King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam 1
Term (252)
Date: 15 Feb. 2026 Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: Dr. Adel Al-Mahdi. Sec #: 02. Serial #: _____

- Answer only five (5) questions. Question 1 is mandatory.
- Mobiles and calculators are NOT allowed in this exam.
- Write all steps clearly.

Question #	Marks	Maximum Marks
1	mandatory	**20**
2		10
3		10
4		10
5		10
6		10
Total		60

Question 1: Solve the following system:

$$\begin{cases} x_1' + 5x_1 + x_2' + 3x_2 &= 0, \\ 2x_1' + x_1 + x_2' + x_2 &= 0, \end{cases}$$

where $x_i = x_i(t),\ i = 1, 2$, and the primes denote the time derivative.

Question 2: Find the **Fourier series** of the function $f(t)$ defined by

$$f(t) = |t|, \quad -\pi \leq t \leq \pi.$$

Question 3: Find the **Fourier cosine series** of the function

$$f(t) = \begin{cases} t, & 0 < t \leq 1, \\ 1, & 1 \leq t < 2. \end{cases}$$

Question 4:

Write the following Fourier series in the cosine phase–angle forms:

$$f(t) = -2\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(nt).$$

Question 5: Find the complex Fourier series of $f(t) = \begin{cases} 0, & -\pi < t < 0, \\ t, & 0 < t < \pi. \end{cases}$

Question 6: If the Fourier series for the odd function

$$f(t) = \begin{cases} 2t + t^2, & -2 \le t \le 0, \\ 2t - t^2, & 0 \le t \le 2, \end{cases}$$

is

$$f(t) = \frac{32}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \sin\left(\frac{(2n-1)\pi t}{2}\right).$$

Use **Parseval's equality** to show that

$$\frac{\pi^6}{960} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}.$$