Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS380: Actuarial Contingencies 1
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Final Exam Term 241
Thursday, December 19, 2024
8.00 AM - 10.30 AM

Name………………………………………………. ID#:_____   Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

6. Only answers supported by work will be considered. Unsupported guesses will not be graded.

7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

8. ***Only answers supported by work will be considered. Unsupported guesses will not be graded***.

9. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Mark	Mark Obtained	Comments
1 - 4	1*4 = 4		
5 - 12	2*8 = 16		
13	3		
14	3		
15	3		
16	3		
17	3		
Total	35		

Extra blank page

PART A

1. Which of the following accurately describes whole life insurance?
   a) Coverage is limited to a specified term with no savings element.
   b) Premiums are paid until death, and benefits are paid only if the insured dies within the policy term.
   c) Premiums are paid for life, and a benefit is guaranteed whenever the insured dies.
   d) The policy includes investment-linked benefits that fluctuate with the market.

2. What is a key feature of endowment insurance?
   a) It only provides a death benefit if the policyholder dies during the term.
   b) It pays a lump sum either on death during the term or on survival to the end of the term.
   c) It offers purely investment-linked returns with no mortality benefits.
   d) It guarantees benefits only to the insured's beneficiaries.

3. What is the main purpose of the equivalence principle in premium calculation?
   a) To ensure the premium equals the insurer's administrative costs.
   b) To set premiums so that the expected present value of premiums equals the expected present value of benefits.
   c) To allow flexibility in premium payments over the policy term.
   d) To calculate profit margins for insurers on each policy.

4. Which of the following is an important consideration when calculating gross premiums?
   a) Mortality assumptions only.
   b) Expected future claims and all expenses incurred by the insurer.
   c) Policyholder age and gender only.
   d) The probability of policy lapses and cancellations.

5. Suppose that Gompertz' law applies with $B = 0.00013$ and $c = 1.03$. Calculate $\frac{d}{dt} {}_t p_{40}$ at $t = 10$.
   a) -0.000567 b) 0.000567   c) -0.00567   d) -0.00567

6. **Table 1** is an extract from a (hypothetical) select life table with a select period of two years. Note carefully the layout – each row relates to a fixed age at selection. Use this table to calculate the probability that a life currently aged 76 who was selected one year ago will die between ages 85 and 87.

   a) 0.66177   b) 0.09433   c) 0.08993   d) 0.8993

Table 1: Extract from a (hypothetical) select life table.

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
75	15 930	15 668	15 286	77
76	15 508	15 224	14 816	78
77	15 050	14 744	14 310	79
⋮	⋮	⋮	⋮	⋮
80			12 576	82
81			11 928	83
82			11 250	84
83			10 542	85
84			9 812	86
85			9 064	87

7. Table 2 gives the values of l_x and A_x, assuming an effective interest rate of 6% per year. Calculate $_{5|}A_{35}$, assuming UDD between integer ages where necessary.
   a) 0.735942 b) 0.012656   c) 0.138719   d) 0.748974
   Table 2

x	l_x	A_x
35	100 000.00	0.151375
36	99 737.15	0.158245
37	99 455.91	0.165386
38	99 154.72	0.172804
39	98 831.91	0.180505
40	98 485.68	0.188492

8. Table 2 gives the values of l_x and A_x, assuming an effective interest rate of 6% per year. Calculate $\bar{A}_{35:\overline{5|}}$, assuming UDD between integer ages where necessary.

   a) 0.735942 b) 0.012656   c) 0.138719   d) 0.748974

9. What is the effective rate of interest per year, to the nearest whole number, given that $\ddot{a}_{50:\overline{10|}} = 8.2066$, $a_{50:\overline{10|}} = 7.8277$, and $_{10}p_{50} = 0.9195$.

   a) 1%   b) 2%   c) 3%   d) 4%

10. Given that $a_{60} = 10.996$, $a_{61} = 10.756$, $a_{62} = 10.509$ and $i = 0.06$, calculate $_2p_{60}$
    a) 0.9822   b) 0.9722   c) 0.9622   d) 0.9522

11. Consider a fully discrete 10-year term insurance issued to a select life aged 50, with sum insured $100000. Calculate the net annual premium assuming an interest rate of 5% per year, $\ddot{a}_{[50]:\overline{10|}} = 8.05665$ and $A^{1}_{[50]:\overline{10|}} = 0.01439$.
    a) 168.57   b)178.57   c)188.57   d)198.57

12. A fully discrete whole life insurance with sum insured $10000 is issued to a select life aged x. The net premium is $134. You are given that $q_{[x]} = 0.00106$ and $i = 0.045$. Calculate $_1V^n$.
    a) 259.17   b)927.51   c)129.57   d)512.97

PART B

13. A group of 100000 independent lives, each aged 65, purchases one-year term insurance. At the start, 20% of the group are preferred lives, with $q_{65} = 0.002$ and 80% of the group are normal lives, with $q_{65} = 0.009$.
    (a) Calculate the standard deviation of the number of survivors at the end of the year.
    (b) Calculate the proportion of preferred lives expected in the survivor group.
    (c) Using a normal approximation, without continuity correction, calculate the 90th percentile of the number of survivors at the end of the year.
    Hint: The number of survivors in each subgroup follows a binomial distribution.

14. Under an endowment insurance issued to a life aged x, let X denote the present value of a unit sum insured, payable at the moment of death or at the end of the n-year term. Under a term insurance issued to a life aged x, let Y denote the present value of a unit sum insured, payable at the moment of death within the n-year term. Given that $V[X] = 0.0052$, $v^n = 0.3$, $_np_x = 0.8$, $E[Y] = 0.04$, calculate $V[Y]$.

15. Each member of a group of 200 independent lives aged x will receive a life annuity-due of $100 per year paid from a trust fund. You are given that

$$\ddot{a}_x = \begin{cases} 13.01704 & \text{at } i = 4\%, \\ 9.59176 & \text{at } i = 8.16\%. \end{cases}$$

Let Y denote the present value random variable for the total annuity payments for all the group members. Using a normal approximation to the distribution of Y, calculate the amount that should be deposited in the trust fund to be 90% certain that the funds will be sufficient to pay the benefits. Assume interest at 4% per year.

16. You are given the following extract from a select life table with a four-year select period. A select individual aged 41 purchased a fully discrete three-year term insurance with a net annual premium of $350. Use an effective rate of interest of 6% per year to calculate
(a) the sum insured,
(b) the standard deviation of L0,
(c) $\Pr[L0 > 0]$.

Table 3:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	l_{x+4}	$x+4$
[40]	100 000	99 899	99 724	99 520	99 288	44
[41]	99 802	99 689	99 502	99 283	99 033	45
[42]	99 597	99 471	99 268	99 030	98 752	46

17. Given Table 3, a select individual aged 41 purchased a three-year term insurance with a sum insured of $200 000, with premiums payable annually throughout the term. Assume an effective rate of interest of 6% per year, and no expenses.
(a) Show that the premium for the term insurance is P = $323.59.
(b) Calculate the mean and standard deviation of the present value of future loss random variable, L_1, for the term insurance.
(c) Calculate the sum insured for a three-year endowment insurance for a select life age 41, with the same premium as for the term insurance, P = $323.59.