AS201: Financial Mathematics
Dept. of Mathematics, KFUPM
Instructor: Ridwan A. Sanusi (PhD)
Major Exam 1
Sept. 27, 2025

Name. . . . . ID: . . . .

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

6. Only answers supported by work will be considered. Unsupported guesses will not be graded.

7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. Write important steps to arrive at the solution of the exam problems.

The test is 100 minutes, GOOD LUCK, and you may begin now!

1. Aya puts 10,000 into a bank account that pays an annual effective interest rate of 4% for ten years, with interest credited at the end of each year. If a withdrawal is made during the first five and one-half years, a penalty of 5% of the withdrawal is made. Aya withdraws K at the end of each of years 4, 5, 6 and 7. The balance in the account at the end of year 10 is 10,000. Calculate K (to the nearest 10).

A. 980
B. 960
C. 940
D. 1000
E. 1010

Solution (Exercise 1.1.4S (Assignment 1)):
Withdrawals at years 4 and 5 incur a 5% penalty, so effective deductions are 1.05K. Withdrawals at years 6 and 7 have no penalty. The balance at year 10 is set to 10,000. Solving the equation:
$[(((11,698.5856 - 1.05K) * 1.04 - 1.05K)*1.04 - K) * 1.04 - K] * (1.04)) = 10,000$
$K = 979.93$.

2. A mutual fund advertises that average annual compound rates of return for periods ending December 31, 2015, are: 10 years 13%, 5 years 17%, 2 years 15%, 1 year 22%. Find the annual rate of return for 2014.

A. 0.062
B. 0.073
C. 0.084
D. 0.095
E. 0.106

Solution (Exercise 1.1.5bii):

$$(1 + R_{2014})(1 + R_{2015}) = (1.15)^2, \ R_{2015} = 22\%.$$

$$\text{So } 1 + R_{2014} = \frac{1.15^2}{1.22} = 1.0840 \Rightarrow R_{2014} \approx 8.40\%.$$

3. Layan can receive either (i) \$100 at time 0, \$200 at time *n*, and \$300 at time 2n; or (ii) \$600 at time 10. At annual effective rate *i* the present values are equal. Given $v^n = 0.75941$, determine *i* (to the nearest 2 digits).

A. 0.02
B. 0.04
C. 0.06
D. 0.08
E. 0.10

Solution (Exercise 1.2.7S (Assignment 1)):

Solution. Let $x = v^n = 0.75941$. Then the PV equality is

$$100 + 200x + 300x^2 = 600\,v^{10} = 600\,x^{10/n}.$$

Compute left side numerically:

$$100 + 200(0.75941) + 300(0.75941)^2 \approx 425.19.$$

Hence

$$x^{10/n} \approx \frac{425.19}{600} \approx 0.70865.$$

Taking logs:

$$\frac{10}{n} = \frac{\ln(0.70865)}{\ln(0.75941)} \approx 1.2516 \quad \Rightarrow \quad n \approx 7.99 \approx 8.$$

Thus $n \approx 8$. Now $v = x^{1/n} = 0.75941^{1/8} \approx 0.96613$ so

$$i = \frac{1}{v} - 1 \approx \frac{1}{0.96613} - 1 \approx \boxed{3.56\%\text{ p.a. (approx.)}}.$$

4. Wafaa deposits X into a savings account at time 0, which pays interest at a nominal rate of *i*, compounded semi-annually. Nisreen deposits 2X into a different savings account at time 0, which pays simple interest at an annual rate of *i*. Wafaa and Nisreen earn the same amount of interest during the last 6 months of the 8^{th} year. Calculate *i*.

A. 0.0835
B. 0.0946
C. 0.0724
D. 0.0613
E. 0.1057

Solution (Exercise 1.4.4S (Assignment 1)):
Wafaa's interest in the last 6 months

$$= X\left(1+\frac{i}{2}\right)^{15}\cdot\frac{i}{2}$$

Nisreen's interest = $X * i$
Set equal:

$$\left(1+\frac{i}{2}\right)^{15}\cdot\frac{i}{2} = i \implies \left(1+\frac{i}{2}\right)^{15} = 2 \implies i = 2(2^{1/15}-1) \approx 0.0946$$

Answer: $\boxed{0.0946}$

---

5. Fajr makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest $\delta_t = 0.01(t^2)$, $t > 0$. The amount of interest earned from time 3 to time 6 is X. Calculate X.

A. 340.2
B. 451.3
C. 562.4
D. 673.5
E. 784.6

Solution (Exercise 1.6.6S):

Solution: Balance at time 3: $100e^{9/100} + X$.
Balance at time 6: $(100e^{9/100} + X)e^{63/100}$.
Interest from 3 to 6:

$$I = (100e^{9/100} + X)(e^{63/100} - 1) = X$$

Solving gives $X \approx 784.6$.

6. Sama deposits 100 into a bank account. Her account is credited interest at a nominal rate convertible semi-annually. At the same time, Noor deposits 100 into a separate account. Noor's account is credited interest at a force of interest of δ. After 7.5 years, the value of each account is 200. Calculate $i - \delta$.

A. 0.001
B. 0.002
C. 0.003
D. 0.004
E. 0.005

Solution (Exercise 1.6.5S):

For Sama:

$$100(1 + i/2)^{15} = 200 \implies i = 2(2^{1/15} - 1)$$

For Noor:

$$100e^{7.5\delta} = 200 \implies \delta = \frac{\ln 2}{7.5}$$

Then:

$$i - \delta \approx 0.094554 - 0.0924196 = 0.0021344$$

7. Suppose that for the coming year inflation is forecast at an annual effective rate of $r = 0.15$ and interest is forecast at annual effective rate $i = 0.10$. What will be the corresponding real, or inflation-adjusted rate of interest for the coming year?

A. 0.043
B. -0.043
C. -0.434
D. 0.434
E. 0.050

Solution (Exercise 1.7.1 (Assignment 1)):

$$i' = \frac{i - r}{1 + r} = \frac{-0.05}{1.15} = -0.043478$$

8. Sara deposits 1000 into a fund. The fund earns:
i) an annual nominal rate of interest of 4% convertible quarterly for the first three years;
ii) a constant annual force of interest of 5% for the next three years; and
iii) an annual nominal discount rate of 6% convertible semi-annually thereafter.
Calculate the amount in the fund at the end of ten years.

(A) 1658
(B) 1667
(C) 1670
(D) 1674
(E) 1677

Solution (EXAM FM SAMPLE QUESTION 321, Difficulty 1.54):

$$1000\left(1+\frac{0.04}{4}\right)^{3x4}\left(e^{0.05(3)}\right)\left(1-\frac{0.06}{2}\right)^{-2x4}$$

$$=1670.42$$

9. Seifeldin's new savings account earns an annual effective interest rate of 3% for each of the first ten years and an annual effective interest rate of 2% for each year thereafter.
Seifeldin deposits an amount *X* at the beginning of each year, starting with year 1, so that the account balance just after the deposit in the beginning of year 26 is 100,000.
Determine which of the following is an equation of value that can be used to solve for *X*.

(A) $$\frac{100,000}{(1.03)^{10}(1.02)^{15}}=X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}}+X\sum_{k=12}^{26}\frac{1}{(1.02)^{k-1}}$$

(B) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}}=X\sum_{k=1}^{10}\frac{1}{(1.03)^{k}}+X\sum_{k=11}^{26}\frac{1}{(1.02)^{k}}$$

(C) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}}=X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}}+X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}$$

(D) $$\frac{100,000}{(1.03)^{10}(1.02)^{15}}=X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}}+X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}$$

(E) $$\frac{100,000}{(1.03)^{10}(1.02)^{16}}=X\sum_{k=1}^{10}\frac{1}{(1.03)^{k}}+X\sum_{k=11}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-10}}$$

Solution (EXAM FM SAMPLE QUESTION 233, Difficulty 7.65):

233. Solution: D

Let t represent the number of years since the beginning of year 1. Since the annual effective interest rate is 3% in each of years 1 through 10, and 2% each year thereafter, the present value of an amount is calculated by multiplying it by a discounting factor of $\frac{1}{(1.03)^t}$ if $0\le t\le 10$, and $\frac{1}{(1.03)^{10}(1.02)^{t-10}}$ if $t>10$.

The balance is initially 0 (the account is new before the first deposit). Deposits of *X* are made at times t = 0, 1, 2, 3,…, 25, or equivalently at time $t=k-1$ for each whole number k from 1 to 26 inclusive.

For the final balance to become 0, a withdrawal of 100,000 at time t = 25 would be needed. Since the net present value of the cash flows (withdrawals minus deposits) must be zero, in a time period from a zero balance to another zero balance, we have

$$\frac{100,000}{(1.03)^{10}(1.02)^{25-10}}-X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}}-X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-1-10}}=0$$

$$\frac{100,000}{(1.03)^{10}(1.02)^{15}}=X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}}+X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}.$$

10. Sadem can receive one of the following two payment streams:
(i) 100 at time 0, 200 at time n years, and 300 at time $2n$ years
(ii) 600 at time 10 years
At an annual effective interest rate of i, the present values of the two streams are equal.
Given $v^n = 0.76$, calculate i.

(A) 3.5%
(B) 4.0%
(C) 4.5%
(D) 5.0%
(E) 5.5%

Solution (EXAM FM SAMPLE QUESTION 16, Difficulty 0):

Equating present values:

$$100 + 200v^n + 300v^{2n} = 600v^{10}$$

$$100 + 200(0.76) + 300(0.76)^2 = 600v^{10}$$

$$425.28 = 600v^{10}$$

$$0.7088 = v^{10}$$

$$0.96617 = v$$

$$1.03501 = 1 + i$$

$$i = 0.035 = 3.5\%.$$

11.The Kingdom of Saudi Arabia sends Mohammed's family allowance payment of 30SAR every month for Mohammed's child. Mohammed deposits the payment in a Riyad bank account on the last day of each month. The account earns interest at the annual rate of 9% compounded monthly and the interest is paid into the account on the last day of each month. If the first payment is deposited on May 31, 2007, what is the account balance on December 31, 2018, including the payment just made and interest paid that day?

(A) 7385.91
(B) 6274.80
(C) 5163.79
(D) 4052.68
(E) 3941.57

Solution (EXAMPLE 2.1):

SOLUTION

The following line diagram illustrates the accumulation in the account from one deposit to the next.

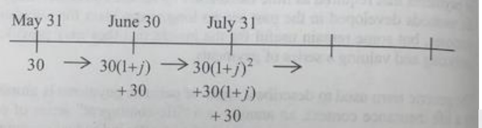


FIGURE 2.1

The one-month compound interest rate is $j = .0075$. The balance in the account on June 30, 2007, including the payment just deposited and the accumulated value of the May 31 deposit is

$$C_2 = 30(1+j) + 30 = 30[(1+j)+1].$$

The balance on July 31, 2007 is

$$C_3 = C_2(1+j) + 30 = 30\left[(1+j)+1\right](1+j) + 30 = 30\left[(1+j)^2 + (1+j) + 1\right].$$

Continuing in this way we see that the balance just after the m^{th} deposit is $C_m = 30\left[(1+j)^{m-1} + \cdots + (1+j)^2 + (1+j) + 1\right]$, which is the accumulation of those first m deposits. By applying the geometric series formula, the balance on December 31, 2018, just after the 140^{th} deposit is

$$30\left[(1+j)^{139} + (1+j)^{138} + \cdots + (1+j) + 1\right]$$

$$= 30\left[\frac{(1+j)^{140} - 1}{(1+j) - 1}\right] = 30\left[\frac{(1.0075)^{140} - 1}{.0075}\right] = 7385.91. \quad \square$$

12. Nouf's grandchild will Begin a four-year college program in one year. Nouf wishes to open an Alinma bank account with a single deposit today so that her grandchild can withdraw 1000SAR each year for four years from the account, with the first withdrawal taking place one year from now, and subsequent withdrawals each year after that. The account has an effective annual interest rate of 6% and the deposit is calculated so that the account balance will be reduced to 0 when the fourth withdrawal is made four years from now.
Determine the amount of the deposit Nouf makes today.

(A) 3465.11
(B) 3576.22
(C) 3687.33
(D) 3798.44
(E) 3809.55

Solution (EXAMPLE 2.6):

Suppose that the amount of the initial deposit is X. If we track the account balance after each withdrawal, we see the following:

Balance after 1^{st} withdrawal:

$$X(1.06)-1000$$

Balance after 2^{nd} withdrawal:

$$[X(1.06)-1000](1.06)-1000 \quad = \quad X(1.06)^2-1000(1.06)-1000$$

Balance after 3^{rd} withdrawal:

$$[X(1.06)^2-1000(1.06)](1.06)-1000$$
$$= X(1.06)^3-1000(1.06)^2-1000(1.06)-1000$$

Balance after 4^{th} withdrawal:

$$X(1.06)^4-1000(1.06)^3-1000(1.06)^2-1000(1.06)-1000.$$

In order for the balance to be 0 just after the 4^{th} withdrawal, we must have

$$X(1.06)^4 \;=\; 1000(1.06)^3+1000(1.06)^2+1000(1.06)+1000,$$

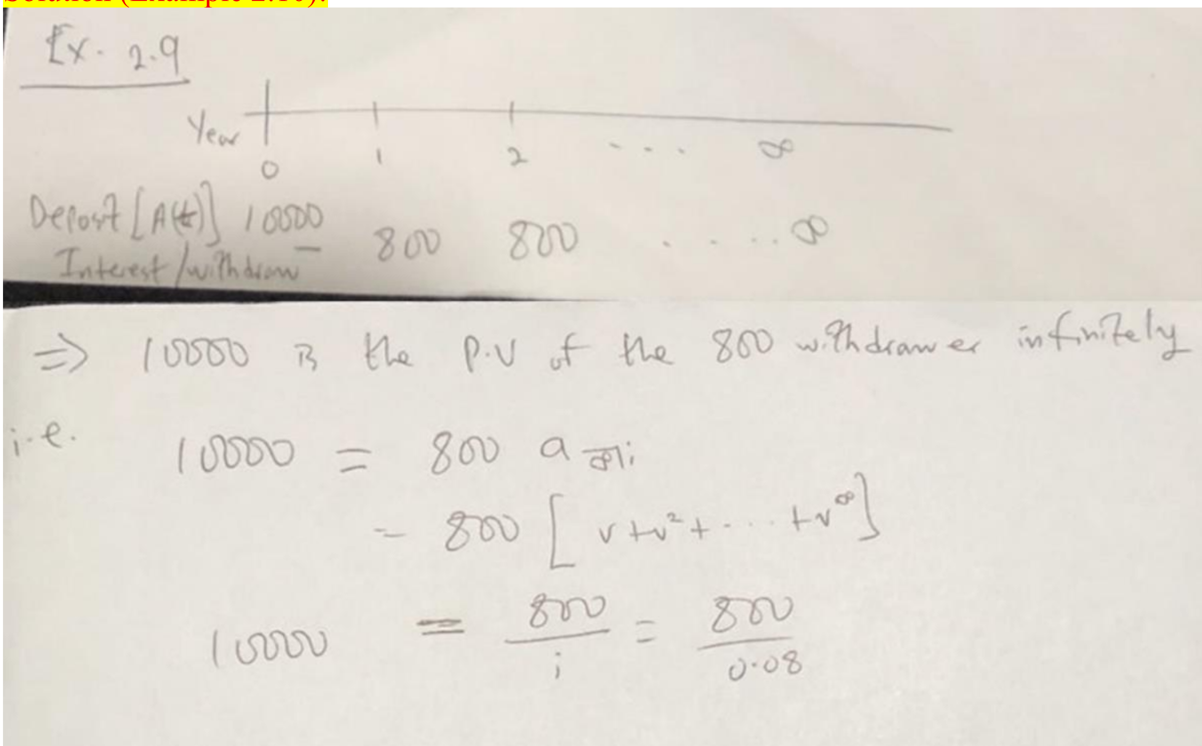
or equivalently,

$$X \;=\; \frac{1000}{1.06}+\frac{1000}{(1.06)^2}+\frac{1000}{(1.06)^3}+\frac{1000}{(1.06)^4}$$
$$= 1000\left[v+v^2+v^3+v^4\right]$$
$$= 3{,}465.11. \qquad \square$$

13. At an annual effective interest rate of 8%, a deposit of \$X will generate interest of \$800 at the end of a year. If that interest payment is "withdrawn" from the account, but the principal is allowed to remain in the account for another year, it will generate another interest payment of \$800 at the end of the second year. Find X if this go on indefinitely and the only amount withdrawn at the end of each year is the interest generated for that year.

A. 8000
B. 8500
C. 9000
D. 9500
E. 10000

Solution (Example 2.10):



14. Azzam began saving money for his retirement by making deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 2001.
Azzam became unemployed and missed making deposits 60 through 72. He then continued making monthly deposits of 200.
How much did Azzam accumulate in his fund, including interest on December 31, 2015, assuming payments continued through to December 1, 2015?

A. 59554.56
B. 49225.10
C. 4614.73
D. 58454.56
E. 53839.83

Solution (Example 2.12):

Example 2.12

Date	Jan 1/01	1/2/01		1/11/05	1/12/05	1/1/06		1/12/06	1/1/07		1/12/15	31/12/15
Time	0	1	...	58	59	60	...	71	72	...	178	179
Deposit	200	200		200	—	—	...	—	200	...	200	—
Deposit #	1	2		59	60	61	...	72	73	...	179	180

121 months

≡ His accumulated value if he did not miss (A) — the accumulated value of the missed payment (B)

$i_{monthly} = \frac{6}{12}\% = 0.005$

A: $200\left[(1.005)^{180} + (1.005)^{179} + \cdots + (1.005)\right]$

$= 200\,\ddot{s}_{\overline{180}|0.005} = 200\,\frac{1.005^{180}-1}{0.005} \times 1.005$

$= 58,454.56$

120 months from 1/12/05 – 31/12/15

B: $200\left[1.005^{121} + 1.005^{120} + \cdots + 1.005^{109}\right]$

$= 200 \times 1.005^{109}\left[1.005^{12} + 1.005^{11} + \cdots + 1\right]$

$= 200 \times 1.005^{109}\, s_{\overline{13}|0.005}$

$\equiv 200 \times 1.005^{108}\, \ddot{s}_{\overline{13}|0.005} = 4,614.73$

$A - B = 53,839.83$

15. Ade's child was born January 1, 2006. Ade receives monthly family allowance payments on the last day of each month, beginning January 31, 2006. The payments are increased by 12% each calendar year to meet cost-of-living increases. Monthly payments are constant during each calendar year at 25 each month in 2006, rising to 28 each month in 2007, 31.36 each month in 2008, and so on. Immediately upon receipt of a payment, Ade deposits it in an account earning $i^{(12)} = .12$ with interest credited on the last day of each month. Find the accumulated amount in the account on the child's 18th birthday.

A. 46626.99
B. 44515.88
C. 43404.77
D. 42393.66
E. 41282.55

Solution (Example 2.19):

The change in payment amount occurs once each year, but the payments are made monthly. The accumulated value on January 1, 2024, the 18th birthday, can be written as the sum of the accumulated values of each of the deposits as

$$25(1.01)^{215} + 25(1.01)^{214} + \cdots + 25(1.01)^{204}$$
$$+25(1.12)(1.01)^{203} + 25(1.12)(1.01)^{202} + \cdots + 25(1.12)(1.01)^{192}$$
$$+25(1.12)^2(1.01)^{191} + 25(1.12)^2(1.01)^{190} + \cdots + 25(1.12)^2(1.01)^{180}$$
$$+\cdots + 25(1.12)^{17}(1.01)^{11} + 25(1.12)^{17}(1.01)^{10} + \cdots + 25(1.12)^{17}.$$

A way of simplifying this sum is to first group the deposits on an annual basis, and for each year find the accumulated value of that year's deposits at the end of that year, as shown in Table 2.2.

TABLE 2.2

Year	Accumulated Value of Deposits on December 31
2006	$25s_{\overline{12}\rvert .01} = X$
2007	$28s_{\overline{12}\rvert .01} = 25(1.12)s_{\overline{12}\rvert .01} = (1.12)X$
2008	$31.36s_{\overline{12}\rvert .01} = 25(1.12)^2 s_{\overline{12}\rvert .01} = (1.12)^2 X$
⋮	
2023	$25(1.12)^{17} s_{\overline{12}\rvert .01} = (1.12)^{17} X$

The 216 monthly deposits are equivalent to 18 geometrically increasing annual deposits of $X, (1.12)X, (1.12)^2 X, \ldots, (1.12)^{17} X$, made at the end (Dec. 31) of years 2006, 2007, 2008, … , 2024. The accumulated value at the time of the final deposit (Dec. 31, 2023) is

$$(1+i)^{17} X + (1+i)^{16}(1.12)X + (1+i)^{15}(1.12)^2 X + \cdots + (1+i)^0 (1.12)^{17} X.$$

This is the accumulated value of a geometric payment annuity-immediate with $n = 18$ payments, geometric growth rate $1+r = 1.12$, annual effective interest rate $i = (1.01)^{12} - 1$, and initial payment $25s_{\overline{12}\rvert .01} = X$. We can use Equation 2.23 to find the accumulated value at the end of 18 years (time of the final payment), with $r = .12$, $1+i = (1.01)^{12}$, and $n = 18$.

$$X \times \left[\frac{(1+i)^{18} - (1+r)^{18}}{i-r}\right] = 25s_{\overline{12}\rvert .01} \times \frac{(1.01)^{216} - (1.12)^{18}}{(1.01)^{12} - 1 - .12}.$$

Since $X = 25s_{\overline{12}\rvert .01} = 317.06$, the accumulated value is 41,282.55. ☐

16. Ronaldo bought an increasing perpetuity-due with annual payments starting at amount 5 and increasing by 5 each year until the payment amount reaches 100. The payments remain at 100 thereafter. The effective annual interest rate is 7.5%. Determine the present value of this perpetuity.

A. 828.5
B. 817.7
C. 806.6
D. 795.5
E. 785.4

Solution (Example 2.23):

SOLUTION

There are at least two ways in which this perpetuity present value can be formulated.

(i) We can separate the first 20 payments (increasing from 5 to 100) from the 21[st] payment and beyond (level at 100). The present value as a perpetuity-due (valued at the time of the first payment) is

$$5(I\ddot{a})_{\overline{20}|.075} + v^{20} \times 100\ddot{a}_{\overline{\infty}|.075} = 5(1.075)(Ia)_{\overline{20}|.075} + v^{20} \times 100\ddot{a}_{\overline{\infty}|.075}$$

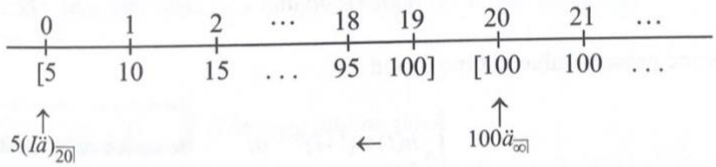
$$= 5(1.075)\left(\frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{.075}\right) + v^{20} \times \frac{100}{d}$$

$$= 447.975 + 337.426 = 785.40.$$

Note that the 20-payment increasing annuity-due can be formulated as

$$(I\ddot{a})_{\overline{20}|} = \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{d} = \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{i/(1+i)} = (1+i) \times \frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{i} = (1+i)(Ia)_{\overline{20}|}.$$

The overall valuation is illustrated in the following time-diagram.



(ii) We can separate the series into a combination of a level perpetuity-due minus a 19 year decreasing annuity-due.

Original series is equivalent to	5	10	15	⋯	95	100	100	100	⋯
Level perp-due combined with	100	100	100	⋯	100	100	100	100	⋯
Decreasing annuity-due	−95	−90	−85	⋯	−5				

Note the negative payments on the decreasing annuity-due. The present value of the two combined series is

$$100\ddot{a}_{\overline{\infty}|} - 5(D\ddot{a})_{\overline{19}|}$$
$$= 100\left(\frac{1}{d}\right) - 5\left[\frac{19 - a_{\overline{19}|}}{d}\right] = 1{,}433.333 - 647.933 = 785.40.$$

17. Messi makes a deposit at the end of each year for 10 years into a fund earning interest at a 4% annual effective rate. The first deposit is equal to X, with each subsequent deposit 9.2% greater than the previous year's deposit. The accumulated value of the fund immediately after the 10th deposit is 5000. Calculate X.

(A) 255
(B) 260
(C) 270
(D) 279
(E) 293

Solution (EXAM FM SAMPLE QUESTION 215, Difficulty 3.22):

215. Solution: D

Equating present values gives

$$\frac{5000}{1.04^{10}} = X\left[\frac{1-\left(\frac{1.092}{1.04}\right)^{10}}{.04-.092}\right]$$

$$3377.82 = X(12.094127)$$

$$X = 279.29$$

18. Saka receives annual payments from a 20-year annuity-immediate. The payment in year 1 is 100 and in each succeeding year the payment is 90% of the prior year's payment. Upon receipt of each payment, Saka invests the payment in a savings account earning interest at a 3% annual effective rate. Calculate the balance in the savings account immediately after Saka invests the last annuity payment.

(A) 696
(B) 717
(C) 739
(D) 1296
(E) 1335

Solution (EXAM FM SAMPLE QUESTION 302, Difficulty 5.58):

302. Solution: D

$$FV = 100\left[\frac{1-\left(\frac{0.9}{1.03}\right)^{20}}{0.03+0.10}\right](1.03)^{20}$$

$$FV = 1295.80$$

Alternatively,

$$FV = 100[(1.03)^{19} + 0.9(1.03)^{18} + \cdots + (0.9)^{19}(1.03)^{0}] = 100\frac{(1.03)^{19} - (0.9)^{20}(1.03)^{-1}}{1-0.9(1.03)^{-1}} = 1295.80.$$

19. An investor deposits 1000 at the beginning of each year for five years in a fund earning a 5% annual effective interest rate. The interest from this fund can be reinvested at a 4% annual effective interest rate. Calculate the total accumulated value at the end of five years.

(A) 5058
(B) 5227
(C) 5436
(D) 5641
(E) 5791

Solution (EXAM FM SAMPLE QUESTION 363, Difficulty 5.01):

363. Solution: E

$$1000(5) + \left[50s_{\overline{5}|0.04} + 50\left[\frac{s_{\overline{5}|0.04} - 5}{0.04}\right]\right] = 5791.22$$

OR

🔹 **Step 1. Understand the setup**

- Deposit: 1000 at **beginning of each year** for 5 years → at times 0, 1, 2, 3, 4.
- Fund interest rate = 5% effective per year.
- Interest **earned** cannot remain in the fund; it is reinvested separately at 4%.
- Principal remains in the fund until withdrawn (after 5 years).
- Task: Find total accumulated value at end of year 5 (fund balance + reinvested interest).

This is essentially a **split-interest problem**: principal accumulates at 5%, but interest flows out each year and grows at 4%.

🔹 **Step 2. Track one deposit (say the deposit at time t)**

Suppose a deposit of 1000 is made at time k (with $k = 0, 1, 2, 3, 4$).

- Principal stays in the fund for $(5-k)$ years, growing at 5%:

$$\text{Principal portion at time } 5 = 1000.$$

(because only the **principal** 1000 remains; interest is siphoned off).

Actually careful: since only interest is siphoned, the **principal** is always 1000 at maturity (not accumulated at 5%).

- Each year until year 5, it produces interest at 5% of 1000 = 50.
- Each interest payment occurs at the **end of each year after deposit**, and is reinvested at 4% until year 5.

So:

For deposit at time k, the interest payments are 50 at times $k+1, k+2, \ldots, 5$. Each is reinvested at 4% until time 5.

🔹 **Step 3. Contribution from one deposit**

Deposit at time k contributes:

$$\text{Value at } 5 = 1000 + \sum_{j=1}^{5-k} 50 \cdot (1.04)^{5-(k+j)}.$$

Explanation:

- The j-th interest payment (50) happens at time $k+j$.
- It grows from time $k+j$ to 5, i.e. $5-(k+j)$ years, at 4%.

🔹 **Step 4. Total contribution**

So total at time 5 is

$$V = \sum_{k=0}^{4}\left[1000 + \sum_{j=1}^{5-k} 50(1.04)^{5-(k+j)}\right].$$

Step 5. Compute explicitly

- For $k=0$: Deposit at time 0 → principal 1000. Interest payments: 50 at times 1–5.

$$1000+50(1.04)^4+50(1.04)^3+50(1.04)^2+50(1.04)^1+50(1.04)^0.$$

- For $k=1$: Deposit at time 1 → principal 1000. Interest: 50 at times 2–5.

$$1000+50(1.04)^3+50(1.04)^2+50(1.04)^1+50(1.04)^0.$$

- For $k=2$:

$$1000+50(1.04)^2+50(1.04)^1+50(1.04)^0.$$

- For $k=3$:

$$1000+50(1.04)^1+50(1.04)^0.$$

- For $k=4$:

$$1000+50(1.04)^0.$$

Step 6. Combine

Total principal = $5\times1000=5000$.

Total reinvested interest =

$$50\left[(1.04)^4+(1.04)^3+(1.04)^2+(1.04)^1+(1.04)^0\right]$$

- $$50\left[(1.04)^3+(1.04)^2+(1.04)^1+(1.04)^0\right]$$
- $$50\left[(1.04)^2+(1.04)^1+(1.04)^0\right]$$
- $$50\left[(1.04)^1+(1.04)^0\right]$$
- $$50\left[(1.04)^0\right].$$

Step 7. Simplify

Coefficient of each power of 1.04:

- $(1.04)^4$: appears once → 50
- $(1.04)^3$: appears twice → 100
- $(1.04)^2$: appears three times → 150
- $(1.04)^1$: appears four times → 200
- $(1.04)^0$: appears five times → 250

So reinvested interest total =

$$50(1.04)^4+100(1.04)^3+150(1.04)^2+200(1.04)^1+250(1.04)^0.$$

Step 8. Final expression

$$V=5000+50(1.04)^4+100(1.04)^3+150(1.04)^2+200(1.04)^1+250.$$

Step 9. Evaluate (exact to 2 decimals)

- $(1.04)^4=1.16985856.$ → term = 58.49
- $(1.04)^3=1.124864.$ → term = 112.49
- $(1.04)^2=1.0816.$ → term = 162.24
- $(1.04)^1=1.04.$ → term = 208.00
- $(1.04)^0=1.$ → term = 250.00

Sum reinvested = 791.22.

So total value

$$V=5000+791.22=\boxed{5791.22}.$$

20. A perpetuity-immediate has annual payments starting at 1, increasing by 1 each year until it reaches 10, then decreasing by 1 each year until it reaches 5, and remaining at 5 thereafter. The present value of this perpetuity at an annual effective interest rate of 9% is X. Calculate X.

(A) 51
(B) 53
(C) 55
(D) 58
(E) 61

Solution (EXAM FM SAMPLE QUESTION 398, Difficulty 3.44):

398. Solution: D

$$(Ia)_{\overline{\infty}|}=\frac{1}{i}+\frac{1}{i^2}=\frac{1}{0.09}+\frac{1}{0.09^2}=134.5679$$

$$X=(Ia)_{\overline{\infty}|}-2v^{10}(Ia)_{\overline{\infty}|}+v^{15}(Ia)_{\overline{\infty}|}$$

$$=134.5679(1-2\times0.42241+0.274538)$$

$$=57.83$$