King Fahd University of Petroleum & Minerals
Department of Mathematics

Semester (252) Math 333 Second Major Exam April 13, 2026 7:00-9:00 pm

Name: ______________________________ I.D. # ____________________

1. Find
   a) $\mathcal{L}\{t\, e^{-t} \cos t\}$.
   b) $\mathcal{L}^{-1}\left\{\frac{S+1}{(S-1)(S^2+1)}\right\}$.

2. Solve the integral equation $y(t) - \int_0^t (t-\tau) y(\tau) d\tau = 1$

3. Consider $f(t) = \begin{cases} 0 & if \quad 0 \le t < \pi \\ 1 & if \quad \pi \le t < 2\pi \\ 0 & if \quad t \ge 2\pi \end{cases}$. Use Laplace transform to solve the initial value problem $y'' = f(t),\ y(0) = y'(0) = 0$. (Write $f(t)$ in terms of Heaviside functions.)

4. Solve $y'' = \delta(t-1),\ y(0) = y(2) = 0$.

5. Suppose the differential equation $y'' - 2x\, y' + 2n\, y = 0$ has the solutions $H_n(x)$. Find the weight $w(x)$ such that $\int_{-\infty}^{\infty} w(x)\, H_n(x)\, H_m(x)\, dx = 0 \quad if\ n \ne m.$

6. Expand $f(x) = x^2,\ 0 < x < 1$ in terms of $J_2(\alpha\, x)$ where $J_2(\alpha) = 0$. Your solution must not contain any integrals.

7. Consider the function $f(x) = \begin{cases} 0 & if \quad -1 < x \le 0 \\ x & if \quad 0 < x < 1 \end{cases}$.
   a) Find a Fourier series for $f(x)$.
   b) What value does the series converge to when $x = -1$ and when $x = 0$?
   c) Evaluate $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
Full Marks	8	7	8	7	3	7	10	50
Score								

You may need $\int x\cos(\alpha x)dx = \frac{\cos(\alpha x)}{\alpha^2} + \frac{x\sin(\alpha x)}{\alpha} + c$, $\int x\sin(\alpha x)dx = \frac{\sin(\alpha x)}{\alpha^2} - \frac{x\cos(\alpha x)}{\alpha} + c$

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots f^{(n-1)}(0)$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{f(t-a)\,\mathcal{U}(t-a)\} = e^{-as}F(s)$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$f * g = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

$$\mathcal{L}\{f * g\} = F(s)\, G(s)$$

$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{sT}}\int_0^T e^{-st} f(t)\, dt$, when $f(t)$ is piecewise continuous on $[0,\infty]$ periodic with period T.

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{p} + b_n \sin\frac{n\pi x}{p}\right]$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}dx$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi x}{p}dx$$

The **Fourier–Bessel series** of a function f defined on the interval $(0, b)$ is given by

(i)

$$f(x) = \sum_{i=1}^{\infty} c_i J_n(\alpha_i x)$$

$$c_i = \frac{2}{b^2 J_{n+1}^2(\alpha_i b)}\int_0^b x J_n(\alpha_i x) f(x)\, dx,$$

where the α_i are defined by $J_n(\alpha b) = 0$.

(ii)

$$f(x) = \sum_{i=1}^{\infty} c_i J_n(\alpha_i x)$$

$$c_i = \frac{2\alpha_i^2}{(\alpha_i^2 b^2 - n^2 + h^2) J_n^2(\alpha_i b)}\int_0^b x J_n(\alpha_i x) f(x)\, dx,$$

where the α_i are defined by $hJ_n(\alpha b) + \alpha b J_n'(\alpha b) = 0$.

(iii)

$$f(x) = c_1 + \sum_{i=2}^{\infty} c_i J_0(\alpha_i x)$$

$$c_1 = \frac{2}{b^2}\int_0^b x f(x)\, dx, \quad c_i = \frac{2}{b^2 J_0^2(\alpha_i b)}\int_0^b x J_0(\alpha_i x) f(x)\, dx,$$

where the α_i are defined by $J_0'(\alpha b) = 0$.

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

$$\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x).$$

The **Fourier–Legendre series** of a function f defined on the interval $(-1, 1)$ is given by

$$f(x) = \sum_{n=0}^{\infty} c_n P_n(x),$$

where

$$c_n = \frac{2n+1}{2}\int_{-1}^{1} f(x) P_n(x)\, dx.$$