Dept. of Mathematics and Statistics

King Fahd University of Petroleum & Minerals

AS201: Financial Mathematics

Dr. Ridwan A. Sanusi

Final Exam Term 251

Dec. 25, 2025

12.30 PM – 2:30 PM

Name…………………………………………………. ID#:_________

Instructions.

1. Mobile phones are not allowed. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so before the exam starts. Nobody will be allowed to leave the room once the exam starts.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. ***Only answers supported by work will be considered. Unsupported guesses will not be graded.***
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***
9. Each question is worth 2.5 points (1 for correct option and 1.5 for correct working), except Q13 and Q17, which are worth 1 point each (for correct option only).

1. Sadem opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semi-annually. Nisreen also opens a bank account with 1000 at the same time as Sadem, but it grows at an annual nominal interest rate of 3% convertible monthly. For each account, interest is credited only at the end of each interest conversion period. Calculate the number of months required for the amount in Sadem's account to be at least double the amount in Nisreen's account.

(A) 276 (B) 282 (C) 285 (D) 286 (E) 288

SOLUTION: 288 (SOA; Difficulty level: 11)

62. Solution: E

Let n = years. The equation to solve is

$$1000(1.03)^{2n} = 2(1000)(1.0025)^{12n}$$

$$2n\ln 1.03 + \ln 1000 = 12n\ln 1.0025 + \ln 2000$$

$$0.029155n = 0.69315$$

$$n = 23.775.$$

This is 285.3 months. The next interest payment to Sadem is at a multiple of 6, which is 288 months.

2. A level monthly contribution of X is required to purchase an annual perpetuity of 5000 that commences five years from today. The contributions are made at the end of each month for 60 months. The last contribution is made at the same time as the first payment from the perpetuity. The annual nominal interest rate is 12% compounded monthly. Calculate X.

(A) 402 (B) 483 (C) 510 (D) 544 (E) 581

SOLUTION: 544 (SOA; Difficulty level: 6.39. Assignment from Group work: Week 4-5: Chapter 2)

361. Solution: D

$$Xs_{\overline{60}|i} = \frac{5000}{d} = \frac{5000}{i}(1+i) = \frac{5000(1.01)^{12}}{(1.01)^{12}-1}$$

$$44,423.95 = 81.67X$$

$$X = 543.94$$

100

3. Sarah borrows X at 12.5% effective and makes level payments for n years.
The interest portion of the final payment is 153.86. The total principal repaid by time n-1 is 6009.12. The principal repaid in first payment is Y. Compute Y.

A. 479.74 B. 1230.88 C. 7240.00 D. 1384.74 E. 905.00

SOLUTION: 479.74 (ASS. 2; EXERCISE 3.2.26S)

Step 1: Find B_{n-1} (balance after payment $n-1$)

Interest in final payment n = $B_{n-1} \times 0.125 = 153.86$

$$B_{n-1} = \frac{153.86}{0.125} = 1230.88$$

Step 2: Find X (original loan)

Principal repaid by time $n-1$ = $X - B_{n-1} = 6009.12$

$$X = 6009.12 + 1230.88 = 7240.00$$

Step 3: Find P (level payment)

Balance after $n-1$ equals present value of last payment:

$$B_{n-1} = P \cdot v, \quad v = \frac{1}{1.125}$$

$$P = 1230.88 \times 1.125 = 1384.74$$

Step 4: Find Y (principal in first payment)

Interest in first payment = $X \times 0.125 = 905.00$

$$Y = P - 905.00 = 1384.74 - 905.00 = 479.74$$

$$\boxed{479.74}$$

4. A 10% bond with face amount 100 is callable on any coupon date from 15.5 years after issue up to the maturity date which is 20 years from issue. Find the minimum annual yield to maturity if the bond is purchased for 120.

A. 0.0388 B. 0.0776 C. 0.0640 D. 0.1280 E. 0.1000

SOLUTION: 0.0776 (ASS. 3; EXERCISE 4.3.1biii)

(iii) Price = 120 (Premium Bond):

If called at maturity (n=40): $120 = 100v_j^{40} + 5a_{\overline{40}|j} \Rightarrow j \approx 0.04$

If called early (n=31): $120 = 100v_j^{31} + 5a_{\overline{31}|j} \Rightarrow j \approx 0.0388$

Minimum yield: $j = 0.0388$ per period

Annual nominal yield $= 2 \times 0.0388 = 0.0776 = 7.76\%$

5. You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.

A. -0.238 B. -0.250 C. 0.250 D. 0.238 E. 0.122

SOLUTION: -0.25 (ASS. 3; EXERCISE 5.2.2S)

Use time-weighted return to find X:

$$\text{Time-weighted return} = \frac{12}{10} \times \frac{X}{12+X} - 1 = 0$$

$$\frac{12}{10} \times \frac{X}{12+X} = 1$$

$$\frac{12X}{10(12+X)} = 1$$

$$12X = 10(12+X)$$

$$12X = 120 + 10X$$

$$2X = 120 \Rightarrow X = 60$$

Calculate dollar-weighted return Y: Using simple interest approximation:

$$10(1+Y) + 60\left(1 + \frac{Y}{2}\right) = 60$$

$$10 + 10Y + 60 + 30Y = 60$$

$$70 + 40Y = 60$$

$$40Y = -10 \Rightarrow Y = -0.25 = -25\%$$

6. Suppose that the term structure of interest rates has the following schedule of spot rates for maturities of 1, 2, 3, and 4 years:

Maturity	1-Year	2-Year	3-Year	4-Year
Spot rate	0.05	0.10	0.15	0.20

Find the one year forward rates of interest for year 3.

A. 0.0512 B. 0.1524 C. 0.2596 D. 0.3634 E. 0.122

Answer: 0.2596 (Example 6.3)

Example 6·3

$s_0(1) = 0.05, \quad s_0(2) = 0.10, \quad s_0(3) = 0.15, \quad s_0(4) = 0.20$

$f_{[0,1]} = s_0(1) = 0.05$

$f_{[1,2]} = \frac{(1+s_0(2))^2}{1+s_0(1)} - 1 = \frac{(1+0.1)^2}{1+005} - 1 = 0.1524$

$f_{[2,3]} = \frac{(1+0.5)^3}{(1+0.1)^2} - 1 = 0.2596$

$f_{[3,4]} = \frac{1.2^4}{1.15^3} - 1 = 0.3634$

7. You are given the following information for 4 bonds. All coupon and yield-to-maturity rates are nominal annual convertible twice per year.

Bond	Time to Maturity	Coupon Rate	YTM
1	$\frac{1}{2}$-year	4%	5%
2	1-year	6%	10%
3	$\frac{1}{2}$-year	4%	15%
4	2-year	8%	15%

Find the associated term structure for zero coupon bonds with maturity of 1.5-year (quotations should be nominal annual rates convertible twice per year).

A. 0.0500 B. 0.1007 C. 0.1515 D. 0.1523 E. 0.2523

SOLUTION: 0.15151 (ASS. 3; EXERCISE 6.1.5)

Step 3: Bond 3 (1.5-year maturity)

Coupon per period = 2

Price using YTM: $P = \frac{2}{1.075} + \frac{2}{(1.075)^2} + \frac{102}{(1.075)^3} = 1.8605 + 1.7307 + 81.9806 = 85.5718$

Use bootstrapping with known $s_{0.5} = 0.05$ and $s_1 = 0.10078$:

$$85.5718 = \frac{2}{1.025} + \frac{2}{(1.05039)^2} + \frac{102}{(1 + s_{1.5}/2)^3}$$

$$85.5718 = 1.9512 + 1.8116 + \frac{102}{(1 + s_{1.5}/2)^3}$$

$$\frac{102}{(1 + s_{1.5}/2)^3} = 85.5718 - 3.7628 = 81.8090$$

$$(1 + s_{1.5}/2)^3 = \frac{102}{81.8090} = 1.2468$$

$$1 + s_{1.5}/2 = 1.2468^{1/3} = 1.07626$$

$$s_{1.5} = 2 \times 0.07626 = 0.15251$$

$$\boxed{0.15151}$$

8. Apply the Macaulay approximation to find the approximate change in bond price for bonds of face amount 100 when there is an increase in yield rate of 10 basis points for a 10-coupon bond with 5% coupons and a yield rate of 10%. What is the relative error in this approximation?

A. 3.2×10^{-5} B. 4.4×10^{-6} C. 2.8×10^{-4} D. 1.1×10^{-7} E. 2.9×10^{-10}

SOLUTION: 4.4 X 10-6 (Example 7.3)

EXAMPLE 7.3 *(Approximating the change in PV using duration)*

Compare the two first-order approximation methods described above by applying them to find the approximate change in bond price for bonds of face amount 100 when there is an increase in yield rate of 10 basis points (.1%) for the following two bonds from Example 7.2:
(a) 10-coupon bond with 5% coupons and a yield rate of 10%
(b) 60-coupon bond with 10% coupons and yield rate of 5%

The exact prices of bonds (a) and (b) are
$100v_{.1}^{10} + 5a_{\overline{10}|.1} = 69.2772$ and
$100v_{.05}^{60} + 10a_{\overline{60}|.05} = 194.6464$.

The exact prices of bonds (a) and (b) at the changed yield rates are
$100v_{.101}^{10} + 5a_{\overline{10}|.101} = 68.7969$ and
$100v_{.051}^{60} + 10a_{\overline{60}|.051} = 191.2204$.

The Macaulay durations of the two bonds are 7.661 for bond (a) and 18.772 for bond (b). The modified durations of the two bonds are $D_{mod}(.1) = \frac{7.661}{1.1} = 6.965$ for bond (a) and $D_{mod}(.05) = \frac{18.772}{1.05} = 17.878$ for bond (b).

Applying the first-order modified approximation, the bond price for bond (a) with yield rate 10.1% will be $= 69.2772 - .001 \times 6.965 \times 69.2772 = 68.7947$. The relative error in this approximation is $\frac{|68.7947 - 68.7969|}{68.7969} = 3.2 \times 10^{-5}$.

Applying the first-order Macaulay approximation, the bond price for bond (a) with yield rate 10.1% will be

$$P(.101) \approx P(.10) \times \left(\frac{1.1}{1.101}\right)^{7.661} = 69.2772 \times .9931 = 68.7966.$$

The relative error in this approximation is
$\frac{|68.7966 - 68.7969|}{68.7969} = 4.4 \times 10^{-6}$.

For bond (b), the first-order modified approximation is 191.167, with relative error 2.8×10^{-4} and the first-order Macaulay approximation is

9. Repeat Question 8 but apply the second-order Macaulay approximation.

A. 3.2×10^{-5} B. 4.4×10^{-6} C. 2.8×10^{-4} D. 1.1×10^{-7} E. 2.9×10^{-10}

SOLUTION: 2.9×10^{-10} (Example 7.3 (Continued))

EXAMPLE 7.3 (continued) *(Second-order approximation)*

In the original Example 7.3, the following bond was considered: 10 coupon bond with 5% coupons and a yield rate of 10%. The bond has face amount 100. The first order modified and Macaulay approximations were applied to approximate the change in present value that occurs as a result of an increase in yield rate of 10 basis points (.1%). Apply the second-order modified and Macaulay approximations and compare them to each other and to the first order approximations.

SOLUTION

Calculations will be done to greater accuracy to help assess the accuracy of the methods. The exact price of the bond is

$$100v_{.1}^{10}+5a_{\overline{10}|.1}=69.27716447$$

The exact price of the bond at the changed yield rate is

$$100v_{.101}^{10}+5a_{\overline{10}|.101}=68.79687762\,.$$

The Macaulay duration of the bond is 7.66086256 and the modified duration of the bond is $D_{\text{mod}}(.1)=\dfrac{7.661}{1.1}=6.964420509$.

The first-order modified approximate price for the bond is 69.7947, with a relative error of 3.2×10^{-5}. The first-order Macaulay approximate price for the bond is 69.7966, with a relative error of 4.4×10^{-6}. The second-order approximations require the values of the modified and Macaulay convexities for the bond.

The Macaulay convexity for the bond is

$$C_{\text{mac}}(i_0)=\frac{\sum_{t=1}^{10}t^2A_t(1+i_0)^{-t}}{\sum_{t=1}^{10}A_t(1+i_0)^{-t}}=\frac{5\times\left(v_{.1}+4v_{.1}^2+\cdots+81v_{.1}^9\right)+105\times100v_{.1}^{10}}{5\times\left(v_{.1}+v_{.1}^2+\cdots+81v_{.1}^9\right)+105v_{.1}^{10}}$$

$$=69.05183414\,.$$

The modified convexity is $C_{\text{mod}}(i_0) = \dfrac{\sum_{t=1}^{10} t(t+1)A_t(1+i_0)^{-t-2}}{\sum_{t=1}^{10} A_t(1+i_0)^{-t}}$

$= 63.39892289$.

Applying the second-order modified approximation, the bond price is

$$P(.1+.001) \doteq P(.1)\times\left(1-.001\times D_{\text{mod}}(.1)+\frac{.001^2}{2}\times C_{\text{mod}}(.1)\right)$$

$$= 69.27716447\times\left(1-.001\times 6.964420509+\frac{.001^2}{2}\times 63.39892289\right)$$

$= 68.79688522$.

The relative error in this approximation is

$$\frac{|68.79688522-68.79687762|}{68.79687762} = 1.1\times 10^{-7}.$$

The relative error in the first order modified approximation was 3.2×10^{-5}.

Applying the second-order Macaulay approximation, the bond price is

$$P(.1+.001) \doteq P(.1)\times\left(\frac{1.1}{1.101}\right)^{D_{\text{mac}}(i_0)}\times\left(1+\left(\frac{.001}{1.1}\right)^2\times\frac{C_{\text{mac}}(i_0)-D_{\text{mac}}^2(i_0)}{2}\right)$$

$$69.27716447\times\left(\frac{1.1}{1.101}\right)^{7.66086256}\times\left(1+\left(\frac{.001}{1.1}\right)^2\times\frac{69.05183414-7.66086256^2}{2}\right)$$

$= 69.79687760$.

The relative error in this approximation is

$$\frac{|68.7979687760-68.79687762|}{68.79687762} = 2.9\times 10^{-10}.$$

10. Suppose that the yield rate and coupon rate on an n-coupon bond are the same. Find the Macaulay duration of a 6-coupon bond with coupon rate 10% per coupon period and yield rate 10% per coupon period.

A. 5.90 B. 6.01 C. 4.79 D. 7.12 E. 8.23

SOLUTION: 4.79 (EXERCISE 7.1.2)

7.1.2

Show Macaulay duration $= \ddot{a}_{\overline{n}|}$ at yield rate when coupon rate = yield rate.
For $n = 6$, 10% per period:

$$\ddot{a}_{\overline{6}|@10\%} = \frac{1-1.10^{-6}}{1-1.10^{-1}} = 4.7908.$$

11. A stock is expected to pay dividends at the end of each year indefinitely. An investor wishes to receive an effective annual return of 5%. Find the stock price if the first dividend is \$2, and subsequent dividends increase by 2% every year.

A. 40 B. 10 C. 66.67 D. 70.12 E. 83

SOLUTION: 66.67 (Example 9.1b)

The price is the present value of a geometrically increasing perpetuity-immediate. The price is $\frac{2}{.05-.02} = 66.67$. □

12. The stock of Deib Osman Corporation is currently valued at 25 per share. An annual dividend has just been paid and the next dividend is expected to be 2 with each subsequent dividend $1 + r$ times the previous one. The valuation is based on an annual interest rate of 12%. What value of r is implied?

A. 0.03 B. 0.04 C. 0.05 D. 0.06 E. 0.07

SOLUTION: 0.04 (EXERCISE 9.1.2a)

$$P = 25, D_1 = 2, i = 0.12.\ 25 = \frac{2}{0.12-g} \Rightarrow g = 0.04, \text{ so } r = 0.04.$$

13. In a structured swap with a financial intermediary, what is the intermediary's primary function?

a) To lend the principal amounts directly to both parties.
b) To guarantee the creditworthiness of both borrowers to their original lenders.
c) To serve as counterparty to both parties in separate offsetting swap agreements.
d) To determine the reference floating rate for all payments.
e) None of the above.

Correct answer: c)

Explanation: The intermediary enters separate agreements with each party, acting as counterparty to both rather than merely facilitating a direct swap between them. This allows the intermediary to manage credit risk and earn a spread.
Reference: "The intermediary agrees with A to accept from A floating-rate interest payments at prime at the same time as paying A fixed interest at 7.2%" and similarly with B in offsetting arrangements.

14. After entering a swap, a firm's effective borrowing rate is lower than the rate it could obtain directly in the market it desires. Which interpretation is most accurate?

A. The firm has altered the market interest rate.
B. The firm has transferred its debt obligation entirely.
C. The firm has synthetically transformed its borrowing exposure.
D. The firm has reduced the total principal it must repay.
E. The firm has eliminated refinancing risk.

Correct answer: c)

Swaps do not change the original borrowing, but instead rearrange cash flows, creating a synthetic fixed or floating position with a lower effective cost.
Reference: The discussion repeatedly emphasizes parallel borrowing combined with swapped cash flows, not debt replacement.

15. Which statement best compares Sukuk al-Ijara and Sukuk al-Mudarabah?

(A) Sukuk al-Ijara offers asset-backed predictable returns suitable for infrastructure, while Sukuk al-Mudarabah offers project-based variable returns with genuine risk-sharing—Mudarabah better embodies Islamic principles but Ijara offers greater commercial practicality.
(B) Sukuk al-Ijara is only for government issuers while Sukuk al-Mudarabah is only for corporate issuers, reflecting different risk profiles and regulatory treatments.
(C) Sukuk al-Ijara is always more Shariah-compliant because it uses tangible assets, while Sukuk al-Mudarabah is controversial due to its profit-sharing nature.
(D) Sukuk al-Ijara and Sukuk al-Mudarabah are economically identical despite different terminology, with the choice depending on marketing preferences rather than substantive differences.
(E) None of the above.

16. Despite being ideal Islamic models, Mudarabah and Musharakah constitute less than 5% of Islamic banking assets. What are the primary barriers to their adoption?

(A) Difficulty modeling profit-sharing risks, higher regulatory capital requirements, consumer preference for predictable payments, and bank preference for guaranteed returns.
(B) Lack of scholarly approval: most Shariah boards consider these structures too risky and discourage their use in modern banking.
(C) Technical complexity that requires advanced financial engineering skills not available in most Islamic financial institutions.
(D) Cultural resistance from Muslim communities who prefer familiar debt-based arrangements over unfamiliar partnership models.
(E) None of the above.

17. How do Murabaha and conventional loans fundamentally differ in their approach to risk?

(A) Murabaha temporarily transfers asset risk to the institution (however briefly), while conventional loans transfer no asset risk—reflecting Islamic finance's emphasis on asset-backing versus conventional finance's credit-based approach.

(B) Murabaha eliminates all risk through careful asset selection, while conventional loans accept risk as an inherent part of lending.
(C) Murabaha transfers all risk to the client immediately upon agreement, while conventional loans gradually transfer risk over the loan term.
(D) Murabaha creates joint risk-sharing where both parties are equally exposed, while conventional loans allocate risk disproportionately based on bargaining power.