AS251- Major Exam 1 - Code 1

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1 Exercise: 8 points

The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- P is the expected price in one year.

Determine which of the following statements about P is TRUE.

1. $P < 100$
2. $P = 100$
3. $100 < P < 105$
4. $P = 105$
5. $P > 105$

$P > F_{0,1} = 105$

2 Exercise: 8 points

Determine which, if any, of the following positions has or have an unlimited loss potential from adverse price movement in the underlying asset, regardless of the initial premium received.

I. Short 1 forward contract

II. Short 1 call option

III. Short 1 put option

1. None
2. I and II only
3. I and III only
4. II and III only
5. The correct answer is not given by (1), (2), (3) or (4)

Short 1 call option:

$$\text{payoff} = -\max\{0, S_T - K\} \searrow -\infty \text{ as } S_T \to \infty$$

Short 1 forward contract:

$$\text{payoff} = F_{0,T} - S_T \searrow -\infty \text{ as } S_T \to \infty$$

Short 1 put option:

$$\text{payoff} = \min\{0, K - S_T\} \longrightarrow 0 \text{ as } S_T \to \infty$$

3 Exercise: 8 points

The dividend yield on a stock and the interest rate used to discount the stock's cash flows are both continuously compounded. The dividend yield is less than the interest rate, but both are positive. The following table shows four methods to buy the stock and the total payment needed for each method. The payment amounts are as of the time payment and have not been discounted to the present

Method	Total payment
Outright purchase	A
Fully leveraged purchase	B
Prepaid forward contract	C
Forward contract	D

Determine which of the following is the correct ranking, from smallest to largest, for the amount of payment needed to acquire the stock

(1) $A < C < D < B$

(2) $D < C < A < B$

(3) $C < A < D < B$

(4) $C < A < B < D$

(5) $A < C < B < D$

$A = S_0$

$D = S_0 e^{(r-\delta)T}$

$C = S_0 e^{-\delta T}$

$B = S_0 e^{rT}$

$e^{-\delta T} < 1 < e^{(r-\delta)T} < e^{rT}$ because $\delta < r$

$\Rightarrow C < A < D < B$

4 Exercise: 8 points

An investor purchases a non-dividend-paying stock and writes a t-year European call option for this stock, with call premium C. The stock price at time of purchase and strike price are both K.

Assume that there are no transaction costs.

The risk-free annual force of interest is a constant r. Let S represent the stock price at time t, where $S > K$.

Determine an algebraic expression for the investor's profit at expiration

(1) Ce^{rt}

(2) $C(1 + rt) - S + K$

(3) $Ce^{rt} + K(1 - e^{rt})$

(4) $(C - S)e^{rt} + K$

(5) $C(1 + r)^t + K$

The investor pays K at t=0 to get the stock. Then, he receives C (premium) at t=0 for the option. Then, he will receive K to sell the stock

$$r = \ln(1+i) \Rightarrow i = e^{r} - 1$$

The profit is $= (C-K)e^{rt} + K$

$$= Ce^{rt} + K(1 - e^{rt})$$

5 Exercise: 8 points

You are given that the stock price is 100, the 1-year forward price is 107, and the 1-year prepaid forward price is 98.

Determine the amount paid at the end of a year on a fully leveraged investment in the stock.

1. 100
2. 104.34
3. 106.22
4. 109.18
5. 128.01

$S_0 = 100;\ F_{0,1} = 107;\ F^P_{0,1} = 98;\ T = 1$

As $F^P_{0,1} = e^{-rT} F_{0,1}$, then $e^{rT} = \frac{F_{0,1}}{F^P_{0,1}}$

The answer is $S_0 e^{rT} = S_0 \frac{F_{0,1}}{F^P_{0,1}} = 100 \times \frac{107}{98}$

$= 109.18$

6 Exercise: 12 points

An investor buys 100 futures contracts. The price of the futures contract is 2,300. The initial margin is 10% and the maintenance margin is 80% of the initial margin. 5% annual effective interest is paid on the margin account.

The futures price changes to 2350 on day 1 and 2280 on day 2. Calculate the size of the margin account on day 2.

1. 20,994.53
2. 21,000.00
3. 21,006.81
4. 21,018.34
5. 21,034.27

Initial margin account

The value of the contracts $100 \times 2{,}300 = 230{,}000\$$

$\Rightarrow$ margin account (buyer) $(t=0) = 23{,}000\ \$$

.) Day 1:

The new price $100 \times 2{,}350 = 235{,}000$

margin account (buyer)

$= 23{,}000 \times 1.05^{1/365} + 5{,}000 = 28{,}003.07$

.) Day 2:

The new price $100 \times 2{,}280 = 228{,}000$

margin account (buyer)

$= 28{,}003.07 \times 1.05^{1/365} - 7{,}000$

$= 21{,}006.81$

7 Exercise: 10 points

The price of a stock is 100. The price of a 6-month prepaid forward on the stock is 96. The stock pays quarterly dividends. The next dividend is payable 3 months from now.

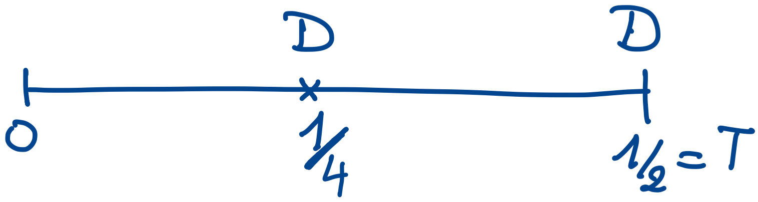
The risk-free rate is 12%. Calculate the amount of each dividend.

1. 1.96
2. 2.05
3. 2.09
4. 2.15
5. 2.19

Stock paying discrete dividends

$$F^P_{0,T=1/2} = S_0 - PV(\text{dividends})$$

$$\Rightarrow 96 = 100 - D\left(e^{-\frac{1}{4}r} + e^{-\frac{1}{2}r}\right)$$



$$\Rightarrow 96 = 100 - D\left(e^{-0.03} + e^{-0.06}\right)$$

$$\Rightarrow D = \frac{100 - 96}{e^{-0.03} + e^{-0.06}} = 2.09\,\$$$

8 Exercise: 10 points

You are given the following:

(i) The current price to buy one share of XYZ stock is 500,

(ii) The stock does not pay dividends.

(iii) The annual risk-free interest rate, compounded continuously, is [redacted] 6%

(iv) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 66.59.

(v) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 18.64.

Calculate the strike price, K.

1. 449
2. 452
3. 480
4. 559
5. 582

$S_0 = 500$; $r = 6\%$; $C(S_0,K) = 66.59$;

$P(S_0,K) = 18.64$; $T = 1$

Put-call parity:

$$C(S_0,K) - P(S_0,K) = S_0 - Ke^{-rT}$$

$$\Rightarrow K = \left(S_0 + P(S_0,K) - C(S_0,K)\right)e^{rT}$$

$$= (500 + 18.64 - 66.59) \times e^{0.06}$$

$$= 480$$

9 Exercise: 14 points

An investor bought a 70-strike European put option on an index with six months to expiration. The premium for this option was 1.

The investor also wrote an 80-strike European put option on the same index with six months to expiration. The premium for this option was 8.

The six-month interest rate is 0%.

Calculate the index price at expiration that will allow the investor to break even (his net profit is 0).

Scenario 1: $0 \leq S_T \leq 70$.

Both puts will be exercised. Then, the profit is

$$(-1+8)(1+0\%)^1 + (70-S_T) - (80-S_T) = -3\$$$

Scenario 2: $70 < S_T \leq 80$.

Only the 80-strike put will be exersiced. The profit becomes $(-1+8)(1+0\%)^1 - (80-S_T) = S_T - 73$, which can be equal to 0 when $S_T = 73$

Scenario 3: $S_T > 80$

No put is exercised. Then, the profit is

$$(-1+8)(1+0\%)^1 = 7\$$$

The final answer is $S_T = 73\$$

10 Exercise: 14 points

100 shares of a non-dividend stock with bid price \$52.25 is shorted. Collateral is 120% of the stock value, the price at which it may be purchased from the market maker. The investor pays 5% effective annual interest on cash borrowed to set up the collateral. The short rebate is 4%. At the end of one year, the position is closed. At that time, the bid price is \$50.75. At both the beginning and the end of the year, the bid-ask spread is 0.5, determine the net profit.

*) At $t=0$, The selling price of the stock is

$$100 \times 52.25 = 5225 \$$$

*) The collateral is $120\% \times 100 \times (52.25 + 0.5) = 6330$

*) You need a loan of $6330 - 5225 = 1105 \$$ to cover the collateral

*) After one year, you should return $1105 \$ \times 1.05 = 1160.25 \$$

*) At $t=1$, to buy the stock, you will pay

$$100 \times (50.75 + 0.5) = 5125 \$$$

*) You will get from the lender is $6330 \times 1.04 = 6583.2 \$$

On the other hand, this amount is

price to buy back the bond + the loan + net profit

$= 5125 + 1160.25 +$ net profit

$\Rightarrow$ net profit $= 6583.2 - 5125 - 1160.25 = 297.95 \$$