King Fahd University of Petroleum & Minerals
College of Computing and Mathematics
Department of Mathematics

COURSE SYLLABUS
MATH 434 – Calculus of Variations and Optimal Control

1. Course Information

Course Code	MATH 434	**Credit Hours**	3
Course Title	Calculus of Variations and Optimal Control	**Pre-requisite**	MATH 208 (Differential Equations and Linear Algebra)
Instructor	Dr. Abdullah Shah	**Office Location**	Building 5, Room# 419
Office Hours	Sunday & Tuesday, 10:00–11:00 AM, or by appointment	**Email**	abdullah.shah.1@kfupm.edu.sa

2. Course Description

Introduction to the calculus of variations. Euler-Lagrange equation, Weierstrass, Legendre and Jacobi necessary conditions. Formulation of optimal control problems. Bolza, Mayer and Lagrange formulations. Variational approach to optimal control. Pontryagin maximum principle.

3. Course Objectives

1. Introduce students to the fundamental concepts of the calculus of variations and enable them to derive first-order optimality conditions for variational and optimal control problems.

2. Familiarize students with the classical necessary conditions of the calculus of variations, including the Euler–Lagrange, Weierstrass, Legendre, and Jacobi conditions.

3. Develop students' ability to formulate optimal control problems in Bolza, Lagrange, and Mayer forms.

4. Enable students to transform optimal control problems from one standard formulation to another.

5. Guide students in constructing the Hamiltonian function associated with an optimal control problem.

6. Develop students' understanding of the Pontryagin Maximum Principle and its application to deriving necessary optimality conditions.

4. Course Learning Outcomes (CLOs)

CLO1. Derive, from first principles, necessary conditions for an extremum, including multivariable cases.

CLO2. Solve Euler–Lagrange equations for calculus of variations problems.

CLO3. Apply the Bolza, Mayer, and Lagrange formulations to optimal control problems.

CLO4. Apply the variational approach to optimal control problems.

CLO5. Apply the Pontryagin Maximum Principle to optimal control problems.

CLO6. Apply continuous-time dynamic programming using the Hamilton–Jacobi–Bellman equation.

CLO7. Solve linear quadratic regulator problems.

5. Course Topics

Week #	Topics
1	Finite and infinite dimensional optimization, Brachistochrone and isoperimetric problems
2	The simplest variational problem and Euler-Lagrange equation
3	The simplest variational problem and Euler-Lagrange equation (Continue)
4	Invariance of Euler-Lagrange equation fixed-endpoint problem for *n* unknown functions
5	Invariance of Euler-Lagrange equation fixed- endpoint problem for *n* unknown functions (Continue)
6	The general variation of a functional endpoints lying on two different curves
7	The general variation of functional endpoints lying on two different curves (Continue)
8	Second variation; Legendre and Jacobi necessary conditions.
9	Broken extremals, the Weierstrass-Erdmann conditions
10	The Pontryagin maximum principle
11	The Pontryagin maximum principle (Continue)
12	Dynamic programming in continuous time and the Hamilton Jacobi-Bellman theory
13	Optimal control to target curves Time-optimal control
14	Fuel-optimal control Linear quadratic regulator
15	Fuel-optimal control Linear quadratic regulator (Continue)

6. Assessment and Grading

The final course grade points will be based on the following distribution:

Assessment Component	Weight
Exam I	20%
Exam II	20%
Assignments and Quizzes	15% (Assignments: 10%; Quizzes: 5%)
Project	10%
Final Exam	35%
Total	**100%**

Letter Grades: Final letter grades will be assigned according to a grading curve, which will be determined based on the overall performance of all students enrolled in the course. The exact cutoffs may change from one term to another, depending on how the scores are distributed.

Exam Questions: Exam questions will be similar in style and level of difficulty to the examples and exercises provided in the textbook/ and lecture materials.

7. Learning Resources

1. Mark Kot (2014), ***A First Course in the Calculus of Variations***, American Mathematical Society.
2. Liberzon, D. (2012), ***Calculus of Variations and Optimal Control Theory: A Concise Introduction***. Princeton University Press.
3. Pinch, E. R. (1993). Optimal Control and the Calculus of Variations. Oxford University Press.
4. Troutman, J. L. (1996). Variational Calculus and Optimal Control: Optimization with Elementary Convexity (2nd ed.) Springer.

8. KFUPM Honor Code

Students of this course are expected to maintain the highest level of academic integrity and ethics. A KFUPM student does not cheat or tolerate those who do. Every student is expected to submit work that is entirely their own, to properly cite all sources used, and to refrain from any form of academic misconduct.

9. Plagiarism and Use of Generative AI

Use of another person's ideas, processes, results, or words requires appropriate credit or reference to the author. Use of Generative AI tools is subject to the instructor's authorization. Submitting the same work in more than one course without the instructor's permission is not allowed.

10. Class Attendance

Students are expected to arrive on time for class, consistent with expectations in professional settings. Recording attendance for another student, signing another student's attendance, or asking another student to record yours will be reported for investigation and may result in severe consequences for both students involved.

11. Examination Policies

Students are expected to adhere to KFUPM policies for all examinations in this course, including quizzes, major exams, midterms, and final exams. The following policies are enacted in this course during examinations:

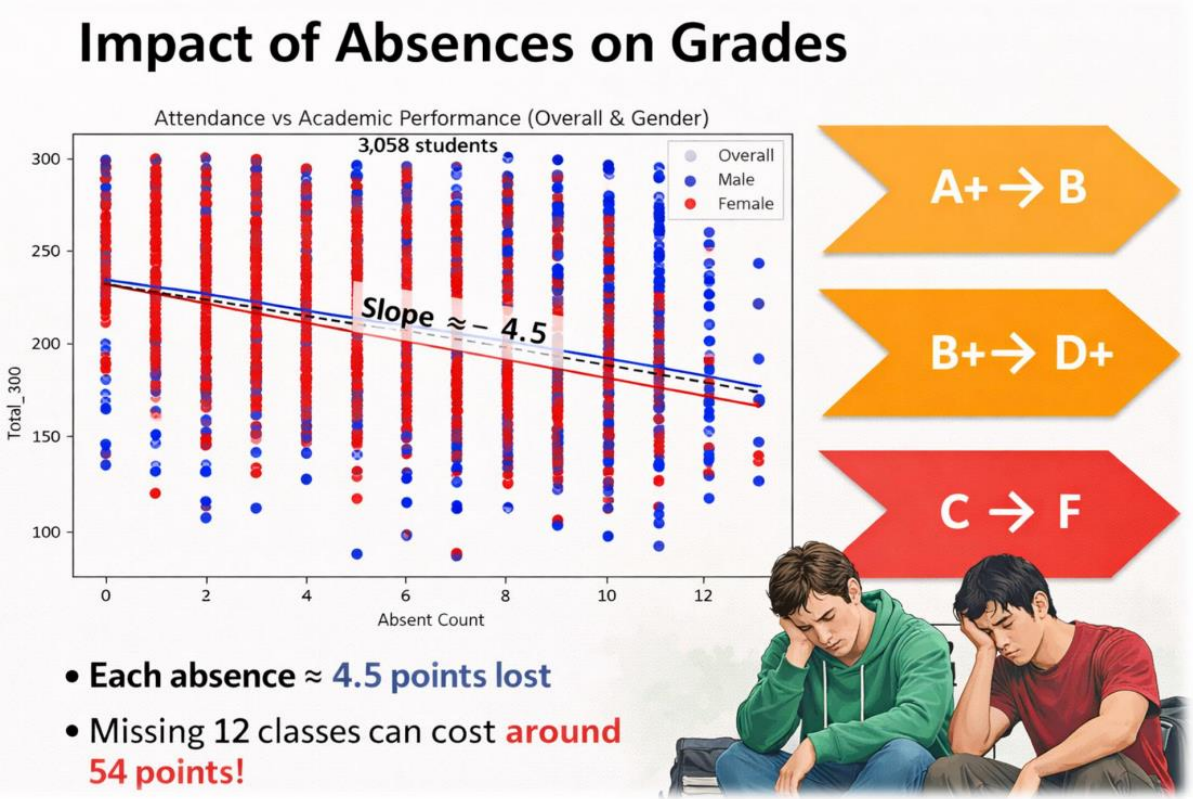
- Students are expected to arrive at the examination hall on time with proper University ID.
- Students are seated according to the instructor's seat assignment and may request a seat change only for legitimate reasons.
- Late arrival at the exam may result in the student being denied admission. Once any student has begun finishing the exam, no further admission will be allowed under any circumstance.
- Students are not permitted to leave the examination room to use the restroom during an exam, except in rare cases involving a documented medical condition (e.g., diabetes, kidney disease). Any excused restroom leave will be fully supervised by a same-gender proctor.
- Students must leave all smart devices outside the examination hall, including smartphones, smartwatches, earbuds, and unauthorized calculators. Possession of any unauthorized device will be considered a cheating attempt and will be handled accordingly.
- Cheating attempts will be reported according to KFUPM policy and will result in a DN grade in the course, along with potential further penalties upon investigation.

12. Use of Mobile Devices in Class:

The use of mobile phones during class is strictly prohibited. Students who wish to use laptops or tablets for notetaking must first obtain permission from the instructor. Any violation will result in a penalty determined by the instructor. Academic Integrity: All KFUPM academic integrity policies apply to this course. For details, students should refer to the Undergraduate Bulletin available on the Registrar's website.

13. Academic Integrity Statement

All KFUPM academic integrity policies apply to this course. For details, students should refer to the Undergraduate Bulletin available on the Registrar's website.



Office Hours:

Courses	SUNDAY	TUESDAY	THURSDAY
MATH434	11:00-11:50	11:00-11:50	11:00-11:50

Remark: Students can also ask for an appointment by email or MS TEAMS.

Good Luck