

# AS251- Final Exam

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## Exercise

Prices for a stock are modeled with a 1-period binomial tree with  $u = 1.2$ ,  $d = 0.7$ , and a period of 3 months.

A European call option on the stock expires in 3 months.

You are given:

1. The stock's initial price is \$50.
2. The stock pays no dividends.
3. The strike price for the call option is \$55.
4. The price of the call option is \$3.10.

Determine the continuously compounded risk-free interest rate.

(a) 5.5555

(b) 5.7555

(c) 5.9555

(d) 6.1555

(e) 5.3555

$$C_u = \max\{0, uS - K\} = \max\{0, 5\} = 5$$

$$C_d = \max\{0, dS - K\} = \max\{0, -20\} = 0$$

$$C = e^{-rh} [p^* C_u + (1-p^*) C_d] = 5p^* e^{-rh}$$

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{rh} - 0.7}{0.5}$$

$$\Rightarrow C = 5 \times \frac{e^{rh} - 0.7}{0.5} \times e^{-rh} = 10(1 - 0.7e^{-rh}) = 10 - 7e^{-rh} = 3.1$$

$$\Rightarrow e^{-rh} = \frac{10 - 3.1}{7} \Rightarrow rh = -\ln\left(\frac{10 - 3.1}{7}\right)$$

$$\Rightarrow r = -\frac{1}{h} \ln\left(\frac{10 - 3.1}{7}\right) = 0.5755$$

$\frac{1}{4}$  ↖

## Exercise

A stock currently has a price of \$45.00 and pays no dividends. One year from now, there is a risk-neutral probability of 50% that the price of the stock will be \$30.00 and a risk-neutral probability of 50% that it will be greater than \$40.00.

The effective annual risk-free interest rate is 4%.

Calculate the price of a one-year European call option with an exercise price of \$40.00. Then,

(a) 4.81

(b) 6.35

(c) 9.81

(d) 10.00

(e) 11.35

$$p^* = 0.5;$$

$$C_d = \max\{0, 30 - K\} = \max\{0, -10\} = 0$$

$$C_u? \quad i = 4\%$$

$$\begin{aligned} S &= ((uS) \times p^* + (dS) \times (1-p^*)) e^{-rh} \\ &= (uS \times 0.5 + 30 \times 0.5) \times 1.04^{-1} \end{aligned}$$

$$\Rightarrow uS = \frac{S \times 1.04}{0.5} - 30 = 63.6$$

$$\Rightarrow C_u = \max\{0, uS - 40\} = 23.6$$

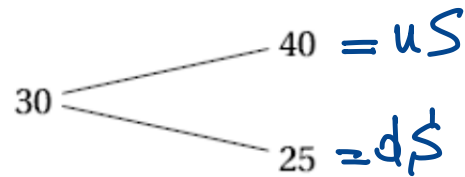
$$\Rightarrow C_0 = e^{-rh} [p^* C_u + (1-p^*) C_d]$$

$$= 1.04^{-1} [0.5 \times 23.6 + 0.5 \times 0]$$

$$= 11.346$$

## Exercise

The price of a non-dividend-paying stock is modeled by the following 1-period binomial tree, with each period being one year:



A European call option expiring in one year on the stock has a strike price of \$30.

The continuously compounded risk-free interest rate is 4%.

Determine the number of shares of stock in the replicating portfolio for the call option.

(a)  $1/3$

(b)  $2/3$

(c)  $4/3$

(d)  $5/3$

(e)  $3/5$

$$\begin{aligned} K &= 30; \quad r = 4\%; \quad \delta = 0 \\ C_u &= 10; \quad C_d = 0; \\ \Delta &= e^{-\delta h} \times \frac{C_u - C_d}{S(u-d)} = e^{-0} \times \frac{10 - 0}{40 - 25} \\ &= \frac{10}{15} = \frac{2}{3} \end{aligned}$$

## Exercise

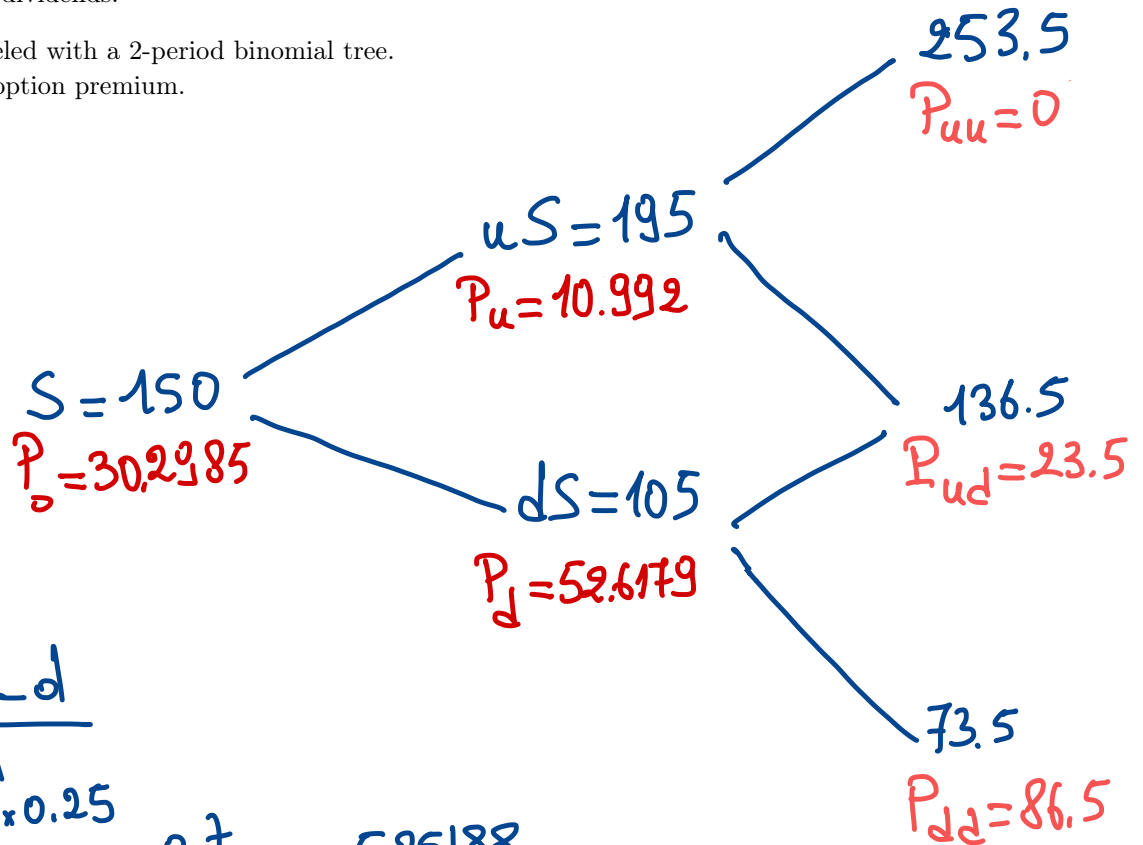
For a 6-month European put option on a stock, you are given:

- The stock price is 150.
- The strike price is 160.
- $u = 1.3$  and  $d = 0.7$ .
- The continuously compounded risk-free rate is 6%.
- There are no dividends.

The option is modeled with a 2-period binomial tree.

Determine the option premium.

- (a) 17.15  
 (b) 21.38  
 (c) 27.23  
 (d) 30.29  
 (e) 37.11



$$p^* = \frac{e^{(r-d)h} - d}{u - d}$$

$$= \frac{e^{0.06 \times 0.25} - 0.7}{1.3 - 0.7} = 0.525188$$

$$P_u = e^{-rh} [P_{uu} p^* + P_{ud} (1 - p^*)] = 10.992$$

$$P_d = e^{-rh} [P_{ud} p^* + P_{dd} (1 - p^*)] = 52.6179$$

$$P_0 = e^{-rh} [P_u p^* + P_d (1 - p^*)] = 30.2985$$

## Exercise

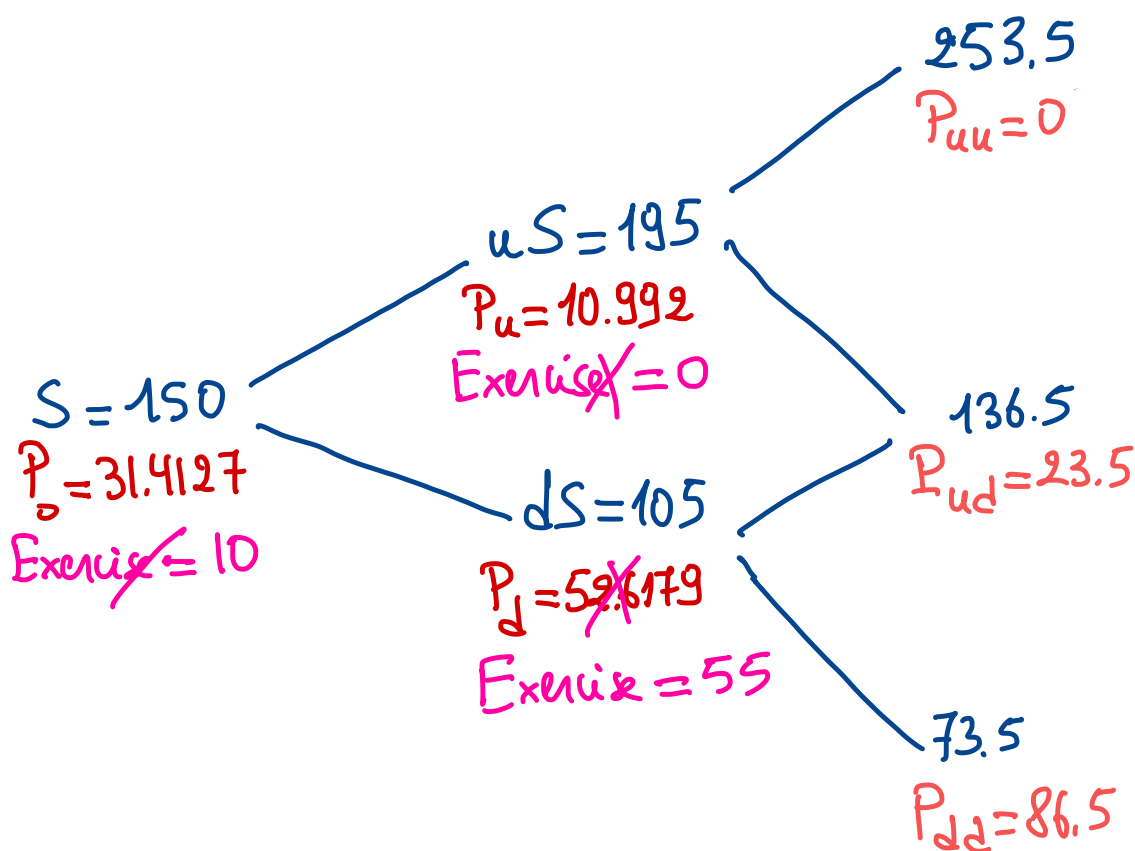
For a 6-month American put option on a stock, you are given:

- The stock price is 150.
- The strike price is 160.
- $u = 1.3$  and  $d = 0.7$ .
- The continuously compounded risk-free rate is 6%.
- There are no dividends.

The option is modeled with a 2-period binomial tree.

Determine the option premium.

- (a) 21.38
- (b) 23.18
- (c) 27.23
- (d) 30.11
- (e) 31.14



$$P_0 = e^{-rh} [10.999 p^* + 55 \times (1 - p^*)]$$

$$= 31.4127$$

## Exercise

The spot exchange rate of dollars for euros is  $x_0 = 1.15$ . A 6-month American call option allows purchase of euros at 1.25 dollar for 1 euro. You are given:

- $r_d = 0.05 \rightarrow r$
- $r_e = 0.04 \rightarrow \delta$
- The annual volatility of the exchange rate is 0.1  $\rightarrow \sigma$

$$h = \frac{1}{4}$$

A 2-period binomial tree based on forward prices is used to value the option. Then  $u + d =$

(a) 2.0075

(b) 2.2375

(c) 2.4575

(d) 2.6975

(e) 2.8775

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = 1.0539$$
$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = 0.95361$$

## Exercise

Future prices of dollars expressed in euros are modeled with a 2-period binomial tree, with each period being 6 months.

You are given:

1. The spot exchange rate is 0.9.  $\rightsquigarrow x$
2. The tree has  $u = 1.1$  and  $d = 0.9$ .
3. The continuously compounded risk-free interest rate for euros is 0.07.  $\rightsquigarrow r$
4. The continuously compounded risk-free interest rate for dollars is 0.05.  $\rightsquigarrow \delta$

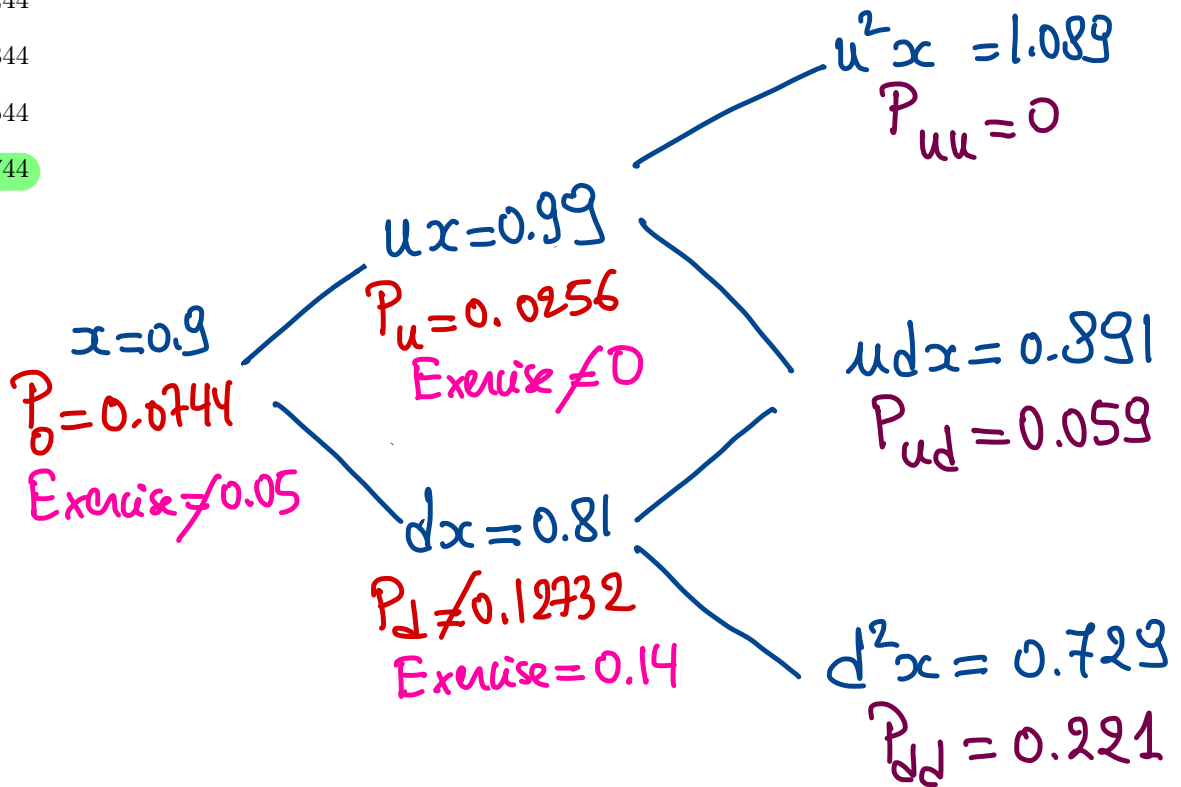
$$h = 0.5$$

$$T = 1$$

A euro-denominated American put option on dollars expiring in 1 year has a strike price of 0.95 euro. Determine the option's premium.

$$K = 0.95$$

- (a) 0.0144
- (b) 0.0244
- (c) 0.0344
- (d) 0.0544
- (e) 0.0744

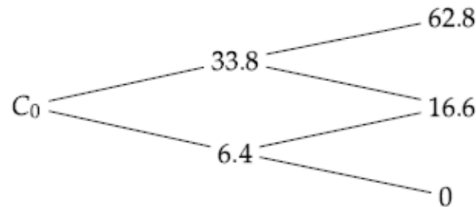


$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = 0.55025$$



## Exercise

Future prices of a stock are modeled with a 2-period binomial tree. The risk-neutral probabilities of up movements at all nodes of the tree are equal. A European option on the stock has the following prices. Determine  $C_0$ , the price of the option at the initial node.



(a) 8.23

(b) 11.54

(c) 16.73

(d) 27.18

(e) 35.12

$$33.8 = e^{-rh} [62.8 p^* + 16.6 (1-p^*)]$$
$$= e^{-rh} [16.6 + 46.2 p^*] \quad (1)$$

$$6.4 = e^{-rh} [16.6 p^*] \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{16.6 + 46.2 p^*}{16.6 p^*} = \frac{33.8}{6.4}$$

$$\Rightarrow (16.6 + 46.2 p^*) \times 6.4 = (16.6 p^*) \times 33.8$$

$$\Rightarrow p^* = 0.4003$$

$$(2) \Rightarrow e^{-rh} = \frac{6.4}{16.6 p^*} = 0.963133$$

$$C_0 = e^{-rh} [p^* \times 33.8 + (1-p^*) \times 6.4]$$
$$= 16.73$$

## Exercise

A non-dividend paying stock has a current value of 100. In each of the next six-month periods, the stock price could rise by 25% or fall by 25%. The risk-free interest rate is 6% per year.

Consider a European call on this stock with an exercise price of 90. Then,  $C_u + C_d =$

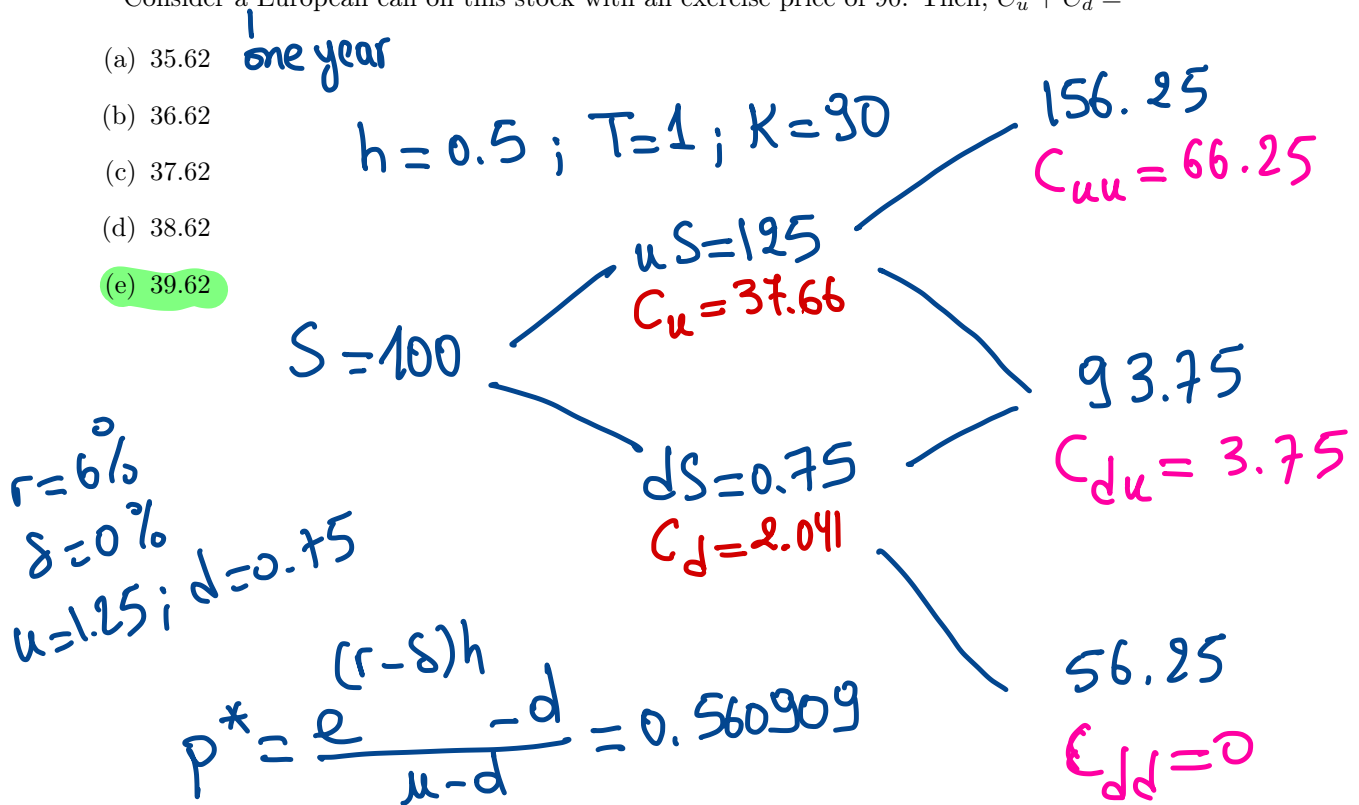
(a) 35.62

(b) 36.62

(c) 37.62

(d) 38.62

(e) 39.62



## Exercise

For an American call option on a stock:

- (i) The stock price is \$50.  $\sim S$
- (ii) The strike price is \$45.  $\sim K$
- (iii) There are 3 months to expiry.  $\sim T = 1/4$
- (iv) The stock is about to pay a dividend of  $D$ .
- (v)  $r = 0.04$ .

Determine the least upper bound of values for  $D$  such that immediate exercise is definitely not optimal.

(a)  $D < 0.447757$

(b)  $D > 0.447757$

(c)  $D < 0.847757$

(d)  $D > 0.847757$

(e)  $D > 1.247757$

Present value of  $K$  is  
$$K(1 - e^{-rT}) = 0.447757$$

Present value of dividends is  $D$

$$D < 0.447757$$

## Exercise

$$S_0 = 80$$

$S_t$  is the price of a non-dividend-paying stock at time  $t$ .  $S_t$  follows a lognormal model.

You are given:

- The continuously compounded annual rate of return on the stock is 0.15.
- The stock's volatility is 0.3.
- $S_0 = 80$ .

Calculate the probability that  $S_4$  is at least 150.

(a)  $1 - N(0.14768)$

(b)  $N(0.14768)$

(c)  $1 - N(0.24768)$

(d)  $1 - N(0.34768)$

(e)  $1 - N(0.44768)$

$$m = \alpha t - 0.5\sigma^2 t = (0.15 - 0.5 \times 0.3^2) \times 4 = 0.42$$

$$v = \sigma\sqrt{t} = 0.3\sqrt{4} = 0.6$$

$$\mathbb{P}\left\{S_4 > 150\right\} = \mathbb{P}\left\{\frac{S_4}{S_0} > \frac{150}{80} = \frac{15}{8}\right\}$$

$$= \mathbb{P}\left\{\ln\left(\frac{S_4}{S_0}\right) > \ln\left(\frac{15}{8}\right)\right\}$$

$$= \mathbb{P}\left\{\frac{\ln(S_4/S_0) - m}{v} > \frac{\ln(15/8) - m}{v}\right\}$$

$$= 1 - N\left(\frac{\ln(15/8) - m}{v}\right)$$

$$= 1 - N\left(\frac{\ln(15/8) - 0.42}{0.6}\right)$$

$$= 1 - N(0.34768)$$

## Exercise

$S_t$  is the price of a non-dividend-paying stock at time  $t$ .  $S_t$  follows a lognormal model. You are given:

1.  $S_0 = 40$ .
2. The stock's continuously compounded expected growth rate is  $\alpha = 0.15$ .
3. The stock's volatility  $\sigma = 0.3$ .

Determine the median price of the stock after one year.

(a) 44.42

(b) 45.42

(c) 46.42

(d) 47.42

(e) 48.42

$$t=1;$$

.) The median of  $S_1/S_0$  is  $e^m$

.) The median of  $S_1$  is

$$S_0 e^m = S_0 e^{\alpha \times t - 0.5 \sigma^2 \times t} = 44.42$$

## Exercise

$S_t$  is the price of a non-dividend-paying stock at time  $t$ .  $S_t$  follows a lognormal model. You are given:

1.  $S_0 = 40$ .
2. The stock's continuously compounded expected growth rate is  $\alpha = 0.15$ .
3. The stock's volatility  $\sigma = 0.3$ .

Determine  $E[\ln(S_4/S_0)]$ .

(a) 0.33

(b) 0.36

(c) 0.39

(d) 0.42

(e) 0.45

$$t=4$$

$\ln(S_4/S_0)$  is normally distributed with

$$\text{mean } m = \alpha \times t - 0.5 \sigma^2 \times t = 0.42$$

## Exercise

A stock's prices follow a lognormal distribution. You are given:

- $\alpha = 0.14$
- $\delta = 0.02$
- $\sigma = 0.3$

Determine the probability that the stock's price at the end of one month will be greater than its current price.

(a)  $1 - N(0.07217)$

(b)  $1 - N(-0.07217)$

(c)  $1 - N(0.37217)$

(d)  $1 - N(-0.37217)$

(e)  $1 - N(-0.47217)$

$$t = 1/12$$

$$m = \alpha t - \delta t - 0.5\sigma^2 t = 6.25 \times 10^{-3}$$

$$v = \sigma\sqrt{t} = 0.0866025$$

$$\begin{aligned} \mathbb{P}\{S_t > S_0\} &= \mathbb{P}\left\{\frac{S_t}{S_0} > 1\right\} \\ &= \mathbb{P}\left\{\ln\left(\frac{S_t}{S_0}\right) > \ln 1 = 0\right\} \\ &= 1 - N\left(\frac{0 - m}{v}\right) \\ &= 1 - N(-0.07216) \end{aligned}$$

## Exercise

A stock's prices follow a lognormal distribution. You are given:

- $\alpha = 0.14$
- $\delta = 0.02$
- $\sigma = 0.3$
- For a standard normal distribution, the 97.5 percentile is 1.96.

Construct a 50% synthetic prediction <sup>interval</sup> interval for the ratio of the stock's price at the end of a month to the current price.

- (a) (0.80917, 0.99243).
- (b) (0.84917, 0.99243).
- (c) (0.80917, 1.19243).
- (d) (0.84917, 1.19243).
- (e) (0.88917, 1.19243).

$$m = 6.25 \times 10^{-3}$$
$$v = 0.0866025$$

The prediction interval is  $\left( e^{m - z_{\alpha/2} \times v}, e^{m + z_{\alpha/2} \times v} \right)$

$$= (0.84917, 1.19243)$$



## Exercise

A stock's price follows a lognormal model. You are given:

1.  $S_0 = 60$
2.  $\alpha = 0.15$
3.  $\sigma = 0.2$
4.  $\delta = 0.05$

A European call option on the stock with strike price 70 expires in 3 months.  
Calculate the probability that the option pays off.

- (a)  $N(-0.34151)$   
(b)  $N(0.34151)$   
(c)  $N(-1.34151)$   
(d)  $N(1.34151)$   
(e)  $N(2.34151)$

$\rightsquigarrow K$        $\rightsquigarrow T = 1/4$

$$\begin{aligned} & \mathbb{P} \{ \text{the option pays off} \} \\ &= \mathbb{P} \{ S_t > K \} = N(\hat{d}_2) \\ \hat{d}_2 &= \frac{\ln(S_0/K) + (\alpha - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}} = -1.34 \end{aligned}$$

## Exercise

A stock's price follows a lognormal model. You are given:

- (i) The stock's initial price is \$40.
- (ii)  $\alpha = 0.10$
- (iii)  $\sigma = 0.15$
- (iv) The stock pays no dividends.

$t=2$   $S_2 < 50$

Calculate the conditional expected value of the stock after 2 years given that it is less than \$50.

Hint:  $N(0.00303) = 0.50121$ ,  $N(0.21516) = 0.58518$ ,  $N(0.4) = 0.65542$  and  $N(0.6) = 0.72575$

- (a) 38.85
- (b) 39.85
- (c) 40.85
- (d) 41.85
- (e) 42.85

$$E[S_2 | S_2 < 50] = S_0 e^{(\alpha - \delta)t} \frac{N(-\hat{d}_1)}{N(-\hat{d}_2)}$$

$$\hat{d}_1 = \frac{\ln(S_0/k) + (\alpha - \delta + 0.5\sigma^2)t}{\sigma\sqrt{t}} = -0.00303$$

$$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{t} = -0.21516$$

$$\Rightarrow E[S_2 | S_2 < 50] = 40 \times e^{0.1 \times 2} \frac{N(0.00303)}{N(0.21516)}$$

$$= 41.85$$

## Exercise

For a non-dividend paying stock, you are given:

1. The stock price follows a lognormal model.
2. The current price is 100.  $\rightarrow S_0$
3. The continuously compounded expected rate of return is 0.1.
4. The volatility is 0.2.

A European call option on the stock expiring in one year has a strike price of 100. Calculate the expected payoff on the call option.

Hint:  $N(0.00303) = 0.50121$ ,  $N(0.21516) = 0.58518$ ,  $N(0.4) = 0.65542$  and  $N(0.6) = 0.72575$

- (a) 5.78
- (b) 9.23
- (c) 11.19
- (d) 14.67
- (e) 28.15

$$\hat{d}_1 = \frac{\ln(S_0/K) + (\alpha + 0.5\sigma^2)t}{\sigma\sqrt{t}} = 0.6$$

$$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{t} = 0.4$$

$$\begin{aligned} \text{Answer } & S_0 e^{\alpha t} N(\hat{d}_1) - K N(\hat{d}_2) \\ &= 100 e^{0.1} N(0.6) - 100 N(0.4) \\ &= 14.67 \end{aligned}$$

## Exercise

A stock's price follows a lognormal distribution. To simulate its price over 10 years, scenarios are generated. In each scenario, the stock price at time  $t$  is generated by generating a standard normal random variable  $Z$ . Then  $S_t$  is set equal to  $S_{t-1}e^{0.1+0.2Z}$  for  $t = 1, 2, \dots, 10$ .

Determine the expected value of the ratio  $\frac{S_{10}}{S_0}$  in this simulation.

(a) 3.320117

(b) 4.320117

(c) 5.320117

(d) 6.320117

(e) 7.320117

$$0.1 + 0.2Z \sim \mathcal{N}(m_1 = 0.1, \sigma_1^2 = 0.2^2)$$

$$\begin{aligned} E\left[\frac{S_{10}}{S_0}\right] &= e^{m_{10} + 0.5\sigma_{10}^2} \\ &= e^{10[m_1 + 0.5\sigma_1^2]} \\ &= e^{10[0.1 + 0.5 \times 0.2^2]} \\ &= 3.32 \end{aligned}$$

## Exercise

$T = 1/4$   $K = 52$

For 3-month 52-strike European call option on a stock, you are given:

- (i) The stock's price follows the Black-Scholes framework.
- (ii) The stock's price is 50.  $\rightarrow S$
- (iii) The stock's volatility is 0.4.  $\rightarrow \sigma$
- (iv) The stock's continuous dividend rate is 4%.  $\rightarrow \delta$
- (v) The continuously compounded risk-free interest rate is 8%.  $\rightarrow r$

Determine the premium of the option.

Hint:  $N(-0.04611)=0.48161$ ,  $N(-0.24611)=0.40280$ ,  $N(0.24611)=0.59720$ ,  $N(0.04611)=0.51839$ .

- (a) 2.31
- (b) 3.31
- (c) 4.31
- (d) 5.31
- (e) 6.31

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = -0.0461$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.2461$$

$$\text{Call} = S e^{-\delta T} N(d_1) - K e^{-rT} N(d_2)$$

$$= 50 e^{-0.04 \times 0.25} N(-0.0461) - 52 e^{-0.08 \times 0.25} N(-0.2461)$$
$$= 3.31$$

## Exercise

$$T = 1/4$$

For 3-month 52-strike European put option on a stock, you are given:

- (i) The stock's price follows the Black-Scholes framework.
- (ii) The stock's price is 50.  $\rightarrow S_0$
- (iii) The stock's volatility is 0.4.  $\rightarrow \sigma$
- (iv) The stock's continuous dividend rate is 4%.  $\rightarrow \delta$
- (v) The continuously compounded risk-free interest rate is 8%.  $\rightarrow r$

Determine the premium of the option.

Hint:  $N(-0.04611)=0.48161$ ,  $N(-0.24611)=0.40280$ ,  $N(0.24611)=0.59720$ ,  $N(0.04611)=0.51839$ .

- (a) 2.78
- (b) 3.78
- (c) 4.78
- (d) 5.78
- (e) 6.78

$$\begin{aligned} \text{Put} &= Ke^{-rT} N(-d_2) - Se^{-\sigma T} N(-d_1) \\ &= 52 e^{-0.08 \times 0.25} N(0.2461) - 50 e^{-0.04 \times 0.25} N(0.0461) \\ &= 4.78 \end{aligned}$$

$$T = 0.5$$

## Exercise

A stock has quarterly dividends, paid at the end of 3 months and 6 months from now. You are given:

- The stock price is  $S = 42$ .
- Quarterly dividends are  $D = 0.75$ .
- The volatility of a prepaid forward on the stock is  $\sigma = 0.3$ .
- A 6-month European put option is written on the stock with strike price  $K = 40$ .
- The continuously compounded risk-free rate is  $r = 0.04$ .

Calculate the put option's premium with the Black-Scholes formula.

Hint:  $N(-0.26149) = 0.39686$ ,  $N(-0.04936) = 0.48032$ ,  $N(-1.68221) = 0.04626$ ,  $N(-1.73221) = 0.04162$

(a) 2.15

(b) 2.25

(c) 2.45

(d) 2.55

(e) 2.75

$$S - PV(\text{divs}) = S - 0.75 \times (e^{-0.04/4} + e^{-0.04/2}) = 40.5223$$

$$d_1 = \frac{\ln\left(\frac{S - PV(\text{divs})}{K}\right) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} = 0.2615$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.04936$$

$$\begin{aligned} P &= Ke^{-rT} N(-d_2) - (S - PV(\text{divs})) N(-d_1) \\ &= 40e^{-0.04 \times 0.5} \times N(-0.04936) - 40.5223 N(-0.2615) \\ &= 2.75 \end{aligned}$$

## Exercise

You are given:

- (i) The spot exchange rate for yen in dollars is 0.009  $\rightarrow x$
- (ii)  $\sigma = 0.05$
- (iii) The continuously compounded risk-free rate for yen is 2%  $\rightarrow \delta$
- (iv) The continuously compounded risk-free rate for dollars is 4%  $\rightarrow r$

Calculate the Black-Scholes price for a 1-year European dollar-denominated call option on yen with a strike price of 0.010  $\rightarrow K$

Hint:  $N(-0.26149) = 0.39686$ ,  $N(-0.04936) = 0.48032$ ,  $N(-1.68221) = 0.04626$ ,  $N(-1.73221) = 0.04162$

(a) 0.0000082

(b) 0.0002082

(c) 0.0030082

(d) 0.0500082

(e) 0.0700082

$$d_1 = \frac{\ln(x/k) + (r - \delta + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = -1.68221$$

$$d_2 = d_1 - \sigma\sqrt{T} = -1.73221$$

$$\begin{aligned} \text{Call} &= x e^{-\delta T} N(d_1) - K e^{-r T} N(d_2) \\ &= 0.009 e^{-0.02 \times 1} N(-1.68221) - 0.010 \times e^{-0.04 \times 1} N(-1.73221) \\ &= 0.00000821 \end{aligned}$$



## Exercise

A futures contract on silver has a price of 10 for delivery at the end of 1 year. Volatility is 0.25. The continuously compounded risk-free rate is 4%.

Calculate the premium for a 1-year European call option on the futures contract with a strike price of 10.

Hint:  $N(0.125) = 0.54974$ ,  $N(-0.125) = 0.45026$ ,  $N(0.15) = 0.55962$ ,  $N(-0.15) = 0.44038$

(a) 0.5558

(b) 0.9558

(c) 1.5558

(d) 1.9558

(e) 2.9558

$$d_1 = \frac{\ln(F/K) + \frac{\sigma^2}{2} T}{\sigma\sqrt{T}} = 0.125$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.125$$

$$\begin{aligned} \text{Call} &= Fe^{-rT} N(d_1) - Ke^{-rT} N(d_2) \\ &= 10e^{-0.04} N(0.125) - 10e^{-0.04} N(-0.125) \\ &= 0.9558. \end{aligned}$$

## Exercise

You are interested in purchasing a call option on a nondividend paying common stock that is currently trading at a price of 100 per share. You are given the following information.

- The standard deviation of the continuously compounded annual rate of return on the stock is 0.4  $\rightarrow \sigma$
- The time to maturity of the call is 3 months.  $\rightarrow T = 1/4$
- 

$$\ln\left(\frac{\text{Current share price}}{\text{Present value of the exercises price}}\right) = -0.08$$

at the risk-free rate.

Calculate the price of a call option using Black-Scholes.

Hint:  $N(-0.3) = 0.38209$ ,  $N(-0.5) = 0.30854$ ,  $N(-0.80379) = 0.21076$ ,  $N(-0.94813) = 0.17153$

(a) 2.3154

(b) 3.2478

(c) 4.7854

(d) 5.4231

(e) 6.2817

$$d_1 = \frac{\ln\left(\frac{S}{Ke^{-rT}}\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} = -0.3$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.5$$

$$\ln\left(\frac{S}{Ke^{-rT}}\right) = -0.08 \Rightarrow \frac{S}{Ke^{-rT}} = e^{-0.08} \Rightarrow Ke^{-rT} = Se^{0.08}$$

$$\begin{aligned} \text{Call} &= Se^{-\delta T} N(d_1) - \underbrace{Ke^{-rT}}_{Se^{0.08}} N(d_2) \\ &= SN(-0.3) - Se^{0.08} N(-0.5) \\ &= 4.785 \end{aligned}$$