AS201- Major Exam 2 (Code 1)

KFUPM, Department of Mathematics

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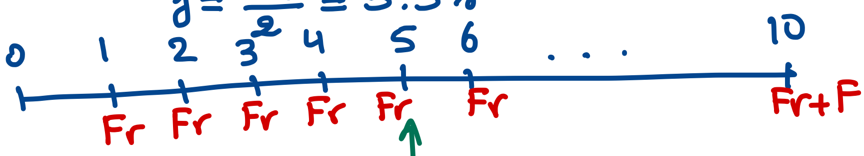
Exercise 1

A 1000 par value 5-year bond with 8% semiannual coupons was bought to yield 7% convertible semiannually. Determine the amount of premium amortized in 6-th coupon payment.

1. 2.00
2. 2.08
3. 4.00
4. 4.21
5. 5.57

$F = C = 1000; \quad n = 5 \times 2 = 10; \quad r = \frac{8\%}{2} = 4\%$

$j = \frac{7\%}{2} = 3.5\%$



$BV_5 = F + F(r-j)a_{\overline{5}|j}$

$BV_6 = F + F(r-j)a_{\overline{4}|j}$

Amount of premium amortized in 6-th coupon payment

$= BV_5 - BV_6$

$= F(r-j)\left[\underbrace{a_{\overline{5}|j} - a_{\overline{4}|j}}_{v+v^2+\cdots+v^5-v-v^2-\cdots-v^4 = v^5}\right]$

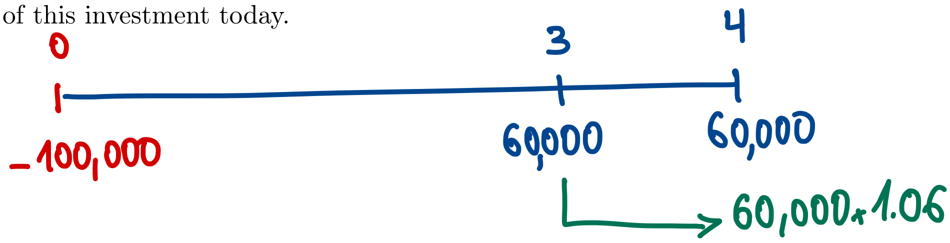
$= F(r-j)v^5 = 1000 \times [0.04 - 0.035] \times 1.035^{-5}$

$= 4.21\$$

Exercise 2

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of years 3 and 4. The cash flows can be reinvested at 6% par annum effective. Using an annual effective interest rate of 5%, calculate the net present value of this investment today.

1. −1398
2. −699
3. 699
4. 1686
5. 2454

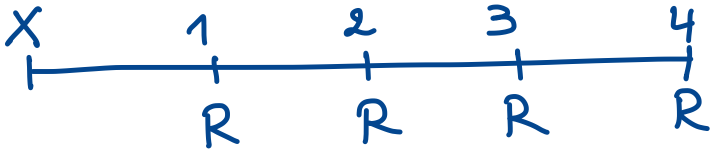


$$NPV = 60000[1+1.06]\times 1.05^{-4} - 100{,}000$$
$$= 1686$$

Exercise 3

Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the second year is 1076.82. Calculate the principal repaid in the first year

1. 414
2. 424
3. 434
4. 444
5. 454



$i = 8\%$

$$OB_2 = R a_{\overline{2}|i} \Rightarrow R = \frac{OB_2}{a_{\overline{2}|i}} = 603.85$$

$$OB_0 = R a_{\overline{4}|i} = 2000$$

$$PR_1 = R - OB_0 i = 603.85 - 2000 \times 0.08$$
$$= 443.85\$$$

Exercise 4

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The prices assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into a account earning interest at an annual effective rate of i.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond. Calculate i.

1. 9.50%
2. 9.75%
3. 10.00%
4. 10.25%
5. 10.50%

$n = 20$; $F = C = 1000$; $r = 0.03$; $j = 0.06$

$Fr = 30\$ \Rightarrow P = C = 1000$

He will invest 30 each 6-months for 20 periods at an annual effective rate i (e: interest per 6-month period)

So, he will get $30\, s_{\overline{20}|e} + 1000$

$= 1000 \times (1+0.07)^{10} = 1967.15$

$\Rightarrow \boxed{s_{\overline{20}|e} = 32.24} \Rightarrow e = 0.0476$

$\Rightarrow i = (1+e)^2 - 1 = 0.0975$

Exercise 5

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually. Calculate X.

1. 1400
2. 1420
3. 1440
4. 1460
5. 1480

$r = 4\%$; $j = 3\%$

Given that the coupon rate is greater than the yield rate, the lowest yield rate is calculated based on a call at the earliest possible date.

$$P = X + X(r-j)a_{\overline{30}|j} \Rightarrow X = \frac{P}{1+(r-j)a_{\overline{30}|j}}$$

$$= \frac{1722.25}{1+(0.04-0.03)a_{\overline{30}|0.03}}$$

$$= 1440$$

Exercise 6

Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

1. 1251
2. 1262
3. 1278
4. 1278
5. 1295

$n = 20 \times 2 = 40$; $Fr = 40$; $C = 1100$

$C_{30} = C_{31} = \cdots = C_{39} = 1200$; $C_{40} = 1100$

$j = 3\%$

$$P_n = C_n + (Fr - C_n \cdot j)\, a_{\overline{n}|j} = C_n + (40 - jC_n)\, a_{\overline{n}|j}$$

·) For $n = 30, 31, \dots, 39$, we have

$$P_n = 1200 + (40 - 0.03 \times 1200)\, a_{\overline{n}|0.03} = 1200 + 4\, a_{\overline{n}|0.03}$$

$$P_{30} = 1200 + 4\, a_{\overline{30}|0.03} = 1278.4$$

$$\vdots$$

$$P_{39} = 1200 + 4\, a_{\overline{39}|0.03} = 1291.23$$

·) For $n = 40$, we have

$$P_{40} = 1100 + (40 - 0.03 \times 1100)\, a_{\overline{40}|0.03} = 1261.8$$

*) On a callable bond, the maximum price of the bond to guarantee a yield rate is the minimum price over all the redemption dates.

If he pays more than 1262, the yield if the maturity date is 40 will be less than 6%.

Exercise 7

You are given the following information about ~~of~~ L that is repaid with a series of 16 annual payments:

- The first payment of 2000 is due one year from now.
- The next seven payments are each 3% larger than the preceding payment.
- From the 9-th to the 16-th payment, each payment will be 3% less than the preceding payment.
- the loan has an annual effective interest rate of 7%.

Calculate L.

1. 20, 689
2. 20, 716
3. 20, 775
4. 21, 147
5. 22, 137

$i=0.07$

$$\underbrace{\overset{1}{2000} \quad \overset{2}{2000\times 1.03} \quad \overset{8}{2000\times 1.03^7}}_{\substack{\text{Geometric with } r_1=0.03 \\ R=2000}} \quad \underbrace{\overset{9}{2000\times 1.03^7\times 0.97} \quad \overset{16}{2000\times 1.03^7\times 0.97^8}}_{\substack{\text{Geometric with } r_2=-0.03 \\ \text{and } R=2000\times 1.03^7\times 0.97}}$$

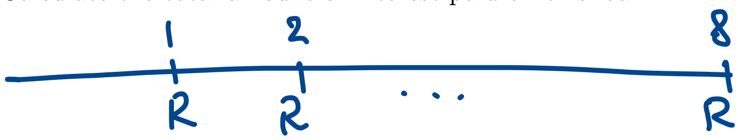
$$L = 2000\times\frac{1-\left(\frac{1+r_1}{1+i}\right)^8}{i-r_1} + v^8\times 2000\times 1.03^7\times 0.97\times\frac{1-\left(\frac{1+r_2}{1+i}\right)^8}{i-r_2}$$

$$= 20688.63$$

Exercise 8

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68. Calculate the total amount of interest paid on this loan.

1. 1239
2. 1647
3. 1820
4. 2319
5. 2924



$i = 4.75\%$

$$\left.\begin{array}{l} OB_4 = R a_{\overline{4}|i} \\ OB_5 = R a_{\overline{3}|i} \end{array}\right\} \quad PR_5 = 699.68 = OB_4 - OB_5 = R\left[a_{\overline{4}|i} - a_{\overline{3}|i}\right]$$

$$\Rightarrow R = \frac{699.68}{a_{\overline{4}|i} - a_{\overline{3}|i}} = \frac{699.68}{\frac{v^3 - v^4}{i}} = 842.39$$

$$OB_0 = R a_{\overline{8}|i} = 5500\$$$

The amount of interest $8R - OB_0 = 1239.01\$$

Exercise 9

Turner buys a new car and finances it with a loan of $22,000$. He will make n monthly payments of 450.30 starting in one month. He will make one larger payment in $n+1$ months to pay off the loan. Payments are calculated using an annual nominal interest rate of 8.4%, convertible monthly. Immediately after the 18-th payment he refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8% , convertible monthly. Calculated the amount of the new monthly payment.

1. 668
2. 693
3. 702
4. 715
5. 742

$L = 22{,}000 \;;\quad R = 450.30\$ \;;\quad j = \frac{8.4\%}{12} = 0.7\%$

$$OB_{18} = OB_0(1+j)^{18} - R\, s_{\overline{18}|j}$$

$$= 22000 \times 1.007^{18} - 450.3\, s_{\overline{18}|0.007}$$

$$= 16337.10$$

new $j = \frac{4.8\%}{12} = 0.4\%$

$$16337.10 = X a_{\overline{24}|0.004}$$

$$\Rightarrow X = \frac{16337.10}{a_{\overline{24}|0.004}} = 715.26$$

Exercise 10

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

- The first bond has an annual nominal yield rate of 5% compounded semiannually and a redemption value of 1200.
- The second bond has an annual nominal yield rate of i compounded semiannually and a redemption value of 800.

Calculate i

1. 2.20%
2. 2.34%
3. 2.53%
4. 4.40%
5. 4.69%

$r = 2.5\%$; $F = 1000$

First bond:

$j = 2.5\%$; $C = 1200$

$$P = C + (Fr - Cj)\, a_{\overline{n}|j}$$
$$= 1200 + (1000 \times 0.025 - 1200 \times 0.025)\, a_{\overline{60}|0.025}$$
$$= 1045.46$$

Second bond:

$C = 800$; $j = \frac{i}{2}$

$$P = 1045.46$$
$$= 800 + (1000 \times 0.025 - 800 \times j)\, a_{\overline{60}|j}$$

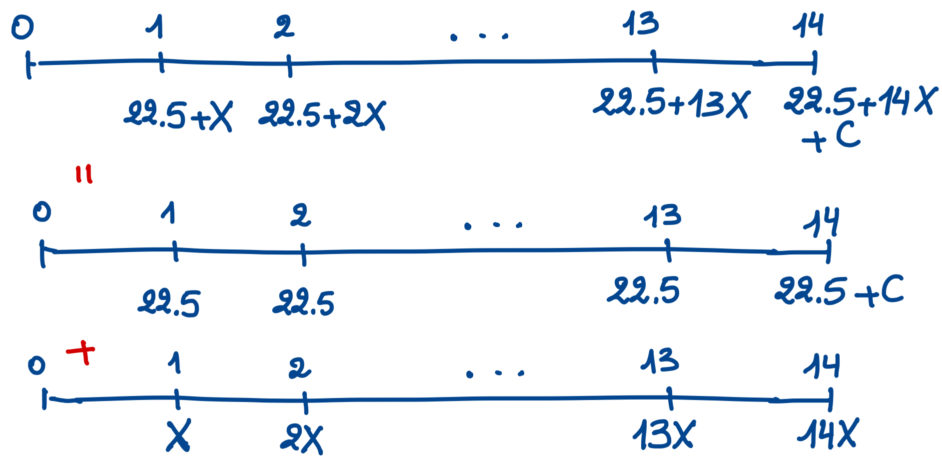
$j = 2.2\% \implies i = 2j = 4.4\%$

Exercise 11

An investor owns a bond that is redeemable for 300 in seven years. The investor has just received a coupon of 22.5 and each subsequent semiannual coupon will be X more than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 1050.50 assuming an annual nominal yield rate of 6% convertible semiannually. Calculate X.

1. 5.04
2. 6.37
3. 6.98
4. 7.54
5. 8.14

$C=300$; $n=7\times 2=14$; $j=0.03$



$$P = Cv^{14} + 22.5\, a_{\overline{14}|0.03} + X\,(Ia)_{\overline{14}|0.03}$$

$$= 452.5 + 79.31X = 1050.5$$

$$\Rightarrow X = \frac{1050.5 - 452.5}{79.31} = 7.54$$

$$(Ia)_{\overline{14}|0.03} = \frac{\ddot{a}_{\overline{14}|0.03} - 14\times 1.03^{-14}}{0.03} = \frac{a_{\overline{13}|0.03} + 1 - 14\times 1.03^{-14}}{0.03}$$

$$= 79.31$$

Exercise 12

John Borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1627.45 at the end of each year. Instead, John repays the loan using a sinking fund that pays an annual effective interest rate of 13%. The deposits to the sinking fund are equal to 1627.45 minus the interest on the loan and are made at the end of each year for 10 years. Calculate the balance in the sinking fund immediately after repayment of the loan.

1. 1527
2. 1557
3. 1587
4. 1627
5. 1657

$OB_0 = 10{,}000$; $i = 10\%$; $n = 10$

*) $i \times 10000 = 1000\$$: Deposit $= 1627.45 - 1000$
$= 627.45$

$i = 13\%$

Sinking fund

$$FV - 10000 = 627.45\, s_{\overline{10}|0.13} - 10000$$
$$= 627.45 \times \frac{1.13^{10} - 1}{0.13} - 10000$$
$$= 1557.47\,\$$$

Exercise 13

You are given the following information about an investment account

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y

1. -0.35%
2. -0.25%
3. -0.15%
4. 0%
5. 0.15%

$0 = \frac{12}{10} \times \frac{X}{12+X} \Rightarrow 12X = 10(12+X)$

$\Rightarrow X = 60$

$i = \frac{X - [10 + X]}{10 + X \times \frac{1}{2}} = \frac{-10}{10 + \frac{X}{2}} = \frac{-10}{40} = -0.25$

Exercise 14

A 30-year loan of 1000 is repaid with payments at the end of each year. Each of the first ten payments equals the amount of interest due. Each of the next ten payments equals 150% of the amount of interest due. Each of the last ten payments is X. The lender charges interest at an effective annual rate of 10%. Calculate X.

1. 67.44
2. 77.44
3. 87.44
4. 97.44
5. 107.44

$OB_0 = 1000 \qquad i = 10\%$

•) $K_1 = K_2 = \cdots = K_{10} = 1000 \times 0.1 = 100\$$

•) For $t = 11, \ldots, 20$

$I_t = OB_{t-1} \times i = 0.1\, OB_{t-1}$

$PR_t = 0.5 I_t = 0.05\, OB_{t-1}$

$\Rightarrow OB_t = OB_{t-1} - PR_t = OB_{t-1} - 0.05\, OB_{t-1}$

$= 0.95\, OB_{t-1}$

So that $OB_{20} = (0.95)^{10} OB_{10} = (0.95)^{10} \times 1000$

$= 598.74\$$

•) For $t = 21, \ldots, 30$:

$$OB_{20} = X a_{\overline{10}|0.1} \Rightarrow X = \frac{OB_{20}}{a_{\overline{10}|0.1}} = 97.44\$$$

Exercise 15

Iggy borrows X for 10 years at an effective annual rate of 6%. If he pays the principal and accumulated interest in one lump sum at the end of 10 years, he would pay 356.54 more in interest than if he repaid the loan with 10 level payments at the end of each year. Calculate X.

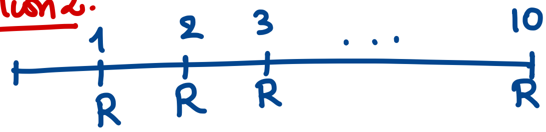
1. 795
2. 805
3. 815
4. 825
5. 835

$OB_0 = X; \quad i = 6\%$

Option 1:

Total $X(1+i)^{10} = 1.06^{10} X$

Option 2:



$OB_0 = X = R\,a_{\overline{10}|0.06} \Rightarrow R = \dfrac{X}{a_{\overline{10}|0.06}}$

Total $10R = \dfrac{10}{a_{\overline{10}|0.06}} X$

$$1.06^{10} X = \frac{10}{a_{\overline{10}|0.06}} X + 356.54$$

$$X\left[1.06^{10} - \frac{10}{a_{\overline{10}|0.06}}\right] = 356.54$$

$$\Rightarrow X = \frac{356.54}{1.06^{10} - \dfrac{10}{a_{\overline{10}|0.06}}} = 825$$

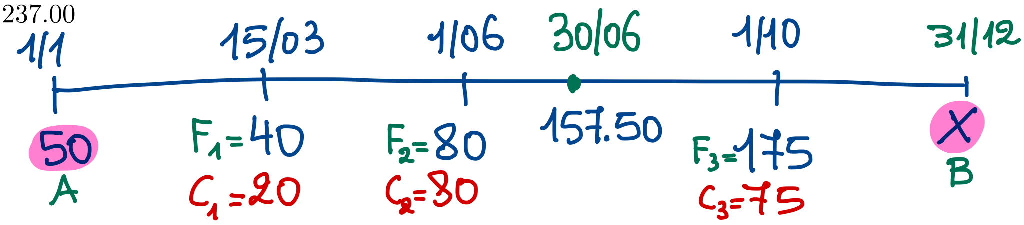
Exercise 16

An investor deposits 50 in an investment account on January 1. The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit	Deposit
March 15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50. On December 31, the value of the account is X. Using the time-weighted method, the equivalent effective annual yield during the first 6 months is equal to the (time-weighted) effective annual yield during the entire 1-year period. Calculate X.

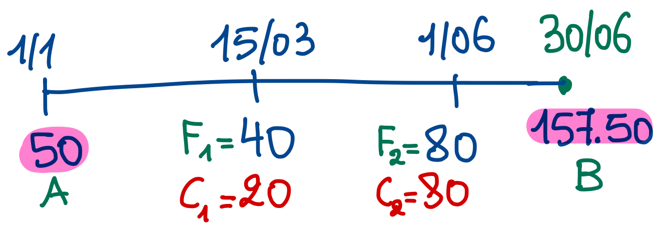
1. 233.50
2. 234.75
3. 235.50
4. 236.25
5. 237.00



Time-weighted rate of return $= \frac{F_1}{A} \times \frac{F_2}{F_1+C_1} \times \frac{F_3}{F_2+C_2} \times \cdots \times \frac{B}{F_n+C_n} - 1$

$\cdot)$ $i = \frac{40}{50} \times \frac{80}{60} \times \frac{175}{160} \times \frac{X}{250} - 1$: return on the entire year

$\cdot)$



$$i_{0.5} = \frac{40}{50} \times \frac{80}{60} \times \frac{157.5}{160} - 1 = 0.05$$

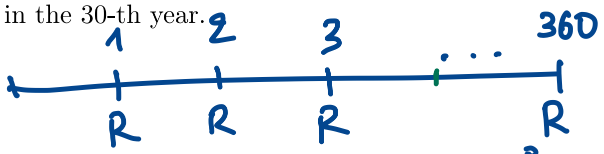
$\cdot)$ $i = (1+i_{0.5})^2 - 1 \Rightarrow \frac{40}{50} \times \frac{80}{60} \times \frac{175}{160} \times \frac{X}{250} = 1.1025$

$$\Rightarrow X = \frac{1.1025 \times 50 \times 60 \times 160 \times 250}{40 \times 80 \times 175} = 236.25$$

Exercise 17

A homebuyer borrows 250, 000 to be repaid over a 30-year period with level monthly payments beginning one month after the loan is made. The interest rate on the loan is a nominal annual rate of 9% compounded monthly. Find the amount of interest paid in the 30-th year.

1. 997
2. 1025
3. 1078
4. 1106
5. 1137



$$OB_0 = 250{,}000 \ ; \quad i = \frac{9\%}{12} = 0.75\%$$

$$OB_0 = R\, a_{\overline{360}|0.0075} \Rightarrow R = \frac{OB_0}{a_{\overline{360}|0.0075}} = 2011.55$$

$$OB_{348} = R\, a_{\overline{12}|0.0075} = 23001.89$$

Amount of interest in the 30-th year is

$$12R - OB_{348} = 1136.71$$

Exercise 18

A project requires an initial capital outlay of 30,000 and will return the following amounts (paid at the end of the next 5 years):

$$14,000 \quad , \quad 12,000 \quad , \quad 6,000 \quad , \quad 4,000 \quad , \quad 2,000.$$

Find the internal rate of return.

1. 10.81%
2. 11.26%
3. 11.78%
4. 12.03%
5. 12.37%

$$0 = 14000v + 12000v^2 + 6000v^3 + 4000v^4 + 2000v^5 - 30,000$$

$$\Rightarrow \text{IRR} = 12.03\%$$

$F=C=100\ ;\ j=0.035$

Exercise 19

A bond of face amount 100 is purchased at a premium of 36 to yield 7%. The amount for amortization of premium in the 5-th coupon is 1.00. What is the term of the bond.

1. 11 years or 22 coupon periods
2. 12 years or 24 coupon periods
3. 12.5 years or 25 coupon periods
4. 13 years or 26 coupon periods
5. 13.5 years or 27 coupon periods

$P - C = P - F$

$= F(r-j)a_{\overline{n}|j} = 36$

is the premium

$$BV_4 = F + F(r-j)\, a_{\overline{n-4}|j}$$
$$BV_5 = F + F(r-j)\, a_{\overline{n-5}|j}$$

$$1 = BV_4 - BV_5 = F(r-j)\big(\underbrace{a_{\overline{n-4}|j}}_{v+\dots+v^{n-4}} - \underbrace{a_{\overline{n-5}|j}}_{v+\dots+v^{n-5}}\big) = F(r-j)v^{n-4}$$

$$\Rightarrow \quad F(r-j) = (1+j)^{n-4}$$

$$P - F = F(r-j)a_{\overline{n}|j} = (1+j)^{n-4} \times \frac{1-(1+j)^{-n}}{j} = \frac{(1+j)^{n-4} - (1+j)^{-4}}{j}$$

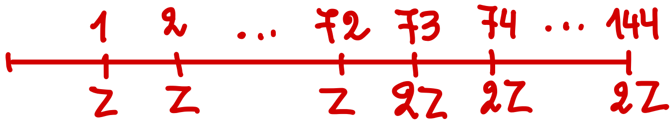
$$\Rightarrow \quad (1+j)^{n-4} = j(P-F) + (1+j)^{-4} = 0.035 \times 36 + 1.035^{-4} = 2.13144$$

$$\overset{\ln}{\Rightarrow} \quad (n-4)\ln(1+j) = \ln(2.13144) \Rightarrow n = \frac{\ln(2.1314)}{\ln(1+j)} + 4$$
$$= \frac{\ln(2.1314)}{\ln(1.035)} + 4$$
$$= 25.999 \simeq 26 \text{ coupon periods}$$

Exercise 20

A loan of 100,000 has payments at the end of each month for 12 years. For the first 6 years the payments are Z each month, and for the final 6 years the payments are $2Z$ each month. Interest is at a nominal rate of 12% compounded monthly. Find the outstanding balance at the end of the first year OB_{12}.

1. 100,141
2. 100,311
3. 100,581
4. 100,791
5. 100,921



$$i = \frac{12\%}{12} = 1\%$$

$$OB_0 = 100{,}000 = Z\, a_{\overline{72}|0.01} + v^{72}\, 2Z\, a_{\overline{72}|0.01}$$

$$= Z\, a_{\overline{72}|0.01}\left(1 + 2v^{72}\right)$$

$$\Rightarrow Z = \frac{100{,}000}{a_{\overline{72}|0.01}\left(1 + 2v^{72}\right)} = 988.89\$$$

$$OB_{12} = OB_0 \times (1 + 0.01)^{12} - Z\, s_{\overline{12}|0.01}$$

$$= 100{,}140.95\$$$

1. With force of interest δ_t:

$$A(t_2) = A(t_1)e^{\int_{t_1}^{t_2} \delta_s ds}$$

2. Annuity immediate

$$a_{\overline{n|}\ i} = \frac{1-(1+i)^{-n}}{i} \quad ; \quad s_{\overline{n|}\ i} = \frac{(1+i)^n - 1}{i}$$

3. Annuity due

$$\ddot{a}_{\overline{n|}\ i} = \frac{1-v^n}{d} \quad ; \quad \ddot{s}_{\overline{n|}\ i} = \frac{(1+i)^n - 1}{d} \quad ; \quad d = \frac{i}{1+i}$$

4. Geometric annuity immediate

$$PV = \frac{1 - \left(\frac{1+r}{1+i}\right)^n}{i-r} \quad ; \quad FV = \frac{(1+i)^n - (1+r)^n}{i-r}$$

5. Increasing arithmetic annuity immediate

$$(Ia)_{\overline{n|}\ i} = \frac{\ddot{a}_{\overline{n|}\ i} - nv^n}{i} \quad ; \quad (Is)_{\overline{n|}\ i} = \frac{\ddot{s}_{\overline{n|}\ i} - n}{i}$$

6. Decreasing arithmetic annuity immediate

$$(Da)_{\overline{n|}\ i} = \frac{n - a_{\overline{n|}\ i}}{i} \quad ; \quad (Ds)_{\overline{n|}\ i} = \frac{n(1+i)^n - s_{\overline{n|}\ i}}{i}$$

7. Present value of perpetuity immediate

$$a_{\overline{\infty|}\ i} = \frac{1}{i}$$

8. Present value of perpetuity due

$$\ddot{a}_{\overline{\infty|}\ i} = \frac{1+i}{i}$$

9. Present value of geometric perpetuity immediate (if $r < i$)

$$PV = \frac{1}{i-r}$$

10. Present value of increasing arithmetic perpetuity immediate

$$(Ia)_{\overline{\infty|}\ i} = \frac{1}{i} + \frac{1}{i^2}$$

11. Loan amortization

$$OB_t = OB_{t-1}(1+i) - K_t \quad ; \quad I_t = OB_{t-1} \times i \quad ; \quad PR_t = K_t - I_t.$$

12. Retrospective form

$$OB_t = OB_0(1+i)^t - K_1(1+i)^{t-1} - K_2(1+i)^{t-2} - \cdots - K_{t-1}(1+i) - K_t$$

13. Prospective form

$$OB_t = K_{t+1} \times v + K_{t+2} \times v^2 + \cdots + K_n \times v^{n-t}$$

14. Amortization with n level payment of amount K each

$$OB_t = Ka_{\overline{n-t|}\ i} \ \ ; \ \ I_t = K(1 - v^{n-t+1}) \ \ ; \ \ PR_t = Kv^{n-t+1}$$

15. Bond price

$$P = C + (Fr - Cj)a_{\overline{n|}\ j}$$

16. Book value

$$BV_{t+1} = BV_t \times (1 + j) - Fr \ \ ; \ \ I_{t+1} = BV_t \times j \ \ ; \ \ PR_{t+1} = Fr - I_{t+1}$$

17. Internal Rate of Return i is the solution of the equation

$$\sum_k C_k v^{t_k} = 0$$

18. Dollar weighted return for a one-year period is

$$\frac{B - [A + \sum_{k=1}^n C_k]}{A + \sum_{k=1}^n C_k(1 - t_k)}$$

19. Time-weighted return for a one-year period

$$\frac{F_1}{A} \times \frac{F_2}{F_1 + C_1} \times \cdots \times \frac{F_n}{F_{n-1} + C_{n-1}} \times \frac{B}{F_n + C_n} - 1$$