Numerical Functional Analysis

MATH 674 | Term 261 (Fall 2026)

Ph.D. in Mathematics | College of Computing and Mathematics (CCM)

Instructor

Name	Dr. Mohammed Alshahrani	**Phone**	+966 13 860 7748
Office	Building 5, Room 201-1	**E-mail**	mshahrani@kfupm.edu.sa
Office hours	Sun/Tue/Thu, 11:00–11:50 | By appointment via TEAMS (online) or in person		
Website	`https://mshahrani.website/`		

Textbook

Required: Bauschke, H. H. & Combettes, P. L. *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*. 2nd ed. CMS Books in Mathematics. Springer, 2017. ISBN 978-3-319-48310-8.

Recommended references

- Atkinson, K. & Han, W. *Theoretical Numerical Analysis: A Functional Analysis Framework*. 3rd ed. Springer Texts in Applied Mathematics 39. New York: Springer, 2009. ISBN 978-1-4419-0457-7.
- Peypouquet, J. *Convex Optimization in Normed Spaces: Theory, Methods and Examples*. SpringerBriefs in Optimization. Cham: Springer, 2015.
- Ryu, E. K. & Yin, W. *Large-Scale Convex Optimization: Algorithms & Analyses via Monotone Operators*. Cambridge University Press, 2022.
- Deuflhard, P. *Newton Methods for Nonlinear Problems: Affine Invariance and Adaptive Algorithms*. Springer Series in Computational Mathematics 35. Berlin: Springer, 2011.

Online lecture notes (open access)

- Arnold, D. N. *Numerical Analysis of Partial Differential Equations* (MATH 8445/8446 lecture notes). University of Minnesota. `https://www-users.cse.umn.edu/~arnold/8445-8446.17-18/notes.pdf`
- Herzog, R. & Müller, B. *Lecture Notes on Infinite-Dimensional Optimization*. Heidelberg University SCOOP, WS 2024/25. `https://scoop.iwr.uni-heidelberg.de/teaching/2024ws/lecture-infinite-dimensio` `infinite-dimensional-optimization-lecture-notes-20241028.pdf`
- Combettes, P. L. — Selected proximal-splitting survey papers and slides. `https://pcombet.math.ncsu.edu/`

Description

Functional-analytic techniques for the numerical solution of operator equations, optimization problems, and discretized differential equations in Banach and Hilbert spaces. Core techniques include operator approximation theory, convergence analysis, projection and iterative methods, convex analysis, first-order and splitting methods, Newton-type methods, and variational principles. Application areas may be chosen from: numerical methods for partial differential equations (finite-element analysis); large-scale convex optimization and operator splitting; optimal control and inverse problems; variational inequalities; or related areas in computational mathematics.

Student Learning Outcomes

After completion of the course, the student should be able to:

1. *(Knowledge.)* Explain the functional-analytic foundations underlying numerical analysis — bounded, compact, and monotone operators; convergence and approximation in Banach and Hilbert spaces.
2. *(Skill.)* Prove stability, consistency, and convergence of projection and iterative methods for linear and nonlinear operator equations, with explicit error estimates.
3. *(Skill.)* Analyze first-order and splitting methods, with convergence analysis in Banach and Hilbert spaces.
4. *(Skill.)* Construct Newton-type schemes (Newton, quasi-Newton, semismooth Newton) for nonlinear operator equations in Banach and Hilbert spaces, including affine-invariance and mesh-independence analysis.
5. *(Skill.)* Formulate variational problems in abstract function spaces, including their numerical discretization and convergence analysis.
6. *(Values, autonomy, responsibility.)* Justify an independent research investigation with a computational component, in written report and oral presentation, adhering to academic-integrity standards.

Grading & Evaluation

Grading Policy

15%	Homework Assignments
20%	Midterm Exam
25%	Research Project
10%	Project Presentation
30%	Final Exam
100%	**Total**

Grade Scale

Grade	Range
A+	$[90\%, 100\%]$
A	$[80\%, 90\%)$
B+	$[75\%, 80\%)$
B	$[70\%, 75\%)$
C+	$[65\%, 70\%)$
C	$[55\%, 65\%)$
D+	$[50\%, 55\%)$
D	$[45\%, 50\%)$
F	$[0\%, 45\%)$

Numerical Computation

Implementation and computational work use the Julia language and packages from its numerical-optimization ecosystem: `ProximalOperators.jl`, `ProximalAlgorithms.jl`, `Optim.jl`, `JuMP.jl`, `Convex.jl`, `JuliaSmoothOptimizers` (`NLPModels`, `JSOSolvers`, `Krylov`, `CUTEst`).

Resources: Lauwens & Downey, *Think Julia* | Julia docs | JuliaSmoothOptimizers | JuliaFirstOrder | learning resources.

Homework Assignments (Paper Reading & Presentation)

Each HW is a **critical review and in-class presentation** of a recent research paper. Paper selection: (a) from the **instructor-curated list** (Appendix A; course website), or (b) **student-proposed** from a recognized venue (*SIAM J. Optim., SIAM J. Numer. Anal., Math. Programming, Numer. Math., IMA J. Numer. Anal., Found. Comput. Math., JOTA*) — instructor approval required one week prior.

Per HW:

- LaTeX critical review, 3–5 pages: summary, technical assessment, comparison to course theory, limitations, extensions.
- In-class presentation: 15 min + 5 min Q&A.

Topics & due dates:

- **HW1** — iterative / projection — **Wk 5**
- **HW2** — first-order / splitting — **Wk 10**
- **HW3** — Newton-type — **Wk 14**

Research Project Guidelines

An **individual** Ph.D.-level project: select an instructor-approved topic that connects a theoretical thread of the course with a computational investigation in Julia. Consult the instructor early if topic selection is difficult.

Project Milestones

Wk 3	One-page proposal (motivation, objective, initial references).
Wk 8	One-page progress report (preliminary results, remaining work).
Wk 14	20-minute in-class presentation with slides.
Wk 15	Final report — ≤ 20 pp, similarity $< 20\%$, with reproducible Julia notebook.

Course Schedule

The schedule is tentative and may be adjusted as the term proceeds. Dates follow the official KFUPM Term 261 (Fall 2026) academic calendar. Abbreviations: **B-C** = Bauschke & Combettes (required); **TNA** = Atkinson & Han; **D** = Deuflhard.

Wk	Dates	Topic	Content	Notes / Due
1	Aug 19–27	FA foundations	Banach/Hilbert review; bounded linear operators; dual spaces	B-C Ch. 1–2
2	Aug 30–Sep 3	Operator approximation	Convex sets; nonexpansive operators; monotone operators (intro); operator approximation	B-C Ch. 3–4, 20
3	Sep 6–10	Projection methods	Operator approximation continued; projection methods (Galerkin-type)	B-C Ch. 28; **HW1 assigned**; **Project proposal due**
4	Sep 13–17	Iterative methods	Iterative methods for linear and nonlinear operator equations: Landweber, fixed-point, collocation	TNA Ch. 5–6
5	Sep 20–24	Stability / convergence	Stability, consistency, convergence framework and rates; lower semicontinuity of functionals	TNA Ch. 2; **HW1 due**; **National Day (Wed–Thu Sep 23–24 — no class)**
6	Sep 27–Oct 1	Convex functions	Convex functions, subdifferential calculus, existence of minimizers	B-C Ch. 8–10, 16
7	Oct 4–8	Duality & Midterm	Fenchel conjugation, Fenchel–Rockafellar duality, KKT conditions	B-C Ch. 13–15, 19, 26; **MIDTERM EXAM (Wks 1–6)**
8	Oct 11–15	VIs → steepest descent	Variational inequalities; steepest descent (analysis)	B-C Ch. 11; **Project progress report due**
9	Oct 18–19	Proximal / CG	Proximal point algorithm; conjugate gradient in Hilbert spaces	B-C Ch. 27
	Oct 20–22	**Midterm Break — no class**		
10	Oct 25–29	Averaged ops / FBS	Averaged operator iterations; Fejér monotonicity; forward-backward splitting	B-C Ch. 5, 29; **HW2 due**
11	Nov 1–5	Splittings	Douglas-Rachford splitting; ADMM	B-C Ch. 29
12	Nov 8–12	Variational & FEM	Abstract variational methods; theoretical aspects of spline and finite-element analysis	TNA Ch. 10, 12
13	Nov 15–19	Newton for op. equations	Newton's method for operator equations: Kantorovich's theorem, affine invariance	D Ch. 1–2
	Nov 22–23	**Autumn Break — no class**		
14	Nov 24–26	Quasi-Newton	Quasi-Newton methods in operator settings; mesh-independence	D Ch. 2–3; **HW3 due**; **Project presentations**
15	Nov 29–Dec 10	Semismooth Newton	Semismooth Newton for variational inequalities and minimization; course wrap-up	D Ch. 4; **Final project report due** (≤ 20 pp, similarity $< 20\%$)
16	Dec 13–24	Final Exam	**FINAL EXAM — See Registrar Website.**	

Academic Integrity

Students are expected to uphold the highest standards of scholarly integrity. All written work must be the student's own; proper citation is required for all sources. Collaboration on homework discussion is permitted, but each student must write their own critical review and present their own analysis. The similarity index of the final project report must be below 20%. Violations will be handled under the KFUPM academic integrity policy.

Appendix A. Candidate Paper List for HWs

A curated starter list. Students may select one paper per HW from the list below, or propose an alternative from a recognized venue (see the Homework section) subject to instructor approval. The list will be kept current on the course website.

HW1 — Iterative and Projection Methods for Operator Equations (due Wk 5)

1. L. Landweber. "An iteration formula for Fredholm integral equations of the first kind." *American Journal of Mathematics* 73:3 (1951), 615–624.
2. L. M. Bregman. "The relaxation method of finding the common point of convex sets and its application to the solution of problems in convex programming." *U.S.S.R. Comp. Math. Math. Phys.* 7:3 (1967), 200–217.
3. H. H. Bauschke and J. M. Borwein. "On projection algorithms for solving convex feasibility problems." *SIAM Review* 38:3 (1996), 367–426.
4. M. Hanke, A. Neubauer and O. Scherzer. "A convergence analysis of the Landweber iteration for nonlinear ill-posed problems." *Numerische Mathematik* 72:1 (1995), 21–37.
5. P. L. Combettes. "The convex feasibility problem in image recovery." *Advances in Imaging and Electron Physics* 95 (1996), 155–270.

HW2 — First-Order / Proximal / Splitting Methods (due Wk 10)

6. R. T. Rockafellar. "Monotone operators and the proximal point algorithm." *SIAM J. Control and Optimization* 14:5 (1976), 877–898.
7. P.-L. Lions and B. Mercier. "Splitting algorithms for the sum of two nonlinear operators." *SIAM J. Numerical Analysis* 16:6 (1979), 964–979.
8. Y. Nesterov. "A method of solving a convex programming problem with convergence rate $O(1/k^2)$." *Doklady Akademii Nauk SSSR* 269 (1983), 543–547.
9. P. L. Combettes and V. R. Wajs. "Signal recovery by proximal forward-backward splitting." *Multiscale Modeling & Simulation* 4:4 (2005), 1168–1200.
10. A. Beck and M. Teboulle. "A fast iterative shrinkage-thresholding algorithm for linear inverse problems." *SIAM J. Imaging Sciences* 2:1 (2009), 183–202.
11. A. Chambolle and T. Pock. "A first-order primal-dual algorithm for convex problems with applications to imaging." *J. Mathematical Imaging and Vision* 40:1 (2011), 120–145.

HW3 — Newton-Type Methods in Infinite Dimensions (due Wk 13)

12. R. S. Dembo, S. C. Eisenstat and T. Steihaug. "Inexact Newton methods." *SIAM J. Numerical Analysis* 19:2 (1982), 400–408.
13. L. Qi and J. Sun. "A nonsmooth version of Newton's method." *Mathematical Programming* 58:1–3 (1993), 353–367.
14. M. Hintermüller, K. Ito and K. Kunisch. "The primal-dual active set strategy as a semismooth Newton method." *SIAM J. Optimization* 13:3 (2002), 865–888.
15. M. Ulbrich. "Semismooth Newton methods for operator equations in function spaces." *SIAM J. Optimization* 13:3 (2002), 805–842.
16. P. Deuflhard and G. Heindl. "Affine invariant convergence theorems for Newton's method and extensions to related methods." *SIAM J. Numerical Analysis* 16:1 (1979), 1–10.