

**KFUPM**  
**Math513 – Final Exam**  
**Duration:** 140 minutes

Name: \_\_\_\_\_ ID: \_\_\_\_\_

**Instructions**

- Write clearly and show all essential steps. You may lose points for messy work.
  - Justify your answers. You may lose points for unsupported answers.
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**Question 1.**

**(25 points)**

Use separation of variables to solve the following Laplace equation:

$$u_{xx} + u_{yy} = 0, \quad 0 < x < a, \quad 0 < y < b,$$

subject to the boundary conditions,

$$\begin{aligned} u(0, y) &= 0, & u(a, y) &= 0, \\ u(x, 0) &= 0, & u(x, b) &= f(x), \end{aligned}$$

where  $f(x)$  is a given function.





**Question 2.****(25 points)**

Use Laplace transform to solve the following wave equation:

$$u_{tt} = u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$

subject to the initial and boundary conditions

$$\begin{aligned} u(x, 0) &= 0, & u_t(x, 0) &= e^{-x}, \\ u(0, t) &= 0, & u(x, t) &\rightarrow 0 \text{ as } x \rightarrow \infty. \end{aligned}$$





**Question 3.**

**(15 points)**

Solve the following initial value problem using Laplace transform:

$$y'' - 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 4.$$



**Question 4.****(10 points)**

The Fourier transform of the following function is:

$$f(t) = \begin{cases} e^{3t}, & t < 0, \\ e^{-2t}, & t > 0 \end{cases}$$

(a)  $\frac{1}{3 - i\omega} + \frac{1}{2 - i\omega}$

(b)  $\frac{1}{3 + i\omega} + \frac{1}{2 + i\omega}$

(c)  $\frac{1}{2 - i\omega} + \frac{1}{3 + i\omega}$

(d)  $\frac{1}{3 - i\omega} + \frac{1}{2 + i\omega}$

(e)  $\frac{1}{3 + i\omega} + \frac{1}{2 - i\omega}$

**Question 5.****(10 points)**

Find the inverse Laplace transform:

$$\mathcal{L}^{-1} \left\{ \frac{(s-1)e^{-s}}{s^4} + e^{-3s} \frac{5s+7}{s^2+5s+4} \right\}.$$

(a)  $H(t-1) \left[ \frac{t^2}{2} - \frac{t^3}{6} \right] + H(t-3) \left[ \frac{2}{3}e^{-t} + \frac{13}{3}e^{-4t} \right]$

(b)  $H(t-1) \left[ \frac{(t-1)^2}{2} - \frac{(t-1)^3}{3} \right] + H(t-3) \left[ \frac{2}{3}e^{-(t-3)} + \frac{13}{3}e^{-4(t-3)} \right]$

(c)  $H(t-1) \left[ \frac{(t-1)^2}{2} - \frac{(t-1)^3}{6} \right] + H(t-3) \left[ \frac{2}{3}e^{-(t-3)} + \frac{13}{3}e^{-4(t-3)} \right]$

(d)  $H(t) \left[ \frac{(t-1)^2}{2} - \frac{(t-1)^3}{6} \right] + H(t-3) \left[ \frac{2}{3}e^{-(t-3)} + \frac{13}{3}e^{-4(t-3)} \right]$

(e)  $H(t-1) \left[ \frac{(t-1)^2}{2} - \frac{(t-1)^3}{6} \right] + \frac{2}{3}e^{-(t-3)} + \frac{13}{3}e^{-4(t-3)}$

**Question 6.****(10 points)**

Let

$$F(s) = \frac{2s^2 + 3s + 1}{s(s+1)(s^2 + 4s + 13)},$$

then

(a)  $f(0) = 2, \quad \lim_{t \rightarrow \infty} f(t) = \frac{1}{13}$

(b)  $f(0) = 0, \quad \lim_{t \rightarrow \infty} f(t) = 0$

(c)  $f(0) = 1, \quad \lim_{t \rightarrow \infty} f(t) = \frac{1}{13}$

(d)  $f(0) = 0, \quad \lim_{t \rightarrow \infty} f(t) = 1$

(e)  $f(0) = 0, \quad \lim_{t \rightarrow \infty} f(t) = \frac{1}{13}$

**Question 7.****(10 points)**

Using Parseval's identity and the fact that

$$\mathcal{F}\{e^{-a|t|}\}(\omega) = \frac{2a}{a^2 + \omega^2} \quad (a > 0),$$

evaluate

$$\int_{-\infty}^{\infty} \frac{d\omega}{(a^2 + \omega^2)^2}.$$

- (a)  $\frac{\pi}{a^2}$
- (b)  $\frac{\pi}{2a^2}$
- (c)  $\frac{\pi}{2a^3}$
- (d)  $\frac{\pi}{a^3}$
- (e)  $\frac{2\pi}{a^3}$

*End of Exam*