AS289- Major Exam 2 (Code 1)

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Exercise 1

Bond A and Bond B are each five-year 1000 face amount bonds. In addition:

- Bond A has an annual coupon rate of 5% paid semiannually.
- Bond B has an annual coupon rate of 3% paid annually.
- The price of Bond B is 100 less than the price of Bond A.
- The annual effective yield rate for Bond A is 4%.

Calculate the annual effective yield rate for Bond B.

1. 4.15%
2. 4.20%
3. 4.25%
4. 4.30%
5. 4.35%

Bond A: $F = C = 1000$; $n_A = 10$; $r_A = 2.5\%$; $j_A = \sqrt{1.04} - 1 \simeq 1.9804\%$

$$\Rightarrow P_A = F_A + F_A(r_A - j_A)\, a_{\overline{n_A}|j_A}$$
$$= 1046.72$$

Bond B: $F = C = 1000$; $n_B = 5$; $r_B = 4\%$

$$P_B = P_A - 100 = 946.72$$

Now, we need to solve the equation

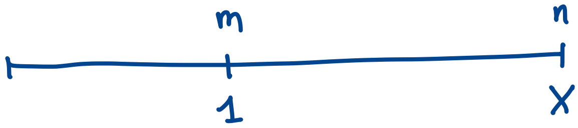
$$946.72 = 1040v^5 + 40(v + v^2 + v^3 + v^4)$$

$$\Rightarrow j_B = 4.2\%$$

Exercise 2

Let $P(0, t)$ be the current price of a zero-coupon bond that will pay 1 at time t. Let X be the value at time n of an investment of 1 made at time m, where $m < n$. Assume all investments earn the same interest rate. Determine X

1. $\dfrac{P(0, m)}{P(0, n)}$
2. $\dfrac{P(0, n)}{P(0, m)}$
3. $P(0, m)P(0, n)$
4. $\dfrac{1}{P(0, m)P(0, n)}$
5. $\dfrac{P(0, n) + P(0, m)}{P(0, m)}$



$$X = 1 \times (1 + i_{m,m+1}) \times (1 + i_{m+1,m+2}) \times \cdots \times (1 + i_{n-1,n})$$

$$= \frac{(1+i_{0,1}) \times (1+i_{1,2}) \times \cdots \times (1+i_{n-1,n})}{(1+i_{0,1}) \times (1+i_{1,2}) \times \cdots \times (1+i_{m-1,m})}$$

$$= \frac{(1+s_n)^n}{(1+s_m)^m}$$

Note that $P(0,t) = \dfrac{1}{(1+s_t)^t}$, then

$$X = \frac{P(0,m)}{P(0,n)}$$

Exercise 3

An 18-year bond, with a price 61% higher than its face value, offers annual coupons with the coupon rate equal to 2.25 times the annual effective yield rate. An n-year bond, with the same face value, coupon rate, and yield rate, sells for a price that is 45% higher than its face value. Calculate n.

1. 10
2. 12
3. 14
4. 17
5. 20

1/ $$P = 1.61F = F + F(r-j)\,a_{\overline{n|}j} = F + F(2.25j - j)\,a_{\overline{18|}j}$$
$$= F\left[1 + 1.25j\,a_{\overline{18|}j}\right]$$
$$\Rightarrow \boxed{1.61 = 1 + 1.25j\,a_{\overline{18|}j}} \Rightarrow 0.61 = 1.25\left[1 - v^{18}\right]$$
$$\Rightarrow v^{18} = 1 - \frac{0.61}{1.25} = \frac{0.64}{1.25} \Rightarrow v = \sqrt[18]{\frac{0.64}{1.25}}$$

2/ $$P = 1.45F = F\left[1 + 1.25j\,a_{\overline{n|}j}\right]$$
$$\Rightarrow 1.45 = 1 + 1.25j\,a_{\overline{n|}j} = 1 + 1.25(1 - v^n)$$
$$\Rightarrow v^n = 1 - \frac{0.45}{1.25} = \frac{0.8}{1.25} \Rightarrow n \ln v = \ln\left(\frac{0.8}{1.25}\right)$$
$$\Rightarrow n = \frac{\ln\left(\frac{0.8}{1.25}\right)}{\ln v} = \frac{18\ln\left(\frac{0.8}{1.25}\right)}{\ln\left(\frac{0.64}{1.25}\right)} = 12$$

Exercise 4

Kate buys a five-year 1000 face amount bond today with a 100 discount. The annual nominal coupon rate is 5% convertible semiannually.

One year later, Wallace buys a four-year bond. It has the same face amount and coupon values as Kate's and is priced to yield an annual nominal interest rate of 10% convertible semiannually. The discount on Wallace's bond is D.

The book value of Kate's bond at the time Wallace buys his bond is B. Calculate $B - D$.

1. 724
2. 738
3. 748
4. 756
5. 838

Kate: $P = 900$; $n = 10$; $r = 2.5\%$; $F = 1000$

$$P = Fv^n + Fra_{\overline{n|}j} \Rightarrow 900 = 1000v^{10} + 25(v + v^2 + \cdots + v^{10})$$

Using Texas BA II Plus

FV=1000; PMT=25; N=10 ; PV=−900

$$\Rightarrow j = 3.7155$$

$$\Rightarrow B = BV_2 = F + F(r-j)a_{\overline{8|}j}$$

$$= 917.19$$

Wallace:

$$P = F + F(r-j)a_{\overline{n|}j} = 1000 + 1000(0.025 - 0.05)a_{\overline{8|}0.05}$$

$$= 838.42$$

$$\Rightarrow D = F - P = 161.58$$

$$\Rightarrow B - D = 755.61\$$$

Exercise 5

A six-year 1000 face amount bond has an annual coupon rate of 8% semiannually. The bond currently sells for 911.37. Calculate the annual effective yield rate

1. 7.29%
2. 8.00%
3. 9.72%
4. 10.00%
5. 10.25%

$F = 1000$, $r = \frac{8\%}{2} = 4\%$; $P = 911.37$; $n = 6 \times 2 = 12$

$$P = Fv^{12} + Fr\left[v + v^2 + \cdots + v^{12}\right]$$

Using Texas BA II Plus

FV=1000; PMT=40; N=12; PV=−911.37

$j = 5\% \Rightarrow$ The effective annual yield rate is $(1+j)^2 - 1 = 10.25\%$

Exercise 6

As of 12/31/2013, an insurance company has a known obligation to pay 1,000,000 on 12/31/2017. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling 822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/2017. The maturity value of the bond equals the par value.

Consider two reinvestment interest rate movement scenarios effective 1/1/2014. Scenario A has interest rates drop by 0.5%. Scenario B has interest rates increase by 0.5%.

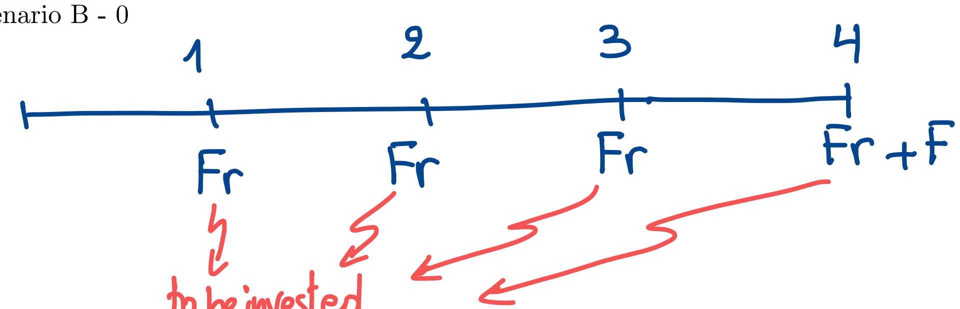
Determine which of the following best describes the insurance company's profit or (loss) as of 12/31/2017 after the liability is paid.

1. Scenario A - 6,613, Scenario B - 11,154
2. Scenario A - (14,763), Scenario B - 14,424
3. Scenario A - (18,913), Scenario B - 19,194
4. Scenario A - (1,313), Scenario B - 1,324
5. Scenario A - 0, Scenario B - 0

$F = 822,703$

$n = 4;\ r = 5\%$

Scenario A:



to be invested at a rate $5\% - 0.5\% = 4.5\%$

The balance is $Fr\, s_{\overline{4}|4.5\%} + F = 998,687\$$

There is a loss of $1,000,000 - 998,687 = 1,313$.

Scenario B:

The balance is $Fr\, s_{\overline{4}|5.5\%} + F = 1,001,323$

There is a profit of $1,001,323 - 1,000,000 = 1,323$

Exercise 7

An n-year bond with annual coupons has the following characteristics:

- The redemption value at maturity is 1890.
- The annual effective yield rate is 6%.
- The book value immediately after the third coupon is 1254.87.
- The book value immediately after the fourth coupon is 1277.38.

Calculate n.

1. 16
2. 17
3. 18
4. 19
5. 20

$C = 1890;\ j = 6\%;\ BV_3 = 1254.87;\ BV_4 = 1277.38$

$BV_4 = BV_3(1+j) - Fr \Rightarrow Fr = BV_3 \times (1+j) - BV_4 = 52.7822$

So, we have

$$BV_3 = C \times v^{n-3} + Fr\, a_{\overline{n-3}|j} = C v^{n-3} + Fr\, \frac{1-v^{n-3}}{j}$$

$$= \left(C - \frac{Fr}{j}\right) v^{n-3} + \frac{Fr}{j}$$

$$\Rightarrow \quad v^{n-3} = \left(BV_3 - \frac{Fr}{j}\right) \Big/ \left(C - \frac{Fr}{j}\right) \simeq 0.371343$$

$$\Rightarrow \quad (n-3)\ln v \simeq \ln(0.371343)$$

$$\Rightarrow \quad n \simeq 3 + \frac{\ln(0.371343)}{\ln v} \simeq 20.$$

Exercise 8

A ten-year 100 par value bond pays 8% coupons semiannually. The bond is priced at 118.20 to yield an annual nominal rate of 6% convertible semiannually.

Calculate the redemption value of the bond.

1. 97
2. 100
3. 103
4. 106
5. 109

$F=100$; $r=4\%$; $P=118.2$; $j=3\%$; $n=20$

$$P = Cv^{20} + Fr\, a_{\overline{20|}j}$$

$$\Rightarrow C = \left[P - Fr\, a_{\overline{20|}j}\right] \times (1+j)^{20}$$

$$= \left[118.2 - 4 \times \frac{1-1.03^{-20}}{0.03}\right] \times 1.03^{20}$$

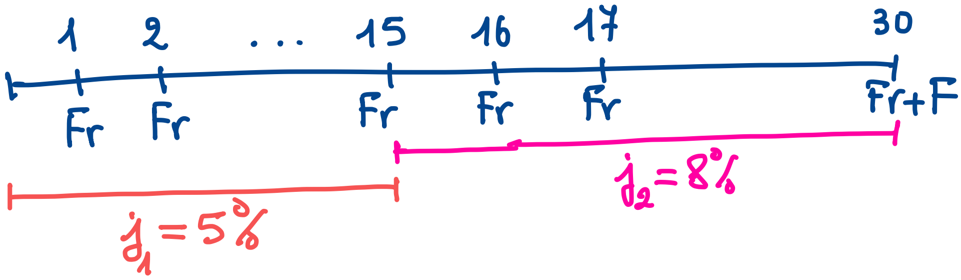
$$= 106$$

Exercise 9

A 30 year 10,000 bond pays 3.5% annual coupons and matures at par. It is purchased to yield 5% annually for the first 15 years and 8% annually thereafter. Calculate the price of the bond.

1. 4978
2. 6090
3. 6590
4. 7371
5. 7858

$n=30$; $F=10{,}000$; $r=3.5\%$;



$$P = Fr\, a_{\overline{15}|j_1} + \left[F v_2^{15} + Fr\, a_{\overline{15}|j_2} \right] v_1^{15}$$

$$= 6590.28$$

Exercise 10

You are told that a bond has a par and redemption value of 1000, a coupon rate of 12% convertible semiannually, and it is priced to yield 10% convertible semiannually. The bond has a term of n years. If the term of the bond is doubled, the price will increase by 50.

Calculate n.

1. 5
2. 7
3. 10
4. 12
5. 14

$F = 1000$; $r = 6\%$; $j = 5\%$. In n years, we have $2n$ coupon payments and in $2n$ years, we have $4n$ coupon payments.

$$\begin{cases} P = F + F(r-j)\, a_{\overline{2n}|j} & (1) \\ P + 50 = F + F(r-j)\, a_{\overline{4n}|j} & (2) \end{cases}$$

$$(2)-(1) \Rightarrow \quad 50 = F(r-j)\left[a_{\overline{4n}|j} - a_{\overline{2n}|j}\right] = F(r-j) \times \frac{v^{2n} - v^{4n}}{j}$$

$$\Rightarrow \underbrace{v^{2n}}_{X} - \underbrace{v^{4n}}_{X^2} = \frac{j \times 50}{F(r-j)} = 0.25$$

$$X^2 - X + 0.25 = 0 \Rightarrow X_1 = \tfrac{1}{2} = v^{2n} \Rightarrow n \simeq \frac{\ln(1/2)}{2 \ln v} = 7.1$$

Exercise 11

You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%

Calculate the one-year annual effective rate for the fifth year implied by this term structure.

$$i_{4,5} = \frac{(1+s_5)^5}{(1+s_4)^4} - 1 = \frac{1.095^5}{1.09^4} - 1 \simeq 11.52\%$$

1. 9.0%
2. 9.5%
3. 10.5%
4. 11.0%
5. 11.5%

Exercise 12

The one-year forward rates, deferred t years, are estimated to be:

Year (t)	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Calculate the spot rate for a zero-coupon bond maturing three years from now.

1. 4%
2. 5%
3. 6%
4. 7%
5. 8%

$$(1+i_{0,1})(1+i_{1,2})(1+i_{2,3}) = (1+s_3)^3$$

$$\Rightarrow s_3 \simeq \sqrt[3]{(1+i_{0,1})(1+i_{1,2})(1+i_{2,3})} - 1$$

$$= 5.987\%$$

Exercise 13

You are given the following information with respect to a bond:

- par amount: 1000
- term to maturity: 3 years
- annual coupon rate: 6% payable annually

Term	Annual spot interest rate
1	7%
2	8%
3	9%

Calculate the annual effective yield rate for the bond if the bond is sold at price equal to its value.

1. 8.12%
2. 8.32%
3. 8.52%
4. 8.72%
5. 8.92%

$F = 1000 \; ; \; r = 6\% \Rightarrow Fr = 60$

We have

$$P = \frac{Fr}{1+s_1} + \frac{Fr}{(1+s_2)^2} + \frac{F+Fr}{(1+s_3)^3} \simeq 926\$.$$

Using Texas BA II Plus

$FV = 1000; \; PMT = 60; \; N = 3; \; PV = -926$

$\Rightarrow j = 8.9192\%$

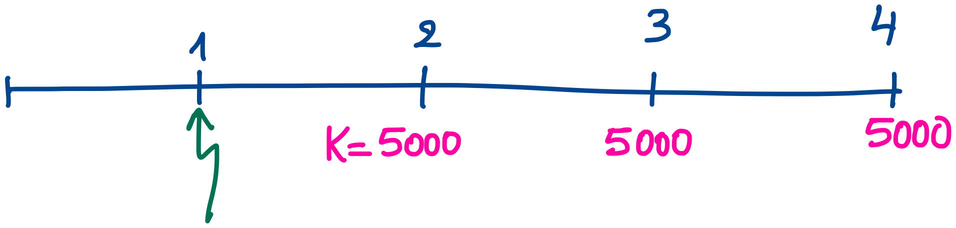
Exercise 14

You are given the following term structure of spot rates

Term (in years)	Spot interest rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

A three-year annuity immediate will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now.

1. 13,094
2. 13,153
3. 13,296
4. 13,321
5. 13,401



$$P = \frac{K}{1+i_{1,2}} + \frac{K}{(1+i_{1,2})(1+i_{2,3})} + \frac{K}{(1+i_{1,2})(1+i_{2,3})(1+i_{3,4})}$$

Note that $(1+i_{0,1})(1+i_{1,2}) \times \cdots \times (1+i_{n-1,n}) = (1+s_n)^n$.

Then $$P = K\left[\frac{1+s_1}{(1+s_2)^2} + \frac{1+s_1}{(1+s_3)^3} + \frac{1+s_1}{(1+s_4)^4}\right]$$

$$\simeq 13,152.5$$

Exercise 15

Consider the following yield curve $s_k = 0.085 + 0.003k - 0.0015k^2$. Find the price of three-year 1000 par bond with 6% annual coupons.

1. 806.5
2. 846.5
3. 906.5
4. 946.5
5. 1006.5

$s_1 = 8.65\%$

$s_2 = 8.5\%$

$s_3 = 8.05\%$

$$P = \frac{60}{1+s_1} + \frac{60}{(1+s_2)^2} + \frac{1060}{(1+s_3)^2}$$

$\simeq 946.5$

Exercise 16

An investor pays 4000 today for a three-year investment that returns cash flows of 1400 at the end of each year. The cash flows can be reinvested at the positive annual effective interest rate of i. Using an annual effective rate of interest of 4%, the net present value of this investment is 0. Calculate i.

1. 3.5%
2. 4.5%
3. 5.0%
4. 7.0%
5. 9.0%

$$FV = 1400\, s_{\overline{3|}i}$$

$$NPV = 1400\, s_{\overline{3|}i} \times 1.04^{-3} - 4000 = 0$$

$$\Leftrightarrow 1400\, s_{\overline{3|}i} = 4000 \times 1.04^{3} = 4{,}499.46$$

Exercise 17

You are given the following information about an investment account:

Date	Value immediately before deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar weighted return is Y.

Calculate Y.

1. −25%
2. −10%
3. 0%
4. 10%
5. 25%

Time-weighted return method:

$$0 = \frac{12}{10} \times \frac{X}{12+X} - 1 \Leftrightarrow 12X - 10(12+X) = 0$$

$$\Leftrightarrow X = 60$$

Dollar-weighted return method:

$$Y = \frac{60 - [10+60]}{10 + 60 \times \frac{1}{2}} = \frac{-10}{40} = -0.25$$

Exercise 18

Mary purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at 100 over the par value of 1100 on any coupon date starting at the end of year 5 and ending six months prior to maturity.

Calculate the minimum yield that Mary could receive, expressed as an annual nominal rate of interest convertible semiannually.

1. 4.7%
2. 4.9%
3. 5.1%
4. 5.3%
5. 5.5%

$r = 2\%$; 20 payments ; $P = 1021.5$

$F = 1100 \Rightarrow Fr = 22$

For $n \in \{10, 11, \ldots, 19\} \Rightarrow C = 1200$

Given that $P < C$, then the minimum yield rate is calculated based on a call at the last posible date

$$\Rightarrow P = Cv^{19} + Fr\, a_{\overline{19}|j} = 1222\, v^{19} + 22\, a_{\overline{18}|j}$$

$$\Rightarrow j = 2.8589\% \Rightarrow (5.7178\%)$$

For $n = 20 \Rightarrow C = 1100$

$$P = 1122\, v^{20} + 22\, a_{\overline{19}|j} \Rightarrow j = 2.4559\%$$

(4.9118%)

Exercise 19

Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The present value of these cash flows is 364.46 at an annual effective rate of interest i.

Calculate i.

1. 10%
2. 11%
3. 12%
4. 13%
5. 14%

$$100 \times (1+i)^{-2} + 200 \times (1+i)^{-1} + 100 = 364.46$$

$$\Longrightarrow 100(1+i)^{-2} + 200(1+i)^{-1} - 264.46 = 0$$

$$\Longrightarrow (1+i)^{-1} = 0.90908$$

$$\Longrightarrow i = 10\%$$

Exercise 20

A 10 year 1000 par bond with 6% semiannual coupons is purchased to yield 5.6% convertible semiannually. How much of the premium is amortized in the seventh period?

1. 1.33
2. 1.36
3. 1.39
4. 1.42
5. 1.45

$n = 20$; $F = 1000$; $r = 3\%$; $j = 2.8\%$

$BV_6 = F + F(r-j)a_{\overline{14}|j} = 1022.9$

$PR_7 = Fr - BV_6 \times j$

$= 1.3588$