is equal to

(a) 0 _____(correct)
(b) 1
(c) -1
(d) 2
(e) -2

2. The general solution of the exact differential equation

$$(2x + y^2e^{xy})\,dx + (e^{xy} + xye^{xy})\,dy = 0$$

is

(a) $ye^{xy} + x^2 = c$ _____(correct)
(b) $xe^{xy} + x^2 = c$
(c) $ye^{xy} + y^2 = c$
(d) $xe^{xy} + y^2 = c$
(e) $ye^{xy} + x^3 = c$

3. The general solution of the differential equation $x\dfrac{dy}{dx} = y + 4x\,e^{\frac{-2y}{x}}$ for $x \neq 0$ is given by

(a) $e^{\frac{2y}{x}} - 8\ln|x| = c$ (correct)

(b) $e^{\frac{2y}{x}} - 4\ln|x| = c$

(c) $e^{\frac{2x}{y}} + 4\ln|x| = c$

(d) $e^{\frac{2x}{y}} + 8\ln|x| = c$

(e) $e^{\frac{2y}{x}} - \ln|x| = c$

4. By using a suitable substitution, we can transform the differential equation $y' + \dfrac{2}{x}y = 4x^3y^4$ into the linear differential equation

(a) $\dfrac{dw}{dx} - \dfrac{6}{x}w = -12x^3$ (correct)

(b) $\dfrac{dw}{dx} - \dfrac{3}{x}w = -12x^3$

(c) $\dfrac{dw}{dx} - \dfrac{6}{x}w = 6x^3$

(d) $\dfrac{dw}{dx} - \dfrac{3}{x}w = 6x^3$

(e) $\dfrac{dw}{dx} - \dfrac{2}{x}w = 4x^3$

5. The solution of the initial-value problem $(2-\cos x)\,y' + y\sin x = 2+\cos x,\ y(0) = 0$ is

(a) $y = \dfrac{2x+\sin x}{2-\cos x}$ (correct)

(b) $y = \dfrac{x+\sin x}{2-\cos x}$

(c) $y = \dfrac{2x+\cos x}{2+\cos x}$

(d) $y = \dfrac{x+\cos x}{2+\cos x}$

(e) $y = \dfrac{2x+\sin x}{2x-\cos x}$

6. The order of the differential equation

$$y''' + 2x(y'')^4 + x^2(y')^6 = 0$$

is

(a) 3 (correct)

(b) 4

(c) 6

(d) 10

(e) 1

7. The general solution of the differential equation $\dfrac{dy}{dx} = x\sqrt{\dfrac{1-y^2}{1-x^2}}$ for $|x| < 1$ and $|y| < 1$ is given by

(a) $y = \sin(c - \sqrt{1-x^2})$ (correct)

(b) $y = 2\sin(c - \sqrt{1-x^2})$

(c) $y = \dfrac{1}{2}\sin(c - \sqrt{1-x^2})$

(d) $y = 3\sin(c - \sqrt{1-x^2})$

(e) $y = \dfrac{1}{3}\sin(c - \sqrt{1-x^2})$

8. By using a suitable substitution, we can transform the differential equation $yy'' + (y')^2 = 0$ into the separable differential equation

(a) $\dfrac{dp}{p} + \dfrac{dy}{y} = 0$ (correct)

(b) $\dfrac{dp}{p} + \dfrac{dy}{2y} = 0$

(c) $\dfrac{dp}{p} - \dfrac{dy}{3y} = 0$

(d) $\dfrac{dp}{2p} - \dfrac{dy}{y} = 0$

(e) $\dfrac{dp}{2p} + \dfrac{dy}{y} = 0$

9. All values of m such that $y = e^{mx}$ is a solution of the differential equation $y'' - 6y' - 7y = 0$ are

   (a) $m = 7$ and $m = -1$ _____(correct)

   (b) $m = 7$ and $m = 2$

   (c) $m = -7$ and $m = 2$

   (d) $m = 7$ and $m = 1$

   (e) $m = 7$ and $m = -2$

10. The sum of all real values of k for which the vectors

    $$\mathbf{v}_1 = (2, 3, 0),\ \mathbf{v}_2 = (4, k, 4),\ \mathbf{v}_3 = (-1, 1, k)$$

    do not form a basis for $\mathbb{R}^3$ is

    (a) 6 _____(correct)

    (b) 5

    (c) 4

    (d) -12

    (e) 12

11. A particle is moving in a straight line with acceleration $a(t) = \ln(5t)$ and an initial velocity $v\left(\frac{1}{5}\right) = \frac{4}{5}$. The velocity at any time $t > \frac{1}{5}$ is given by

(a) $v(t) = t\,\ln(5t) - t + 1$ _____(correct)

(b) $v(t) = 2t\,\ln(5t) - t + 1$

(c) $v(t) = 3t\,\ln(5t) - t + 1$

(d) $v(t) = t\,\ln(5t) - 2t + \frac{6}{5}$

(e) $v(t) = t\,\ln(5t) + 2t + \frac{2}{5}$

12. Which one of the following statements is true about the subset V of $\mathbb{R}^3$ defined by

$$V = \{(x_1,\, x_2,\, x_3) : x_3 = x_1 + x_2\}\,?$$

(a) V is a subspace of $\mathbb{R}^3$ _____(correct)

(b) V is not closed under addition

(c) V is not closed under addition but not closed under multiplication by scalars

(d) V is closed under addition but not closed under multiplication by scalars

(e) V is not closed under addition but closed under multiplication by scalar

13. The solution of the system

$$\begin{aligned} x_1 + 3x_2 + 3x_3 + 3x_4 &= 0 \\ 2x_1 + 7x_2 + 5x_3 - x_4 &= 0 \\ 2x_1 + 7x_2 + 4x_3 - 4x_4 &= 0 \end{aligned}$$

is the set of all scalars multiples of a vector $\mathbf{u}$ where $\mathbf{u} = (a, b, c, d)$, then $\dfrac{a}{c} =$

(a) 2 _____ (correct)

(b) 4

(c) -2

(d) -4

(e) 0

14. The general solution of the differential equation

$$\frac{dy}{dx} = \frac{x + y + 2}{2x + 2y - 1}$$

is given by

(a) $2y - x - \dfrac{5}{3}\ln|3x + 3y + 1| = c$ _____ (correct)

(b) $2y + x - \dfrac{5}{3}\ln|3x + 3y + 1| = c$

(c) $2x + y + \dfrac{5}{3}\ln|3x + 3y - 1| = c$

(d) $x + y + \dfrac{5}{3}\ln|3x + 3y - 1| = c$

(e) $2x + 2y - \dfrac{5}{3}\ln|2x + 2y - 1| = c$

15. A certain radioactive substance decomposes at a rate directly proportional to the quantity of the substance present. If 50% of a certain quantity has decomposed in 20 years, how many years will it take for 80% of the original quantity to decompose?

(a) $\dfrac{20 \ln 5}{\ln 2}$ (correct)

(b) $\dfrac{30 \ln 5}{\ln 2}$

(c) $\dfrac{15 \ln 5}{\ln 2}$

(d) $\dfrac{40 \ln 5}{\ln 2}$

(e) $\dfrac{24 \ln 5}{\ln 2}$