King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 208
Final Exam
Term 252
12 May 2026

EXAM COVER

Number of versions: 8
Number of questions: 21

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12 May 2026

Net Time Allowed: 120 Minutes

MASTER VERSION

1. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 - xy^3 + y^2 = c$ ______ (correct)

(b) $x + x^2 + 2xy^3 + y^2 = c$

(c) $x + x^2 + 3xy^3 + y^2 = c$

(d) $x^2 + x^3 - xy^3 + y^2 = c$

(e) $x^2 + x^3 - xy^3 + 4y^2 = c$

2. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\frac{7}{8}$ ______ (correct)

(b) $\frac{1}{2}$

(c) 0

(d) $\frac{5}{8}$

(e) $\frac{3}{4}$

3. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\sin x + c)\ \cos x$ ________________________________(correct)

(b) $y = (\sin^2 x + c)\ \cos x$

(c) $y = (\cos x + c)\ \sin x$

(d) $y = (\cos^2 x + c)\ \sin x$

(e) $y = (2\sin x + c)\ \cos^2 x$

4. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 1 ________________________________(correct)

(b) 0

(c) 2

(d) -2

(e) 3

5. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ ____(correct)

(b) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

6. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 3 ____(correct)

(b) 4

(c) 2

(d) 1

(e) 0

7. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$ ______ (correct)

(b) $c_{n+1} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$

(c) $c_{n+2} = \dfrac{2-n}{n+2} c_n,\ n \geq 3$

(d) $c_{n+1} = \dfrac{2-n}{n+2} c_n,\ n \geq 2$

(e) $c_{n+2} = \dfrac{3-n}{n+1} c_n,\ n \geq 4$

8. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$ ______ (correct)

(b) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

9. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$ ____ (correct)

(b) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

10. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 12 ____ (correct)

(b) 10

(c) 8

(d) 4

(e) 0

11. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $-1 + \sin x \ln|\sec x + \tan x|$ ________________ (correct)

(b) $x + \sin x \ln|\sec x + \tan x|$

(c) $x^2 + \cos x \ln|\sec x + \tan x|$

(d) $\cos x + x \ln|\sec x + \tan x|$

(e) $x\cos x + \sin x \ln|\sec x + \tan x|$

12. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

13. If the three vectors $V_1 = (4,3,2)$, $V_2 = (2,5,-1)$ and $V_3 = (k,-4,2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{4}{13}$ ______________________________ (correct)

(b) $-\dfrac{4}{11}$

(c) $\dfrac{5}{13}$

(d) $-\dfrac{5}{13}$

(e) $\dfrac{3}{11}$

14. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3+x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by

$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}\,c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \dfrac{1}{3}x^{5/2} + \dfrac{1}{30}x^{7/2}$ ______________________________ (correct)

(b) $x^{3/2} - \dfrac{1}{5}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(c) $x^{3/2} - \dfrac{1}{4}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(d) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{7}{30}x^{7/2}$

(e) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{1}{15}x^{7/2}$

15. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y=0$ is given by

(a) $y(x)=c_1e^x+c_2xe^x+c_3e^x\,\cos(2x)+c_4e^x\,\sin(2x)$ ______________ (correct)

(b) $y(x)=c_1e^x+c_2xe^x+c_3e^{-x}\,\cos(2x)+c_4e^{-x}\,\sin(2x)$

(c) $y(x)=c_1e^{-x}+c_2xe^{-x}+c_3e^x\,\cos(2x)+c_4e^x\,\sin(2x)$

(d) $y(x)=c_1e^{-x}+c_2xe^{-x}+c_3e^{-2x}\,\cos(x)+c_5e^{-2x}\,\sin x$

(e) $y(x)=c_1e^x+c_2xe^x+c_3\,\cos(2x)+c_4\,\sin(2x)$

16. If $A=\begin{bmatrix}1&3&4\\0&1&6\\0&0&1\end{bmatrix}$, then $e^{At}=\begin{bmatrix}e^t&f(t)&h(t)\\0&e^t&g(t)\\0&0&e^t\end{bmatrix}$, where $f(1)+g(1)+h(1)=$

(a) $22\,e$ ______________ (correct)

(b) $16\,e$

(c) $14\,e$

(d) $20\,e$

(e) $17\,e$

17. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$ ____ (correct)

(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

18. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix
$A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) -1 ____ (correct)

(b) 1

(c) 0

(d) 2

(e) -2

19. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 + 3x^3 \ln|x| = 27x^3$ ____(correct)
(b) $3y^2 + 3x^3 \ln|x| = 27x^3$
(c) $y^3 + 2x^3 \ln|x| = 27x^3$
(d) $9y + 3x^3 \ln|x| = 27x^3$
(e) $y^3 - 4x^3 \ln|x| = 27x^3$

20. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) 0 ____(correct)
(b) -2
(c) 2
(d) 3
(e) -3

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2 \end{bmatrix} \right) e^t$ ____________________(correct)

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t+2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t+1 \\ 2t+1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1+2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2+t \\ t^2 \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 1 **CODE 1**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

   (a) 1
   (b) 3
   (c) 2
   (d) 0
   (e) -2

2. If the recurrence relation of the Frobenius series solution of the differential equation

   $$2x^2y'' - 3xy' + (3+x)y = 0$$

   that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by
   $c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

   (a) $x^{3/2} - \dfrac{1}{3}x^{5/2} + \dfrac{1}{30}x^{7/2}$
   (b) $x^{3/2} - \dfrac{1}{4}x^{5/2} + \dfrac{1}{30}x^{7/2}$
   (c) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{7}{30}x^{7/2}$
   (d) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{1}{15}x^{7/2}$
   (e) $x^{3/2} - \dfrac{1}{5}x^{5/2} + \dfrac{1}{30}x^{7/2}$

3. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{2-n}{n+2}\,c_n,\ n \geq 3$
(b) $c_{n+2} = \dfrac{3-n}{n+1}\,c_n,\ n \geq 4$
(c) $c_{n+1} = \dfrac{1-n}{n+2}\,c_n,\ n \geq 2$
(d) $c_{n+1} = \dfrac{2-n}{n+2}\,c_n,\ n \geq 2$
(e) $c_{n+2} = \dfrac{1-n}{n+2}\,c_n,\ n \geq 2$

4. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $x\cos x + \sin x \ln|\sec x + \tan x|$
(b) $-1 + \sin x \ln|\sec x + \tan x|$
(c) $x^2 + \cos x \ln|\sec x + \tan x|$
(d) $\cos x + x \ln|\sec x + \tan x|$
(e) $x + \sin x \ln|\sec x + \tan x|$

5. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\cos x + c)\ \sin x$
(b) $y = (\cos^2 x + c)\ \sin x$
(c) $y = (2\sin x + c)\ \cos^2 x$
(d) $y = (\sin x + c)\ \cos x$
(e) $y = (\sin^2 x + c)\ \cos x$

6. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $16\,e$
(b) $20\,e$
(c) $22\,e$
(d) $14\,e$
(e) $17\,e$

7. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y=0$ is given by

(a) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$

(b) $y(x) = c_1e^{x} + c_2xe^{x} + c_3\cos(2x) + c_4\sin(2x)$

(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

(d) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$

(e) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

8. The largest indicial root at $x=0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{7}{8}$

(d) $\frac{5}{8}$

(e) 0

9. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 0
(b) 1
(c) 4
(d) 3
(e) 2

10. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$
(b) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$
(c) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$
(d) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$
(e) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

11. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

12. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix

$A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) -1

(b) 2

(c) -2

(d) 0

(e) 1

13. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 - 4x^3 \ln|x| = 27x^3$
(b) $y^3 + 2x^3 \ln|x| = 27x^3$
(c) $9y + 3x^3 \ln|x| = 27x^3$
(d) $y^3 + 3x^3 \ln|x| = 27x^3$
(e) $3y^2 + 3x^3 \ln|x| = 27x^3$

14. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 4
(b) 8
(c) 12
(d) 10
(e) 0

15. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) 3
(b) 0
(c) -3
(d) -2
(e) 2

16. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$
(b) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$
(c) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$
(d) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$
(e) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

17. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

18. If the three vectors $V_1 = (4, 3, 2)$, $V_2 = (2, 5, -1)$ and $V_3 = (k, -4, 2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{4}{11}$

(b) $\dfrac{5}{13}$

(c) $-\dfrac{4}{13}$

(d) $-\dfrac{5}{13}$

(e) $\dfrac{3}{11}$

19. The solution of the exact differential equation

$$(1 + 2x - y^3)\, dx + (2y - 3xy^2)\, dy = 0$$

is given by

(a) $x^2 + x^3 - xy^3 + 4y^2 = c$
(b) $x + x^2 + 3xy^3 + y^2 = c$
(c) $x + x^2 + 2xy^3 + y^2 = c$
(d) $x + x^2 - xy^3 + y^2 = c$
(e) $x^2 + x^3 - xy^3 + y^2 = c$

20. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2 \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t+1 \\ 2t+1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t+2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1+2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2+t \\ t^2 \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 2 **CODE 2**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) 0
(b) -2
(c) 2
(d) 1
(e) -1

2. The solution of the initial-value problem $xy^2 \dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $3y^2 + 3x^3 \ln|x| = 27x^3$
(b) $y^3 + 3x^3 \ln|x| = 27x^3$
(c) $y^3 - 4x^3 \ln|x| = 27x^3$
(d) $y^3 + 2x^3 \ln|x| = 27x^3$
(e) $9y + 3x^3 \ln|x| = 27x^3$

3. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $14\,e$
(b) $16\,e$
(c) $20\,e$
(d) $22\,e$
(e) $17\,e$

4. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\sin^2 x + c)\,\cos x$
(b) $y = (\cos^2 x + c)\,\sin x$
(c) $y = (\sin x + c)\,\cos x$
(d) $y = (\cos x + c)\,\sin x$
(e) $y = (2\sin x + c)\,\cos^2 x$

5. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) -3
(b) 0
(c) -2
(d) 3
(e) 2

6. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 1
(b) -2
(c) 0
(d) 2
(e) 3

7. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y''+3x(x-2)y'+7(x+5)y=0$$

is

(a) 2
(b) 0
(c) 1
(d) 3
(e) 4

8. For $A=\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP=\begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P=\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(b) $P=\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(c) $P=\begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(d) $P=\begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(e) $P=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

9. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

10. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

11. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $-1 + \sin x \ln|\sec x + \tan x|$

(b) $x + \sin x \ln|\sec x + \tan x|$

(c) $\cos x + x \ln|\sec x + \tan x|$

(d) $x \cos x + \sin x \ln|\sec x + \tan x|$

(e) $x^2 + \cos x \ln|\sec x + \tan x|$

12. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

13. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 12

(b) 8

(c) 10

(d) 0

(e) 4

14. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{1-n}{n+2} c_n$, $n \geq 2$

(b) $c_{n+1} = \dfrac{2-n}{n+2} c_n$, $n \geq 2$

(c) $c_{n+2} = \dfrac{3-n}{n+1} c_n$, $n \geq 4$

(d) $c_{n+1} = \dfrac{1-n}{n+2} c_n$, $n \geq 2$

(e) $c_{n+2} = \dfrac{2-n}{n+2} c_n$, $n \geq 3$

15. If the three vectors $V_1 = (4, 3, 2)$, $V_2 = (2, 5, -1)$ and $V_3 = (k, -4, 2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{4}{13}$

(b) $-\dfrac{5}{13}$

(c) $\dfrac{5}{13}$

(d) $-\dfrac{4}{11}$

(e) $\dfrac{3}{11}$

16. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\dfrac{5}{8}$

(b) $\dfrac{3}{4}$

(c) 0

(d) $\dfrac{1}{2}$

(e) $\dfrac{7}{8}$

17. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3+x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by
$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \dfrac{1}{4}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(b) $x^{3/2} - \dfrac{1}{3}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(c) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{1}{15}x^{7/2}$

(d) $x^{3/2} - \dfrac{1}{5}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(e) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{7}{30}x^{7/2}$

18. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1-2i \end{bmatrix}$, then $X\left(\dfrac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

19. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 + 2xy^3 + y^2 = c$
(b) $x + x^2 - xy^3 + y^2 = c$
(c) $x^2 + x^3 - xy^3 + 4y^2 = c$
(d) $x^2 + x^3 - xy^3 + y^2 = c$
(e) $x + x^2 + 3xy^3 + y^2 = c$

20. The general solution of differential equation $(D-1)^2(D^2 - 2D + 5)\,y = 0$ is given by

(a) $y(x) = c_1e^x + c_2xe^x + c_3\cos(2x) + c_4\sin(2x)$
(b) $y(x) = c_1e^x + c_2xe^x + c_3e^x\cos(2x) + c_4e^x\sin(2x)$
(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^x\cos(2x) + c_4e^x\sin(2x)$
(d) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$
(e) $y(x) = c_1e^x + c_2xe^x + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t+1 \\ 2t+1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t+2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2 \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1+2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2+t \\ t^2 \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 3 **CODE 3**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

2. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) 1

(b) -2

(c) 0

(d) 2

(e) -1

3. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $\cos x + x \ln|\sec x + \tan x|$

(b) $x \cos x + \sin x \ln|\sec x + \tan x|$

(c) $x + \sin x \ln|\sec x + \tan x|$

(d) $x^2 + \cos x \ln|\sec x + \tan x|$

(e) $-1 + \sin x \ln|\sec x + \tan x|$

4. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

5. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{1-n}{n+2}\, c_n,\ n \geq 2$
(b) $c_{n+1} = \dfrac{2-n}{n+2}\, c_n,\ n \geq 2$
(c) $c_{n+2} = \dfrac{3-n}{n+1}\, c_n,\ n \geq 4$
(d) $c_{n+1} = \dfrac{1-n}{n+2}\, c_n,\ n \geq 2$
(e) $c_{n+2} = \dfrac{2-n}{n+2}\, c_n,\ n \geq 3$

6. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y = 0$ is given by

(a) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$
(b) $y(x) = c_1e^{x} + c_2xe^{x} + c_3\cos(2x) + c_4\sin(2x)$
(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$
(d) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$
(e) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$

7. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 - 4x^3 \ln|x| = 27x^3$
(b) $3y^2 + 3x^3 \ln|x| = 27x^3$
(c) $y^3 + 3x^3 \ln|x| = 27x^3$
(d) $y^3 + 2x^3 \ln|x| = 27x^3$
(e) $9y + 3x^3 \ln|x| = 27x^3$

8. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 - xy^3 + y^2 = c$
(b) $x^2 + x^3 - xy^3 + y^2 = c$
(c) $x^2 + x^3 - xy^3 + 4y^2 = c$
(d) $x + x^2 + 3xy^3 + y^2 = c$
(e) $x + x^2 + 2xy^3 + y^2 = c$

9. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 3
(b) 0
(c) 1
(d) 2
(e) -2

10. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 2
(b) 4
(c) 3
(d) 1
(e) 0

11. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $16\,e$
(b) $14\,e$
(c) $22\,e$
(d) $20\,e$
(e) $17\,e$

12. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by $c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}\,c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{7}{30}x^{7/2}$
(b) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{1}{15}x^{7/2}$
(c) $x^{3/2} - \dfrac{1}{3}x^{5/2} + \dfrac{1}{30}x^{7/2}$
(d) $x^{3/2} - \dfrac{1}{4}x^{5/2} + \dfrac{1}{30}x^{7/2}$
(e) $x^{3/2} - \dfrac{1}{5}x^{5/2} + \dfrac{1}{30}x^{7/2}$

13. If the three vectors $V_1 = (4, 3, 2)$, $V_2 = (2, 5, -1)$ and $V_3 = (k, -4, 2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{5}{13}$

(b) $-\dfrac{4}{13}$

(c) $-\dfrac{4}{11}$

(d) $\dfrac{5}{13}$

(e) $\dfrac{3}{11}$

14. The general solution of the differential equation

$$\frac{dy}{dx} + y \tan x = \cos^2 x$$

is given by

(a) $y = (\sin^2 x + c)\ \cos x$

(b) $y = (\cos^2 x + c)\ \sin x$

(c) $y = (2 \sin x + c)\ \cos^2 x$

(d) $y = (\sin x + c)\ \cos x$

(e) $y = (\cos x + c)\ \sin x$

15. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

16. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) 0

(b) $\frac{5}{8}$

(c) $\frac{3}{4}$

(d) $\frac{7}{8}$

(e) $\frac{1}{2}$

17. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

18. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

19. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) -2
(b) 3
(c) 2
(d) 0
(e) -3

20. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 8
(b) 10
(c) 12
(d) 4
(e) 0

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1+2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2+t \\ t^2 \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t+1 \\ 2t+1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t+2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2 \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 4 **CODE 4**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 4
(b) 10
(c) 8
(d) 0
(e) 12

2. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

3. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\frac{7}{8}$

(b) $\frac{5}{8}$

(c) 0

(d) $\frac{3}{4}$

(e) $\frac{1}{2}$

4. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y = 0$ is given by

(a) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$

(b) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$

(d) $y(x) = c_1e^{x} + c_2xe^{x} + c_3\cos(2x) + c_4\sin(2x)$

(e) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

5. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

6. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by
$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)} c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \frac{1}{3}x^{5/2} + \frac{1}{30}x^{7/2}$

(b) $x^{3/2} - \frac{1}{5}x^{5/2} + \frac{1}{30}x^{7/2}$

(c) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{1}{15}x^{7/2}$

(d) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{7}{30}x^{7/2}$

(e) $x^{3/2} - \frac{1}{4}x^{5/2} + \frac{1}{30}x^{7/2}$

7. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 3
(b) 0
(c) 1
(d) 2
(e) 4

8. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{1-n}{n+2}c_n,\ n \geq 2$
(b) $c_{n+2} = \dfrac{2-n}{n+2}c_n,\ n \geq 3$
(c) $c_{n+1} = \dfrac{1-n}{n+2}c_n,\ n \geq 2$
(d) $c_{n+2} = \dfrac{3-n}{n+1}c_n,\ n \geq 4$
(e) $c_{n+1} = \dfrac{2-n}{n+2}c_n,\ n \geq 2$

9. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $x + \sin x \ln|\sec x + \tan x|$

(b) $x\cos x + \sin x \ln|\sec x + \tan x|$

(c) $x^2 + \cos x \ln|\sec x + \tan x|$

(d) $-1 + \sin x \ln|\sec x + \tan x|$

(e) $\cos x + x \ln|\sec x + \tan x|$

10. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

11. If the three vectors $V_1 = (4,3,2)$, $V_2 = (2,5,-1)$ and $V_3 = (k,-4,2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\frac{3}{11}$

(b) $\frac{5}{13}$

(c) $-\frac{4}{13}$

(d) $-\frac{4}{11}$

(e) $-\frac{5}{13}$

12. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1-2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

13. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) 2
(b) -3
(c) 3
(d) -2
(e) 0

14. The solution of the exact differential equation

$$(1 + 2x - y^3)\, dx + (2y - 3xy^2)\, dy = 0$$

is given by

(a) $x + x^2 + 2xy^3 + y^2 = c$
(b) $x^2 + x^3 - xy^3 + 4y^2 = c$
(c) $x^2 + x^3 - xy^3 + y^2 = c$
(d) $x + x^2 - xy^3 + y^2 = c$
(e) $x + x^2 + 3xy^3 + y^2 = c$

15. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 1
(b) 0
(c) 2
(d) 3
(e) -2

16. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix

$A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) 1
(b) -1
(c) 0
(d) -2
(e) 2

17. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

18. The general solution of the differential equation

$$\frac{dy}{dx} + y \tan x = \cos^2 x$$

is given by

(a) $y = (2\sin x + c)\,\cos^2 x$

(b) $y = (\sin^2 x + c)\,\cos x$

(c) $y = (\cos x + c)\,\sin x$

(d) $y = (\cos^2 x + c)\,\sin x$

(e) $y = (\sin x + c)\,\cos x$

19. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $22\,e$
(b) $14\,e$
(c) $16\,e$
(d) $20\,e$
(e) $17\,e$

20. The solution of the initial-value problem $xy^2 \dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 - 4x^3 \ln|x| = 27x^3$
(b) $9y + 3x^3 \ln|x| = 27x^3$
(c) $3y^2 + 3x^3 \ln|x| = 27x^3$
(d) $y^3 + 3x^3 \ln|x| = 27x^3$
(e) $y^3 + 2x^3 \ln|x| = 27x^3$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1 + 2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2 + t \\ t^2 \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t + 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t + 1 \\ 2t + 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 5 **CODE 5**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has __21__ questions.

Important Instructions:

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1. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

2. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) 3

(b) -2

(c) -3

(d) 0

(e) 2

3. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3 + x)y = 0$$

that corresponds to the indicial root $r = \frac{3}{2}$ is given by
$c_0 = 1$, $c_n = -\frac{1}{n(2n+1)}c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \frac{1}{3}x^{5/2} + \frac{1}{30}x^{7/2}$
(b) $x^{3/2} - \frac{1}{4}x^{5/2} + \frac{1}{30}x^{7/2}$
(c) $x^{3/2} - \frac{1}{5}x^{5/2} + \frac{1}{30}x^{7/2}$
(d) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{1}{15}x^{7/2}$
(e) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{7}{30}x^{7/2}$

4. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 2
(b) 1
(c) 3
(d) 0
(e) -2

5. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

6. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

7. If the three vectors $V_1 = (4,3,2)$, $V_2 = (2,5,-1)$ and $V_3 = (k,-4,2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\dfrac{5}{13}$

(b) $-\dfrac{4}{11}$

(c) $\dfrac{3}{11}$

(d) $-\dfrac{5}{13}$

(e) $-\dfrac{4}{13}$

8. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

9. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 + 2x^3 \ln|x| = 27x^3$
(b) $y^3 - 4x^3 \ln|x| = 27x^3$
(c) $9y + 3x^3 \ln|x| = 27x^3$
(d) $3y^2 + 3x^3 \ln|x| = 27x^3$
(e) $y^3 + 3x^3 \ln|x| = 27x^3$

10. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix

$A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) -2
(b) 1
(c) -1
(d) 2
(e) 0

11. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 10
(b) 12
(c) 8
(d) 4
(e) 0

12. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y = 0$ is given by

(a) $y(x) = c_1e^x + c_2xe^x + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$
(b) $y(x) = c_1e^x + c_2xe^x + c_3\cos(2x) + c_4\sin(2x)$
(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^x\cos(2x) + c_4e^x\sin(2x)$
(d) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$
(e) $y(x) = c_1e^x + c_2xe^x + c_3e^x\cos(2x) + c_4e^x\sin(2x)$

13. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

14. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $-1 + \sin x \ln|\sec x + \tan x|$

(b) $\cos x + x \ln|\sec x + \tan x|$

(c) $x \cos x + \sin x \ln|\sec x + \tan x|$

(d) $x^2 + \cos x \ln|\sec x + \tan x|$

(e) $x + \sin x \ln|\sec x + \tan x|$

15. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\sin^2 x + c)\,\cos x$

(b) $y = (2\sin x + c)\,\cos^2 x$

(c) $y = (\cos x + c)\,\sin x$

(d) $y = (\sin x + c)\,\cos x$

(e) $y = (\cos^2 x + c)\,\sin x$

16. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $20\,e$

(b) $17\,e$

(c) $22\,e$

(d) $16\,e$

(e) $14\,e$

17. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{2-n}{n+2} c_n,\ n \geq 3$

(b) $c_{n+1} = \dfrac{2-n}{n+2} c_n,\ n \geq 2$

(c) $c_{n+2} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$

(d) $c_{n+2} = \dfrac{3-n}{n+1} c_n,\ n \geq 4$

(e) $c_{n+1} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$

18. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 1

(b) 4

(c) 3

(d) 0

(e) 2

19. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 + 3xy^3 + y^2 = c$
(b) $x^2 + x^3 - xy^3 + 4y^2 = c$
(c) $x^2 + x^3 - xy^3 + y^2 = c$
(d) $x + x^2 - xy^3 + y^2 = c$
(e) $x + x^2 + 2xy^3 + y^2 = c$

20. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\dfrac{3}{4}$
(b) $\dfrac{5}{8}$
(c) $\dfrac{7}{8}$
(d) 0
(e) $\dfrac{1}{2}$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t + 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t + 1 \\ 2t + 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1 + 2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2 + t \\ t^2 \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 6 **CODE 6**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\cos x + c)\ \sin x$
(b) $y = (\sin^2 x + c)\ \cos x$
(c) $y = (2\sin x + c)\ \cos^2 x$
(d) $y = (\sin x + c)\ \cos x$
(e) $y = (\cos^2 x + c)\ \sin x$

2. The number of regular singular points of the differential equation

$$x^3(x^2 - 25)(x-2)^2 y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 2
(b) 0
(c) 3
(d) 4
(e) 1

3. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $-1 + \sin x \ln|\sec x + \tan x|$

(b) $x + \sin x \ln|\sec x + \tan x|$

(c) $x \cos x + \sin x \ln|\sec x + \tan x|$

(d) $x^2 + \cos x \ln|\sec x + \tan x|$

(e) $\cos x + x \ln|\sec x + \tan x|$

4. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

5. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) -2
(b) 0
(c) 2
(d) 3
(e) -3

6. If the three vectors $V_1 = (4, 3, 2)$, $V_2 = (2, 5, -1)$ and $V_3 = (k, -4, 2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\dfrac{3}{11}$
(b) $-\dfrac{5}{13}$
(c) $-\dfrac{4}{13}$
(d) $\dfrac{5}{13}$
(e) $-\dfrac{4}{11}$

7. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $9y + 3x^3 \ln|x| = 27x^3$
(b) $y^3 - 4x^3 \ln|x| = 27x^3$
(c) $y^3 + 2x^3 \ln|x| = 27x^3$
(d) $y^3 + 3x^3 \ln|x| = 27x^3$
(e) $3y^2 + 3x^3 \ln|x| = 27x^3$

8. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y = 0$ is given by

(a) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\,\cos(x) + c_5e^{-2x}\,\sin x$
(b) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{x}\,\cos(2x) + c_4e^{x}\,\sin(2x)$
(c) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{x}\,\cos(2x) + c_4e^{x}\,\sin(2x)$
(d) $y(x) = c_1e^{x} + c_2xe^{x} + c_3\,\cos(2x) + c_4\,\sin(2x)$
(e) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{-x}\,\cos(2x) + c_4e^{-x}\,\sin(2x)$

9. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3+x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by
$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}\,c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{7}{30}x^{7/2}$
(b) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{1}{15}x^{7/2}$
(c) $x^{3/2} - \frac{1}{5}x^{5/2} + \frac{1}{30}x^{7/2}$
(d) $x^{3/2} - \frac{1}{3}x^{5/2} + \frac{1}{30}x^{7/2}$
(e) $x^{3/2} - \frac{1}{4}x^{5/2} + \frac{1}{30}x^{7/2}$

10. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $17\,e$
(b) $20\,e$
(c) $14\,e$
(d) $16\,e$
(e) $22\,e$

11. The solution of the exact differential equation

$$(1 + 2x - y^3)\, dx + (2y - 3xy^2)\, dy = 0$$

is given by

(a) $x + x^2 - xy^3 + y^2 = c$
(b) $x^2 + x^3 - xy^3 + 4y^2 = c$
(c) $x + x^2 + 3xy^3 + y^2 = c$
(d) $x^2 + x^3 - xy^3 + y^2 = c$
(e) $x + x^2 + 2xy^3 + y^2 = c$

12. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) 0
(b) 1
(c) -1
(d) -2
(e) 2

13. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

14. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) 0

(b) $\frac{5}{8}$

(c) $\frac{7}{8}$

(d) $\frac{3}{4}$

(e) $\frac{1}{2}$

15. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 3
(b) 1
(c) -2
(d) 2
(e) 0

16. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$
(b) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$
(c) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$
(d) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$
(e) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

17. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

18. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

19. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 4
(b) 12
(c) 10
(d) 8
(e) 0

20. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+1} = \dfrac{1-n}{n+2} c_n$, $n \geq 2$
(b) $c_{n+2} = \dfrac{3-n}{n+1} c_n$, $n \geq 4$
(c) $c_{n+2} = \dfrac{2-n}{n+2} c_n$, $n \geq 3$
(d) $c_{n+1} = \dfrac{2-n}{n+2} c_n$, $n \geq 2$
(e) $c_{n+2} = \dfrac{1-n}{n+2} c_n$, $n \geq 2$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t+2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1+2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2+t \\ t^2 \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t+2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2 \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t+1 \\ 2t+1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2+2t \\ t^2+2t \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 7 **CODE 7**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

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8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The solution of the initial-value problem $xy^2\dfrac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 - 4x^3 \ln|x| = 27x^3$
(b) $3y^2 + 3x^3 \ln|x| = 27x^3$
(c) $y^3 + 3x^3 \ln|x| = 27x^3$
(d) $y^3 + 2x^3 \ln|x| = 27x^3$
(e) $9y + 3x^3 \ln|x| = 27x^3$

2. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 3
(b) 1
(c) -2
(d) 2
(e) 0

3. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y''+3x(x-2)y'+7(x+5)y=0$$

is

(a) 1
(b) 2
(c) 3
(d) 0
(e) 4

4. If $y=\sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y''+xy'-y=0$ about the ordinary point $x=0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2}=\dfrac{1-n}{n+2}c_n,\ n\geq 2$
(b) $c_{n+2}=\dfrac{2-n}{n+2}c_n,\ n\geq 3$
(c) $c_{n+2}=\dfrac{3-n}{n+1}c_n,\ n\geq 4$
(d) $c_{n+1}=\dfrac{1-n}{n+2}c_n,\ n\geq 2$
(e) $c_{n+1}=\dfrac{2-n}{n+2}c_n,\ n\geq 2$

5. A particular solution of the differential equation $y'' + y = \sec^2 x$ is given by $y_p(x) =$

(a) $-1 + \sin x \ln|\sec x + \tan x|$

(b) $x^2 + \cos x \ln|\sec x + \tan x|$

(c) $x + \sin x \ln|\sec x + \tan x|$

(d) $\cos x + x \ln|\sec x + \tan x|$

(e) $x \cos x + \sin x \ln|\sec x + \tan x|$

6. If the three vectors $V_1 = (4, 3, 2)$, $V_2 = (2, 5, -1)$ and $V_3 = (k, -4, 2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{4}{13}$

(b) $\dfrac{3}{11}$

(c) $-\dfrac{4}{11}$

(d) $\dfrac{5}{13}$

(e) $-\dfrac{5}{13}$

7. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y=0$ is given by

(a) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{-2x}\cos(x) + c_5e^{-2x}\sin x$

(b) $y(x) = c_1e^{-x} + c_2xe^{-x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

(c) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{x}\cos(2x) + c_4e^{x}\sin(2x)$

(d) $y(x) = c_1e^{x} + c_2xe^{x} + c_3e^{-x}\cos(2x) + c_4e^{-x}\sin(2x)$

(e) $y(x) = c_1e^{x} + c_2xe^{x} + c_3\cos(2x) + c_4\sin(2x)$

8. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2\begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3\begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) -3

(b) -2

(c) 3

(d) 2

(e) 0

9. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$

(d) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$

(e) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

10. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

$$y_p = Axe^{3x} + B\cos x + C\sin x,$$

where $5B + 3A + 5C =$

(a) 4

(b) 12

(c) 10

(d) 0

(e) 8

11. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\cos x + c)\ \sin x$

(b) $y = (\sin^2 x + c)\ \cos x$

(c) $y = (\cos^2 x + c)\ \sin x$

(d) $y = (2\sin x + c)\ \cos^2 x$

(e) $y = (\sin x + c)\ \cos x$

12. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

13. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\frac{7}{8}$
(b) $\frac{3}{4}$
(c) $\frac{1}{2}$
(d) 0
(e) $\frac{5}{8}$

14. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$
(b) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$
(c) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$
(d) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$
(e) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

15. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 + 2xy^3 + y^2 = c$
(b) $x + x^2 - xy^3 + y^2 = c$
(c) $x + x^2 + 3xy^3 + y^2 = c$
(d) $x^2 + x^3 - xy^3 + 4y^2 = c$
(e) $x^2 + x^3 - xy^3 + y^2 = c$

16. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $14\,e$
(b) $16\,e$
(c) $20\,e$
(d) $22\,e$
(e) $17\,e$

17. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3+x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by

$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}\, c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \frac{1}{4}x^{5/2} + \frac{1}{30}x^{7/2}$

(b) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{1}{15}x^{7/2}$

(c) $x^{3/2} - \frac{1}{5}x^{5/2} + \frac{1}{30}x^{7/2}$

(d) $x^{3/2} - \frac{1}{3}x^{5/2} - \frac{7}{30}x^{7/2}$

(e) $x^{3/2} - \frac{1}{3}x^{5/2} + \frac{1}{30}x^{7/2}$

18. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix

$A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

(a) 1

(b) 2

(c) 0

(d) -2

(e) -1

19. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

20. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1 + 2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2 + t \\ t^2 \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t + 1 \\ 2t + 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t + 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 8 **CODE 8**

Math 208
Final Exam
Term 252
12 May 2026
Net Time Allowed: 120 Minutes

Name	
ID	**Sec**

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $K = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 2$ of the matrix

   $A = \begin{bmatrix} 0 & -2 & -1 \\ -3 & -1 & -3 \\ 2 & 2 & 3 \end{bmatrix}$, then $\alpha =$

   (a) 2
   (b) 0
   (c) 1
   (d) -2
   (e) -1

2. By using the method of undetermined coefficients, a particular solution of the differential equation $y'' - 3y' = 8e^{3x} + 4\sin x$ is given by

   $$y_p = Axe^{3x} + B\cos x + C\sin x,$$

   where $5B + 3A + 5C =$

   (a) 8
   (b) 10
   (c) 0
   (d) 12
   (e) 4

3. The general solution of differential equation $(D-1)^2(D^2-2D+5)\,y=0$ is given by

   (a) $y(x)=c_1e^x+c_2xe^x+c_3e^x\,\cos(2x)+c_4e^x\,\sin(2x)$
   (b) $y(x)=c_1e^{-x}+c_2xe^{-x}+c_3e^{-2x}\,\cos(x)+c_5e^{-2x}\,\sin x$
   (c) $y(x)=c_1e^x+c_2xe^x+c_3\,\cos(2x)+c_4\,\sin(2x)$
   (d) $y(x)=c_1e^x+c_2xe^x+c_3e^{-x}\,\cos(2x)+c_4e^{-x}\,\sin(2x)$
   (e) $y(x)=c_1e^{-x}+c_2xe^{-x}+c_3e^x\,\cos(2x)+c_4e^x\,\sin(2x)$

4. A particular solution of the differential equation $y''+y=\sec^2 x$ is given by $y_p(x)=$

   (a) $x^2+\cos x\ln|\sec x+\tan x|$
   (b) $x\cos x+\sin x\ln|\sec x+\tan x|$
   (c) $\cos x+x\ln|\sec x+\tan x|$
   (d) $-1+\sin x\ln|\sec x+\tan x|$
   (e) $x+\sin x\ln|\sec x+\tan x|$

5. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then $e^{At} = \begin{bmatrix} e^t & f(t) & h(t) \\ 0 & e^t & g(t) \\ 0 & 0 & e^t \end{bmatrix}$, where $f(1) + g(1) + h(1) =$

(a) $22\,e$

(b) $17\,e$

(c) $16\,e$

(d) $14\,e$

(e) $20\,e$

6. If the rank of the matrix $\begin{bmatrix} 1 & 1 & 3 & 3 & 0 \\ -1 & 0 & -2 & -1 & 1 \\ 2 & 3 & 7 & 8 & K \\ -2 & 4 & 0 & 6 & 6 \end{bmatrix}$ is equal to 2, then $K =$

(a) 2

(b) 3

(c) -2

(d) 1

(e) 0

7. A fundamental matrix of the system $X' = \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} 5e^{4t} & e^{-t} \\ e^{4t} & e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} 5e^{4t} & 0 \\ e^{4t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{4t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

(e) $\begin{bmatrix} 5e^{4t} & 0 \\ 2e^{4t} & e^{-t} \end{bmatrix}$

8. The number of regular singular points of the differential equation

$$x^3(x^2-25)(x-2)^2y'' + 3x(x-2)y' + 7(x+5)y = 0$$

is

(a) 2
(b) 0
(c) 3
(d) 4
(e) 1

9. The solution of the exact differential equation

$$(1 + 2x - y^3)\,dx + (2y - 3xy^2)\,dy = 0$$

is given by

(a) $x + x^2 + 3xy^3 + y^2 = c$
(b) $x^2 + x^3 - xy^3 + 4y^2 = c$
(c) $x + x^2 + 2xy^3 + y^2 = c$
(d) $x^2 + x^3 - xy^3 + y^2 = c$
(e) $x + x^2 - xy^3 + y^2 = c$

10. Consider the system $X' = AX$, $X(0) = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, where A is a 2×2 matrix with real entries. If A has an eigenvalue $\lambda = 5 + 2i$ with corresponding eigenvector $K = \begin{bmatrix} -1 \\ 1 - 2i \end{bmatrix}$, then $X\left(\frac{\pi}{2}\right) =$

(a) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$
(b) $\begin{bmatrix} -1 \\ 8 \end{bmatrix} e^{\frac{5\pi}{2}}$
(c) $\begin{bmatrix} 2 \\ -8 \end{bmatrix} e^{\frac{5\pi}{2}}$
(d) $\begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{\frac{5\pi}{2}}$
(e) $\begin{bmatrix} 2 \\ -6 \end{bmatrix} e^{\frac{5\pi}{2}}$

11. The largest indicial root at $x = 0$ for the differential equation

$$8xy'' + y' + 2y = 0$$

is

(a) $\frac{3}{4}$
(b) 0
(c) $\frac{1}{2}$
(d) $\frac{5}{8}$
(e) $\frac{7}{8}$

12. The solution of the initial-value problem $xy^2\frac{dy}{dx} = y^3 - x^3$, $y(1) = 3$ is given by

(a) $y^3 + 2x^3 \ln|x| = 27x^3$
(b) $y^3 + 3x^3 \ln|x| = 27x^3$
(c) $9y + 3x^3 \ln|x| = 27x^3$
(d) $y^3 - 4x^3 \ln|x| = 27x^3$
(e) $3y^2 + 3x^3 \ln|x| = 27x^3$

13. The general solution of $X' = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{11t} + c_2 \begin{bmatrix} 1 \\ \beta \\ 0 \end{bmatrix} e^{8t} + c_3 \begin{bmatrix} 1 \\ 0 \\ \alpha \end{bmatrix} e^{8t},$$

then $\alpha - \beta =$

(a) -2
(b) 2
(c) -3
(d) 3
(e) 0

14. Consider the non-homogeneous system $X' = AX + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. If the general solution of the associated homogeneous system is

$$X_c = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} e^{2t},$$

then a particular solution $X_p =$

(a) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

15. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power series solution of the differential equation $(x^2+1)y'' + xy' - y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation:

(a) $c_{n+2} = \dfrac{2-n}{n+2} c_n,\ n \geq 3$

(b) $c_{n+2} = \dfrac{3-n}{n+1} c_n,\ n \geq 4$

(c) $c_{n+2} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$

(d) $c_{n+1} = \dfrac{2-n}{n+2} c_n,\ n \geq 2$

(e) $c_{n+1} = \dfrac{1-n}{n+2} c_n,\ n \geq 2$

16. The general solution of the system

$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

is given by

(a) $X = c_1 \begin{bmatrix} 5 \\ 1 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(b) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{6t}$

(c) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t}$

(d) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{8t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-10t}$

(e) $X = c_1 \begin{bmatrix} 5 \\ 2 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{10t}$

17. If the recurrence relation of the Frobenius series solution of the differential equation

$$2x^2y'' - 3xy' + (3+x)y = 0$$

that corresponds to the indicial root $r = \dfrac{3}{2}$ is given by
$c_0 = 1$, $c_n = -\dfrac{1}{n(2n+1)}\, c_{n-1}$, $n \geq 1$, then the first three terms in the solution are given by

(a) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{7}{30}x^{7/2}$

(b) $x^{3/2} - \dfrac{1}{4}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(c) $x^{3/2} - \dfrac{1}{5}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(d) $x^{3/2} - \dfrac{1}{3}x^{5/2} + \dfrac{1}{30}x^{7/2}$

(e) $x^{3/2} - \dfrac{1}{3}x^{5/2} - \dfrac{1}{15}x^{7/2}$

18. If the three vectors $V_1 = (4,3,2)$, $V_2 = (2,5,-1)$ and $V_3 = (k,-4,2)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\dfrac{5}{13}$

(b) $\dfrac{3}{11}$

(c) $-\dfrac{4}{13}$

(d) $-\dfrac{4}{11}$

(e) $-\dfrac{5}{13}$

19. For $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$, if P is a diagonalizing matrix such that $P^{-1}AP = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$, then

(a) $P = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$

(e) $P = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix}$

20. The general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

is given by

(a) $y = (\sin^2 x + c)\ \cos x$

(b) $y = (2\sin x + c)\ \cos^2 x$

(c) $y = (\cos x + c)\ \sin x$

(d) $y = (\cos^2 x + c)\ \sin x$

(e) $y = (\sin x + c)\ \cos x$

21. The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ has an eigenvalue $\lambda = 1$ of defect 2. If we choose $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ such that $(A - I)^3 V_3 = 0$ and $(A - I)^2 V_3 \neq 0$, then the general solution of the system $X' = AX$ is

(a) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(b) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t \\ 1 + 2t \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ t^2 + t \\ t^2 \end{bmatrix} \right) e^t$

(c) $X = \left(c_1 \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 \end{bmatrix} \right) e^t$

(d) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2t + 1 \\ 2t + 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

(e) $X = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2t + 2 \\ 2t + 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t^2 + 2t \\ t^2 + 2t \end{bmatrix} \right) e^t$

Q	MASTER	1	2	3	4	5	6	7	8
1	A	A 4	E 18	D 12	E 10	E 8	D 3	C 19	E 18
2	A	A 14	B 19	E 18	D 5	D 20	C 6	B 4	D 10
3	A	E 7	D 16	E 11	A 2	A 14	A 11	C 6	A 15
4	A	B 11	C 3	E 9	E 15	B 4	D 17	A 7	D 11
5	A	D 3	B 20	A 7	B 9	A 5	B 20	A 11	A 16
6	A	C 16	A 4	D 15	A 14	D 17	C 13	A 13	D 4
7	A	E 15	D 6	C 19	A 6	E 13	D 19	C 15	B 12
8	A	C 2	E 5	A 1	A 7	D 12	B 15	E 20	C 6
9	A	D 6	E 12	C 4	D 11	E 19	D 14	D 17	E 1
10	A	E 9	A 8	C 6	E 8	C 18	E 16	B 10	C 17
11	A	A 17	A 11	C 16	C 13	B 10	A 1	E 3	E 2
12	A	A 18	B 9	C 14	E 17	E 15	C 18	E 5	B 19
13	A	D 19	A 10	B 13	E 20	B 9	B 8	A 2	E 20
14	A	C 10	A 7	D 3	D 1	A 11	C 2	B 12	A 9
15	A	B 20	A 13	C 5	A 4	D 3	B 4	B 1	C 7
16	A	A 5	E 2	D 2	B 18	C 16	B 12	D 16	C 8
17	A	D 12	B 14	E 17	D 12	C 7	E 9	E 14	D 14
18	A	C 13	B 17	B 8	E 3	C 6	E 5	E 18	C 13
19	A	D 1	B 1	D 20	A 16	D 1	B 10	E 9	A 5
20	A	B 8	B 15	C 10	D 19	C 2	E 7	C 8	E 3
21	A	A 21	C 21	E 21	C 21	C 21	C 21	D 21	C 21

Answer Counts

V	A	B	C	D	E
1	6	3	4	5	3
2	6	7	2	2	4
3	2	2	7	5	5
4	6	2	2	5	6
5	3	3	6	5	4
6	2	6	5	4	4
7	4	4	4	3	6
8	4	2	6	4	5

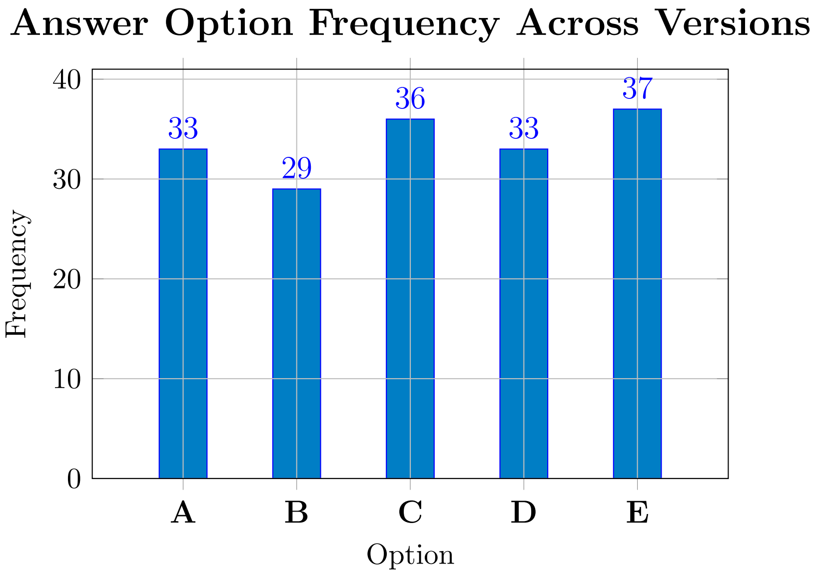

Figure 1: Frequency of Each Answer Option (Excluding MASTER)