KFUPM

Math513 – Second Exam

Duration: 2 hours

Name: ____________________ **ID:** ____________________

Instructions

- Write clearly and show all essential steps. You may lose points for messy work.
- Justify your answers. You may lose points for unsupported answers.

Question 1. **(10 points)**

Find the first three nonzero terms in the Legendre-Fourier expansion of

$$f(x) = \begin{cases} -1, & -1 < x < 0, \\ x, & 0 < x < 1. \end{cases}$$

Question 2. **(10 points)**

Use d'Alembert's formula to solve the following wave equation:

$$u_{tt} = 4u_{xx}, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the following initial conditions:

$$u(x, 0) = \sin(3x), \quad u_t(x, 0) = \sin(2x) - \sin(x).$$

Question 3. **(25 points)**

Solve the following wave equation:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \quad 0 < r < L, \quad t > 0,$$

subject to the following boundary and initial conditions:

$$\begin{aligned} &u(L, t) = 0, \quad t > 0, \\ &u(r, 0) = \beta, \quad u_t(r, 0) = 0, \quad 0 < r < L. \end{aligned}$$

Question 4. **(30 points)**

Solve the following heat equation:

$$u_t = a^2 u_{xx}, \quad 0 < x < L, \quad t > 0,$$

subject to the following boundary and initial conditions:

$$\begin{aligned} u_x(0,t) &= -\beta, \quad u(L,t) = T_0, \quad t > 0, \\ u(x,0) &= T_0, \quad 0 < x < L. \end{aligned}$$

End of Exam