King Fahd University of Petroleum & Minerals
Department of Mathematics

Semester (252) Math 333 First Major Exam February 24, 2026 8:30-10:00 pm

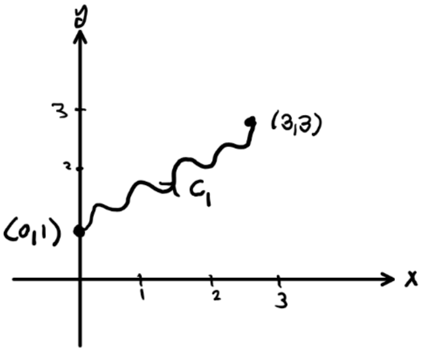
Name: ____ I.D. # ____________________

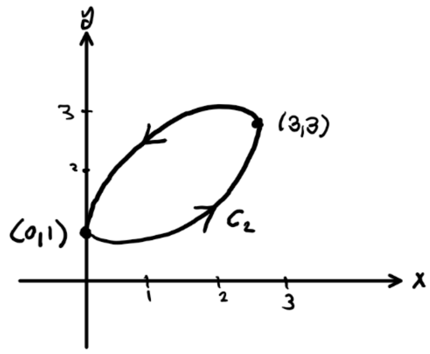
Points (Q1 = 2, Q2 = 4, Q3 = 5, Q4 = 5, Q5 = 5, Q6 = 4)

1. Find the length of the curve traced by $\vec{r}(t) = \sqrt{2}\, t\, i + e^{t}\, j + e^{-t}\, k,\ \ 0 \le t \le 1.$

2. Consider the vector field $\vec{F}(x,y) = \langle 2xy + y^2, x^2 + 2xy + 1 \rangle$.

   a) Show that the field is conservative and find a potential function for it.
   b) Evaluate $\int_{(0,1)}^{(3,3)} \vec{F}.d\vec{r}$ along the curve C_1 shown, and along the curve C_2 shown below.

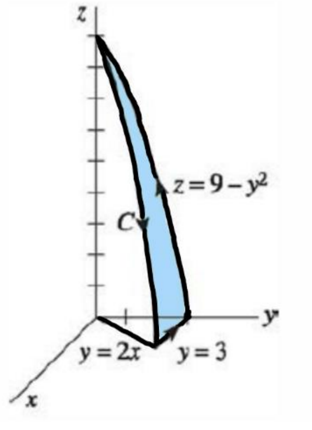




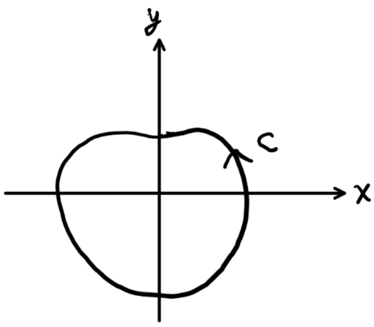
3. ***Use Green's theorem*** to evaluate $\oint_C y^2 x\, dx + 3\cos y\ dy$, where C is the positively oriented boundary of the region in the first quadrant determined by the curves $y = x^2$ and $y = x^3$.

4. Consider the vector field $\vec{F}(x,y,z) = y^2\, i + x\, z^3\, j + (z-1)^2\, k$. ***Use the Divergence theorem*** to find the outward flux $\iint_S \vec{F} \cdot \vec{n}\, dS$ across the surface of the region that is enclosed by $x^2 + y^2 = 4$ and the planes $z = 1$ and $z = 2$.

5. Consider the vector field $\vec{F}(x,y,z) = \langle x^2\, y, x + y^2, x\, y^2\, z \rangle$, and the shaded surface shown. ***Use Stokes' theorem*** to evaluate $\oint_C \vec{F}.d\vec{r}$.



6. Consider the vector field $\vec{F}(x,y,z) = (3x - \frac{y}{x^2+y^2})\, i + (-y + \frac{x}{x^2+y^2})\, j$. Evaluate $\oint_C \vec{F}.d\vec{r}$, where C is the positively oriented limaçon shown.



Formulas

$$s = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\, dt = \int_a^b \|\mathbf{r}'(t)\|\, dt.$$

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C (\mathbf{F} \cdot \mathbf{T})\, dS = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS,$$

$$\iint_S (\mathbf{F} \cdot \mathbf{n})\, dS = \iiint_D \text{div } \mathbf{F}\, dV.$$