Dept. of Mathematics and Statistics King Fahd University of Petroleum & Minerals AS251: Mathematics of Financial Derivatives

Dr. Ridwan A. Sanusi Major Exam 1 Term 242 Thursday, February 13, 2025 8.00 AM - 9.30 AM

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Instructions.

- 1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
- 2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
- 3. Only materials provided by the instructor can be present on the table during the exam.
- 4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
- 5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
- 6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
- 7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
- 8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. *Write important steps to arrive at the solution of the exam problems*.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Mark	Mark Obtained	Comments
PART A			
1-2	0.5*2		
2-14	1*12		
PART B			
1	2		
2	3		
BONUS	3		
Total	18		

Extra blank page

PART A

- 1. Which of the following is a speculation?
- A. An investor expects the price of a dollar relative to the euro to appreciate and buys a call option.
- B. A US company receives a payment in Euros in 6 months and pays for an option to sell euros for dollars at the end of 6 months.
 - C. A corn farmer short sells corn three months before the harvest.
- 2. Determine which of the following is NOT a typical reason for why derivatives are used.
 - A. Hedging
 - B. Reduce the likelihood of interrupting
 - C. Reduce transaction costs
 - D. Satisfy regulatory, tax, and accounting requirements
 - E. As a form of insurance
- 3. 3-year zero-coupon bond with a maturity value of 1000 has an annual effective interest rate of 6%. The bond is short-sold with collateral of 110% of its value. After one year, the position is closed, and the bond is bought back. At that time, the annual effective interest rate of the bond is 7%. The interest on the loan to provide collateral is 6%. Net profit is 7.32. Determine the repo rate.
- (A) 0.5
- (B) 0.05
- (C) 0.005
- (D) 0.55
- (E) 0.505

- 4. 100 shares of a non-dividend-paying stock with bid price of 52.25 are shorted. Collateral is 120% of the stock's value, and the price at which it may be purchased from the market maker is \$2.75. The investor pays 5% effective annual interest on cash borrowed to set up the collateral. The short rebate is 4%. At the end of one year, the position is closed. At that time, the bid price is 50.75, and the bid-ask spread is 0.5. Determine the net profit.
- (A) 5225
- (B) 55.25
- (C) 253.20
- (D) 5125
- (E) 297.95

5. For a non-dividend-paying stock index, the current price is 1100 and the 6-month forward price is 1150. Assume the price of the stock index in 6 months will be 1210.

Which of the following is true regarding forward positions in the stock index?

- (A) Long position gains 50
- (B) Long position gains 60
- (C) Long position gains 110
- (D) Short position gains 60
- (E) Short position gains 110
- 6. Investors in a certain stock demand to be compensated for risk. The current stock price is 100. The stock pays dividends at a rate proportional to its price. The dividend yield is 2%. The continuously compounded risk-free interest rate is 5%. Assume there are no transaction costs.

Let X represents the expected value of the stock price 2 years from today. Assume it is known that X is a whole number. Determine which of the following statements is true about X.

- (A) The only possible value of X is 105.
- (B) The largest possible value of X is 106.
- (C) The smallest possible value of X is 107.
- (D) The largest possible value of X is 110.
- (E) The smallest possible value of X is 111.
- 7. The current price of a non-dividend-paying stock is 100. The annual effective risk-free interest rate is 4%, and there are no transaction costs.

The stock's two-year forward price is mispriced at 108. So to exploit this mispricing, an investor can short a share of the stock for 100 and simultaneously take a long position in a two-year forward contract. The investor can then invest the 100 at the risk-free rate, and finally buy back the share of stock at the forward price after two years.

Determine which term best describes this strategy.

- (A) Hedging
- (B) Forward
- (C) Arbitrage
- (D) Call Option
- (E) Put Option
- 8. Determine which of the following is NOT a distinguishing characteristic of futures contracts, relative to forward contracts.
- (A) Contracts are settled daily, and marked-to-market.
- (B) Contracts are more liquid, as one can offset an obligation by taking the opposite position.
- (C) Contracts are more customized to suit the buyer's needs.
- (D) Contracts are structured to minimize the effects of credit risk.
- (E) Contracts have price limits, beyond which trading may be temporarily halted.

9. An investor enters a long position in a futures contract on an index (F) with a notional value of 200F, expiring in one year. The index pays an annual continuously compounded dividend yield of 4%, and the annual continuously compounded risk-free interest rate is 2%.

At the time of purchase, the index price is 1100. Three months later, the investor has sustained a loss of 100. Assume the margin account earns an interest rate of 0%.

Let S be the price of the index at the end of month three. Calculate S.

- (A) 1078 (B) 1085
- (C) 1094
- (D) 1105
- (E) 1110

- 10. An investor purchases a 6-month call option on a stock with a strike price of 50. The investor pays 3.35 as a premium. At the end of 6 months, the price of the stock is 60. The risk-free annual effective interest rate is 5%. Calculate the investor's profit.
- A. 10
- B. 65.673
- C. 6.5673
- D. 100
- E. 0.66

- 11. Which type of option allows exercise at any time until the expiry of the option?
- (A) European
- (B) Bermudan
- (C) American
- (D) Exotic

12. For a certain stock, Investor A purchases a 45-strike call option while Investor B purchases a 135-strike put option. Both options are European with the same expiration date. Assume that there are no transaction costs.

If the final stock price at expiration is S, Investor A's payoff will be 12. Calculate Investor

B's payoff at expiration, if the final stock price is S.
(A) 0
(B) 12
(C) 36
(D) 57
(E) 78
13. The market price of Stock A is 50. A customer buys a 50-strike put contract on Stock A for 500. The put contract is for 100 shares of A. Calculate the customer's maximum possible loss.
(A) 0
(B) 5
(C) 50
(D) 500
(E) 5000
14. An investor purchased Option A and Option B for a certain stock today, with strike prices 70 and 80, respectively. Both options are European one-year put options. Determine which statement is true about the moneyness of these options, based on a particular stock price.
(A) If Option A is in-the-money, then Option B is in-the-money.
(B) If Option A is at-the-money, then Option B is out-of-the-money.
(C) If Option A is in-the-money, then Option B is out-of-the-money.
(D) If Option A is out-of-the-money, then Option B is in-the-money.

(E) If Option A is out-of-the-money, then Option B is out-of-the-money.

PART B

- 1. For a stock, you are given:
 - (i) It pays quarterly dividends of 0.20.
 - (ii) It has just paid a dividend.
 - (iii) Its price is 50.
 - (iv) The continuously compounded risk-free interest rate is 5%.

Calculate the forward price for an agreement to deliver 100 shares of the stock six months from now.

- 2. Suppose the S&P 500 futures index is currently 1237.25 and the initial margin is 10%. The notional value is 250 per index. You wish to enter into 20 S&P 500 futures contracts.
- (a) Find the initial margin balance.
- (b) Suppose you earn a continuously compounded rate of 4% on your margin balance. Your position is marked to market weekly, and the maintenance margin is 75% of the initial margin. What is the greatest S&P 500 index futures price 1 week from today at which you receive a margin call?

BONUS QUESTION:

Consider the following two strategies:

- **I.** Buy a one-year put option with a strike price of 50.
- **II.** Short a one-year forward contract.

You are given:

- (i) The continuously compounded risk-free interest rate is 0.04.
- (ii) The forward price of the underlying asset is 55.
- (iii) The premium for the put option is 4.

Let S be the price at the end of one year.

Determine the range of S for which the profit on strategy I is higher.