

Dept. of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS251: Mathematics of Financial Derivatives
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Major Exam 2 Term 242
Wednesday, March 19, 2025
9.00 PM - 11.00 PM

Name..... ID#:_____ Serial #:____

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
- 6. Only answers supported by work will be considered. Unsupported guesses will not be graded.**
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. **Write important steps to arrive at the solution of the exam problems.**

The test is 120 minutes, GOOD LUCK, and you may begin now!

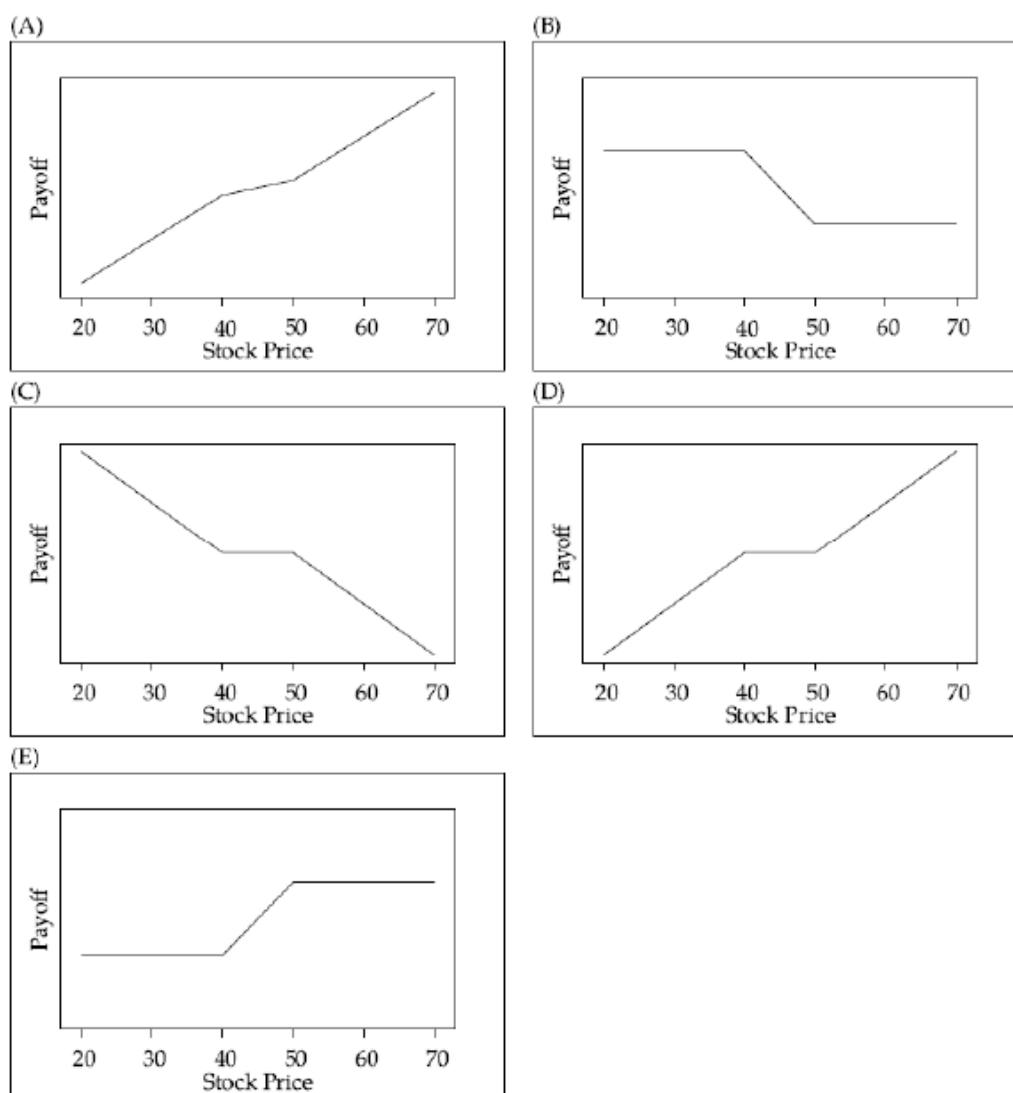
Question	Total Mark	Mark Obtained	Comments
PART A			
1-8	0.5*8		
9-15	1*7		
PART B (Answer 3 out of 4)			
1	3		
2	3		
3	3		
4	3		
Total	20		

Extra blank page

PART A

1. Determine which of the following statements about futures and forward contracts is false.
 - (A) Frequent marking-to-market and settlement of a futures contract can lead to pricing differences between a futures contract and an otherwise identical forward contract.
 - (B) Over-the-counter forward contracts can be customized to suit the buyer or seller, whereas futures contracts are standardized.
 - (C) Users of forward contracts are better able to protect themselves from credit risks than users of futures contracts.
 - (D) Forward contracts can be used to synthetically switch a portfolio invested in stocks into bonds.
 - (E) The holder of a long futures contract must place a fraction of the cost with an intermediary and provide assurances on the remaining purchase price.

2. An investor has a long position in a non-dividend-paying stock, and additionally, has a long collar on this stock consisting of a 40-strike put and a 50-strike call. Determine which of these graphs represents the payoff diagram for the overall position at the time of expiration of the options.

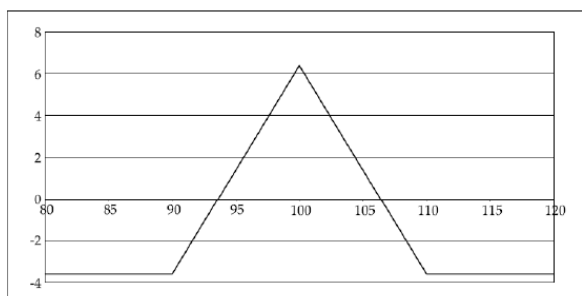


3. Stock ABC has the following characteristics:

- The current price to buy one share is 100.
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

Strike Price	Call option price	Put option price
90	14.63	0.24
100	6.80	1.93
110	2.17	6.81

A butterfly spread on this stock has the following profit diagram.



The annual risk-free interest rate compounded continuously is 5%. Determine which of the following will **NOT** produce this profit diagram.

- (A) Buy a 90 put, buy a 110 put, sell two 100 puts
 - (B) Buy a 90 call, buy a 110 call, sell two 100 calls
 - (C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call
 - (D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts
 - (E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.
4. XYZ stock pays no dividends and its current price is 100. Assume the put, the call, and the forward on XYZ stock are available and are priced so there are no arbitrage opportunities. Also, assume there are no transaction costs. The current risk-free annual effective interest rate is 1%. Determine which of the following strategies currently has the highest net premium.
- (A) Long a six-month 100-strike put and short a six-month 100-strike call
 - (B) Long a six-month forward on the stock
 - (C) Long a six-month 101-strike put and short a six-month 101-strike call
 - (D) Short a six-month forward on the stock
 - (E) Long a six-month 105-strike put and short a six-month 105-strike call
5. Which of the following statements about options are true?
- I. Put/call parity implies that puts and calls should trade at the same price, when the stock price equals the exercise price.
 - II. Early exercise of an American call option only makes sense when there is positive cash flow prior to maturity on the underlying asset.
 - III. As price volatility increases, call option prices rise, and put option prices fall.
- (A) I only (B) II only (C) III only (D) I and II only (E) I and III only

6. The PS index has the following characteristics:
- One share of the PS index currently sells for 1,000.
 - The PS index does not pay dividends.
- Anas wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025. The annual effective risk-free interest rate is 5%. Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.
- (A) Buy the put and sell the call, receive 23.81
 (B) Buy the put and sell the call, spend 23.81
 (C) Buy the put and sell the call, no cost 23.81
 (D) Buy the call and sell the put, receive 23.81
 (E) Buy the call and sell the put, spend 23.81.
7. An investor purchased Option A and Option B for a certain stock today, with strike prices 70 and 80, respectively. Both options are European one-year put options. Determine which statements is true about the moneyness of these options, based on a particular stock price.
- (A) If Option A is in-the-money, then Option B is in-the-money.
 (B) If Option A is at-the-money, then Option B is out-of-the-money.
 (C) If Option A is in-the-money, then Option B is out-of-the-money.
 (D) If Option A is out-of-the-money, then Option B is in-the-money.
 (E) If Option A is out-of-the-money, then Option B is out-of-the-money.
8. Which of the following statements about options are true?
- I. Put/call parity implies that puts and calls should trade at the same price, when the stock price equals the exercise price.
 II. Early exercise of an American call option only makes sense when there is positive cash flow prior to maturity on the underlying asset.
 III. As price volatility increases, call option prices rise, and put option prices fall.
- (A) I only (B) II only (C) III only (D) I and II only (E) I and III only
9. In the one-period binomial options pricing model, which of the following statements is true?
- (A) The risk-neutral probability is the actual probability of the stock price moving up or down.
 (B) The risk-neutral probability is used to ensure that the expected return on the option equals the risk-free rate.
 (C) The binomial model assumes that the stock price can take on any value within a range.
 (D) The binomial model requires the stock price to follow a lognormal distribution.
 (E) The risk-neutral probability is irrelevant for pricing options.

10. Ali Burger, LLC has an exclusive contract to supply Burger to the organizers of the annual Burger eating contest. The contract states that the contest organizers will take delivery of 10,000 Burger in one year at the market price. It will cost Ali Burger 1,000 to provide 10,000 Burger and today's market price is 0.12 for one Burger. The continuously compounded annual risk-free interest rate is 6%.

Ali Burger has decided to hedge as follows:

Buy 10,000 0.12-strike put options for 84.30 and sell 10,000 0.14-strike call options for 74.80. Both options are one-year European.

Ali Burger believes the market price in one year will be somewhere between 0.10 and 0.15 per Burger.

Determine which of the following intervals represents the range of possible profit one year from now for Ali Burger.

- (A) -200 to 100
- (B) -110 to 190
- (C) -100 to 200
- (D) 190 to 390
- (E) 200 to 400

11. You are given the following:

- (i) The current price to buy one share of XYZ stock is 500.
- (ii) The stock does not pay dividends.
- (iii) The annual risk-free interest rate, compounded continuously, is 6%.
- (iv) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 66.59.
- (v) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 18.64.

Using put-call parity, calculate the strike price, K.

- (A) 449 (B) 452 (C) 480 (D) 559 (E) 582

12. You are given:

- (i) The price of a stock is 35.
 - (ii) The stock pays continuous dividends proportional to its price at an annual rate of 0.04.
 - (iii) The continuously compounded risk free interest rate is 0.06.
 - (iv) A one-year American call option on the stock has a strike price of 32.
- Determine the lowest possible price for this call option.

- A) 3.4912 B) 4.3219 C) 1.2394 D) 9.1423 E) 2.9314

13. You are given:

- (i) The price of a stock is 35.
 - (ii) The stock pays continuous dividends at the annual rate of 0.04.
 - (iii) The continuously compounded risk free interest rate is 0.06.
 - (iv) A one-year American put option on the stock has a strike price of 34.
- Determine the lowest possible price for this put option.

A) 0 B) -1.6076 C) 1.6076 D) -2.9800 E) 2.9800

14. A stock price can go up by 20% or down by 15% over the next year. The current stock price is greater than 70. You own a one-year put on the stock. The put has an exercise price of 78.26. The effective annual risk-free interest rate is 11.25%. The stock pays no dividends.

If the put is exercised today, the amount received will be X. The price of the put today (unexercised) is also X.

Calculate the current stock price.

(A) 74.50 (B) 75.00 (C) 76.50 (D) 78.00 (E) 78.25

15. A nondividend paying stock's price is currently \$50.00. It is known that at the end of two months, it will be either \$54.00 or \$46.00. The risk-free interest rate is 9.0% per annum with continuous compounding.

Calculate the value of a two-month European call option with a strike price of \$48 on this stock.

A) 3.5137 B) 0.5945 C) 0.9200 D) 5.3156 E) 1.0800

16. A nondividend paying stock's price is 45. At the end of six months it can go either up 6.0% or down 6.0%.

You have sold 1,000 six-month European call options on this stock with a strike price of 46. Calculate the number of shares of stock that you would need to purchase in order to create a riskless hedge.

A) 314.81 B) 315.81 C) 316.81 D) 317.81 E) 318.81

PART B

(Answer any 3 questions)

1. You are interested in borrowing \$10,000 for one year by using a box spread.

You are given the following option prices:

Strike price	Call premium	Put Premium
30	8.06	1.31
40	3.13	5.80

The continuously compounded risk-free interest rate is 0.06.

One unit of the box spread consists of a long 30-40 bull spread of calls and a long 40-30 bear spread of puts.

Calculate the number of units of the box spread needed to achieve the financing goal, and determine whether they are bought or sold.

2. You are given

(i) The spot exchange rate is 95¥/\$1.

(ii) The continuously compounded risk-free rate in yen is 1%.

(iii) The continuously compounded risk-free rate in dollars is 5%.

(iv) A 1-year dollar denominated European call option on yen with strike \$0.01 costs \$0.0011.

Determine the premium of a 1-year dollar denominated European put option on yen with strike \$0.01.

3. You are given the following prices for American call options:

Strike price	Option price
40	5
50	4
55	3

To exploit the mispricing, you sell one 50-strike option and buy x 40-strike options and y 55-strike options, with x and y selected to create an arbitrage.

Determine the range of possible values for x .

4. Future prices of a stock are modeled with a 1-period binomial tree based on forward prices, the period being 3 months.

You are given:

- (i) The stock pays no dividends.
- (ii) The stock's initial price is 25.
- (iii) The continuously compounded risk-free interest rate is 4%.

20.24. A European call option expiring in 3 months has strike price 24.

Determine the change in the price of the call if the stock's volatility increases from 20% to 30%.