AS251- Major Exam 2 - Code 1

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1 Exercise

Consider a 1-year European option call on a non-dividend paying stock, where

- The stock price is currently 50,
- The strike price is 55,
- The stock price at the end of the year is 40 or 60,
- $r = 0.05$.

Then, the neutral-risk probability is

(a) 0.61818

(b) 0.62818

(c) 0.63818

(d) 0.64818

(e) 0.65818

$$u = \frac{60}{50} = 1.2 \; ; \quad d = \frac{40}{50} = 0.8$$

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.05} - 0.8}{1.2 - 0.8}$$

$$= 0.62818$$

2 Exercise

Consider a 1-year European option call on a non-dividend paying stock, where

- The stock price is currently 50,
- The strike price is 55,
- The stock price at the end of the year is 40 or 60,
- $r = 0.05$.

Then

(a) $\Delta = 0.25$ and $B = -9.51229$

(b) $\Delta = 0.25$ and $B = -9.81229$

(c) $\Delta = 0.5$ and $B = -9.51229$

(d) $\Delta = 0.5$ and $B = -9.81229$

(e) $\Delta = 0.25$ and $B = 9.81229$

$$K = 55 \implies \begin{cases} C_u = 5 \\ C_d = 0 \end{cases}$$

$$\Delta = e^{-\delta h} \frac{C_u - C_d}{S(u-d)} = \frac{5-0}{50(1.2-0.8)} = 0.25$$

$$B = e^{-rh} \times \frac{u C_d - d C_u}{u-d} = e^{-0.05} \times \frac{-0.8 \times 5}{0.4} = -9.51229$$

3 Exercise

In a one-period binomial tree for stock prices, you are given:

- The period of the tree is 6 months.
- S_0 is the initial price of the stock.
- The price of the stock may be $0.8S_0$ or $1.2S_0$ at the end of the period.

Determine the volatility of the stock price.

(a) $\sigma = 20.6\%$

(b) $\sigma = 22.6\%$

(c) $\sigma = 24.6\%$

(d) $\sigma = 26.6\%$

(e) $\sigma = 28.6\%$

$$\sigma = \frac{\ln(u/d)}{2\sqrt{h}} = \frac{\ln(1.2/0.8)}{2\sqrt{0.5}} = 0.286$$

4 Exercise

Assume a binomial tree based on forward prices is used. Let $\omega = e^{\sigma\sqrt{h}}$. Then, the **risk-neutral probability** is

(a) $p^* = \dfrac{1-\omega}{1-\omega^2}$

(b) $p^* = \dfrac{1-\omega}{1+\omega^2}$

(c) $p^* = \dfrac{1}{1-\omega^2}$

(d) $p^* = \dfrac{1}{1+\omega^2}$

(e) $p^* = \dfrac{\omega}{1-\omega^2}$

$$p^* = \frac{1-w}{1-w^2} = \frac{1}{1+w}$$

5 Exercise

A 3-month put option on stock is modeled as a binomial tree, where

- The stock price is 75
- The strike price is 80
- $r = 0.08$
- $\delta = 0.02$
- $\sigma = 0.3$

$S = 75$; $K = 80$; $h = 1/4$

Determine the premium of the put option.

(a) $P = 6.823$

(b) $P = 7.023$

(c) $P = 7.223$

(d) $P = 7.423$

(e) $P = 7.623$

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = 1.1794$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = 0.8737$$

$$P_u = \max\{0, K - uS\} = \max\{0, -8.455\} = 0$$

$$P_d = \max\{0, K - dS\} = \max\{0, 14.47\} = 14.47$$

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.462588$$

$$\Rightarrow P = e^{-rh}\left[p^* P_u + (1-p^*) P_d\right]$$

$$= 7.622$$

6 Exercise

Future prices of a stock are modeled with a 1-period binomial tree based on forward prices, where period being 1 year. You are given:

- The stock price is 30.
- The continuously compounded risk-free interest rate is 5%.
- The stock pays no dividends.
- $\sigma = 0.25$.
- For a European call option on the stock expiring in one year, the replicating portfolio has 0.9 shares of stock.

Determine the premium of the call option.

(a) 4.92

(b) 5.18

(c) 5.34

(d) 5.63

(e) 5.97

$h=1;\ S=30;\ r=0.05;\ \delta=0;\ \sigma=0.25$

$\Delta = 0.9$

$u = e^{(r-\delta)h+\sigma\sqrt{h}} = 1.35;\ d = e^{(r-\delta)h-\sigma\sqrt{h}} = 0.8187$

*) $\Delta = e^{-\delta h}\dfrac{C_u - C_d}{S(u-d)} \Rightarrow C_u - C_d = e^{\delta h}\Delta S(u-d)$

$= 14.3451$

$C_d = 0 \Rightarrow C_u = 14.3451$

*) $p^* = \dfrac{e^{(r-\delta)h} - d}{u-d} = 0.43774$

Then $C = e^{-rh}\left[p^* C_u + (1-p^*)C_d\right]$

$= 5.97$

7 Exercise

For a 1-year European call option on a non-dividend paying stock, a 1-period binomial tree is constructed. You are given:

- The stock price is 45.
- The tree has $u = 1.2$ and $d = 0.8$.
- The continuously compounded risk-free interest rate is 4%.
- A replicating portfolio has 0.5 shares of stock.

$h = 1;\ \delta = 0;\ S = 45$

$u = 1.2;\ d = 0.8;\ r = 0.04$

$\Delta = 0.5$

Determine the strike price of the option.

(a) $K = 44$

(b) $K = 45$

(c) $K = 46$

(d) $K = 47$

(e) $K = 48$

$C_u - C_d = e^{\delta h} \Delta S(u-d) = 9$

$C_d = 0 \Rightarrow C_u = 9 = uS - K$

$\Rightarrow K = uS - 9$

$= 1.2 \times 45 - 9$

$= 45$

8 Exercise

You are given:

- The price of a stock is \$70.
- The stock pays continuous dividends at the annual rate of 8%.
- The continuously compounded risk-free interest rate is 4%.
- A 1-year American put option on the stock has a strike price of \$69.

Determine the lowest possible price for this put option.

(a) 1.27

(b) 1.37

(c) 1.47

(d) 1.57

(e) 1.67

$S=70;\ \delta=0.08;\ r=0.04;\ K=69;\ T=1$

$$C_{Eur}-P_{Eur}=Se^{-\delta T}-Ke^{-rT}$$

$$\Rightarrow P_{Eur}=C_{Eur}-Se^{-\delta T}+Ke^{-rT}$$

$$=C_{Eur}-70e^{-0.08}+69e^{-0.04}$$

$$=C_{Eur}+1.676$$

$$\Rightarrow P_{Amer}\geq P_{Eur}=C_{Eur}+1.676\geq 1.676$$

9 Exercise

An American call option has a strike price of \$50. The risk-free rate is 5%. There are 2 months left to expiry. The present value of dividends over the 2-month period is D.

Determine the lowest value of D such that exercising the option early could be rational.

(a) 0.204

(b) 0.285

(c) 0.328

(d) 0.394

(e) 0.415

$K=50$; $r=0.05$; $T=1/6$; $PV_{0,T}(\text{divs})=D$

Exercise the option early could be rational if

$$PV_{0,T}(\text{divs}) \geq K - PV_{0,T}(K)$$
$$\geq K - Ke^{-rT}$$
$$\geq 50 - 50e^{-0.05\times 1/6} = 0.415$$

10 Exercise

A stock is about to pay a dividend. You are given:

- The stock's current price is 110.
- The stock pays dividends of 2 quarterly.
- The continuously compounded risk-free rate is 0.05.

A European call option on the stock expiring in 4 months with strike price 100 is worth 11.54. A European put option with the same conditions is worth 3.87. Then, the component "value of interest" of the European call option is

(a) 0.73

(b) 1.06

(c) 1.21

(d) 1.32

(e) 1.65

Value of interest

$= K(1-e^{-rT})$

$= 100(1-e^{-0.05\times 1/3})$

$= 1.65$

11 Exercise

For stocks without dividends, an American put option is necessarily worth the same as a European put option.

(a) True

(b) False

12 Exercise

For a non-dividend paying stock with price 21, we have

- The continuously compounded risk-free interest rate is 4%.
- A European 3-month put option with strike price 20 costs 1.00.
- A European 6-month put option on the stock with strike price 20.30 costs 0.90.

Assume you sell the 3-month put option and buy 6-month put option. If the stock price is 19 at 3 months and 21 after 6 months, determine your net profit.

(a) 0.801

(b) 0.901

(c) 1.001

(d) 1.101

(e) 1.201

$$(1-0.9)\times e^{0.04\times\frac{6}{12}} - 20\times e^{0.04\times\frac{3}{12}} + 21$$

$$= 0.901$$

13 Exercise

- A 1-year European call with strike 40 costs 10.
- A 1-year European call with strike 45 costs 4.
- The continuously compounded risk-free interest rate is 0.08.

Assume you buy one 45-strike call and sell $\frac{4}{5}$ 40-strike calls. After one year, the stock price is 46. Determine your net profit.

(a) 0.333

(b) 0.533

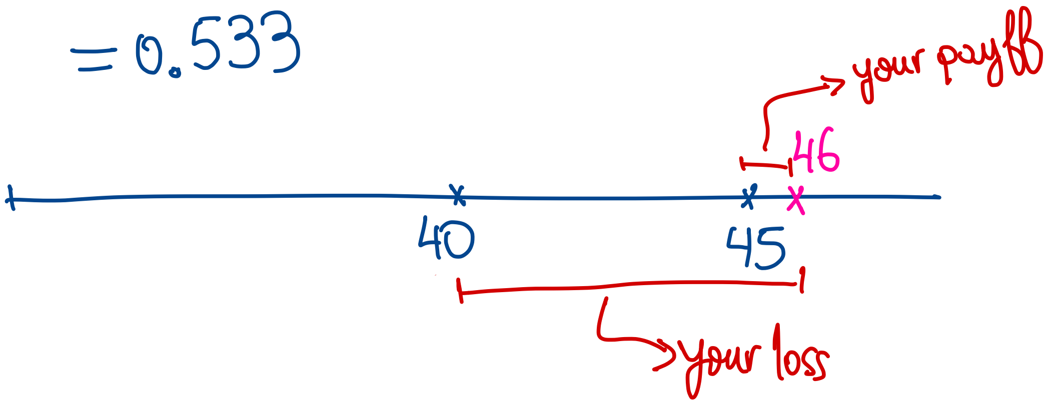
(c) 0.733

(d) 0.933

(e) 1.133

$$\left(10\times\frac{4}{5}-4\right)\times e^{0.08}-\frac{4}{5}(46-40)+(46-45)$$

$$=0.533$$



14 Exercise

A stock's price is 45. The stock will pay a dividend of 1 after 2 months. A European put option with a strike of 42 and an expiry date of 3 months has a premium of 2.71. The continuously compounded risk-free rate is 5%. Determine the premium of a European call option on the stock with the same strike and expiry.

(a) 2.24

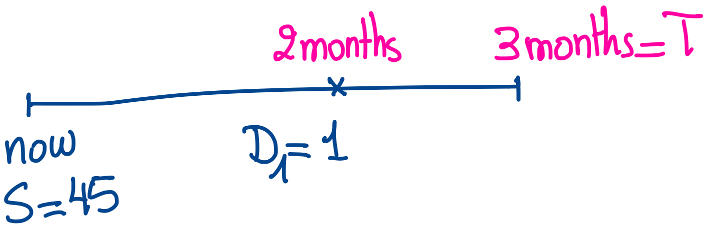
(b) 3.24

(c) 4.24

(d) 5.24

(e) 6.24

$$C - P = S - PV_{0,T}(\text{Divs}) - Ke^{-rT}$$



$K = 42$; $P = 2.71$; $r = 0.05$

$$C = P + S - PV_{0,T}(\text{Divs}) - Ke^{-rT}$$

$$= 2.71 + 45 - 1 \times e^{-0.05 \times 1/4} - 42e^{-0.05 \times 1/4}$$

$$= 5.24$$

15 Exercise

You are given:

(i) The spot exchange rate for dollars to pounds is \$1.4.

(ii) The continuously compounded risk-free rate for dollars is $r = 5\%$.

(iii) The continuously compounded risk-free rate for pounds is $r_e = 8\%$.

A 9-month European put option allows selling 1 pound at the rate of \$1.50. A 9-month dollar denominated call option with the same strike costs \$0.0223. Determine the premium of the 9-month dollar denominated put option.

(a) 0.04862

(b) 0.14862

(c) 0.24862

(d) 0.34862

(e) 0.44862

$x = 1.4; \quad r = 5\%; \quad r_e = 8\%; \quad T = \frac{9}{12} = 0.75$

$K = 1.5; \quad C = 0.0223$

$C - P = x_0 e^{-r_e T} - K e^{-rT}$

$\Rightarrow P = C - x_0 e^{-r_e T} + K e^{-rT}$

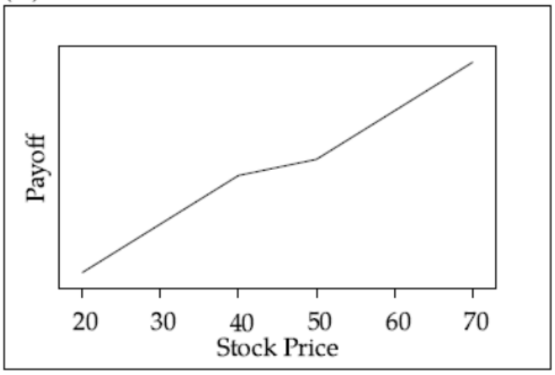
$= 0.0223 - 1.4 \times e^{-0.08 \times 0.75} + 1.5 e^{-0.05 \times 0.75}$

$= 0.14862$

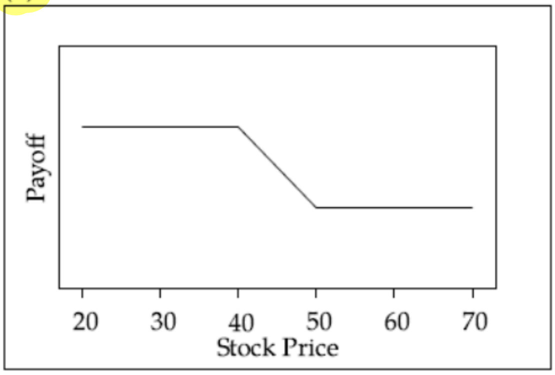
16 Exercise

Determine which of these graphs represents the payoff diagram for 40-50 bear spread at the time of expiration of the options.

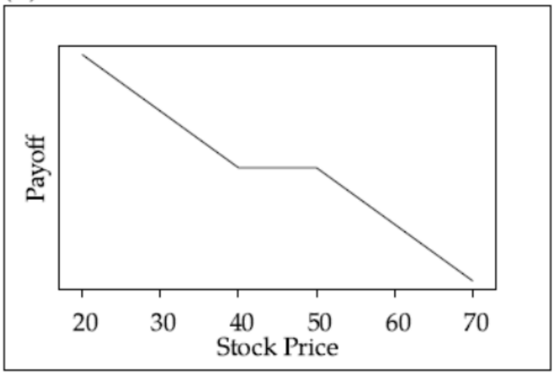
(A)



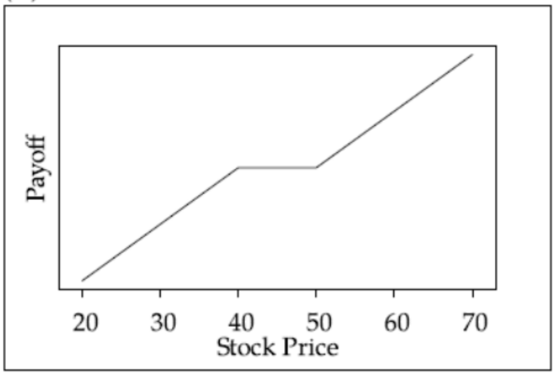
(B)



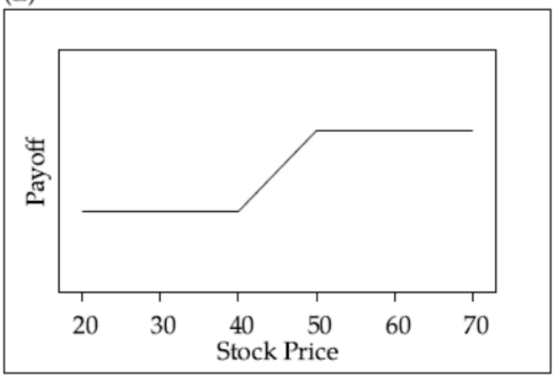
(C)



(D)



(E)



17 Exercise

Consider a box spread consisting of the following 1-year European options: a long call and short put with strike price \$40 and a short call and a long put with strike price \$60. The continuously risk-free rate is 4%. What is the price of this box spread.

(a) 12.22

(b) 15.22

(c) 17.22

(d) 19.22

(e) 21.22

Long call + short put with $K=40$

$\parallel$

Buy a call + sell a put

40

Buy the stock for 40\$ using put

Buy the stock for 40\$ using call

$\Rightarrow$ Long synthetic forward with $K=40$

Short call + long put with $K=60$

$\Rightarrow$ short synthetic forward with $K=60$.

$r=4\%$

The price is $(60-40)\times e^{-0.04}$

$= 19.22\ \$.$

18 Exercise

Determine which of the following strategies creates a ratio spread, assuming all options are European with the same expiration date.

(A) Buy a one-year call, and sell a three-year call with the same strike price

(B) Buy a one-year call, and sell a three-year call with a different strike price

(C) Buy a one-year call, and buy three one-year calls with a different strike price

(D) Buy a one-year call, and sell three one-year puts with a different strike price

(E) Buy a one-year call, and sell three one-year calls with a different strike price

19 Exercise

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025. The annual effective risk-free interest rate is 5%. Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

(A) Buy the call and sell the put, receive 23.81

(B) Buy the call and sell the put, spend 23.81

(C) Buy the put and sell the call, receive 23.81

(D) Buy the put and sell the call, spend 23.81

(E) Buy the put and sell the call, no cost

Using Put-call parity equation, we have

$$C - P = S_0 - Ke^{-rT}$$

C: premium of the call

P: premium of the put

K: the strike price of both: call and put

S_0: the current price of the stock

$$C - P = 1000 - 1025\,(1+0.05)^{-1}$$

P: buy a put

C: sell a call

$$= 23.80\ \$$$

Buy a call + sell a put $\Rightarrow$ spend 23.80

Sell a call + buy a put $\Rightarrow$ receive 23.80

As Sam wants to buy the index, then, he should buy a call and sell a put spending 23.80

20 Exercise

For a bull spread with calls, the net payoff is $S_T - K_2$ when $K_1 < S_T \leq K_2$

(a) True

(b) False

The net payoff is $S_T - K_1$ when $K_1 < S_T \leq K_2$