Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS389: Actuarial Science Problem Lab II

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MAJOR 1, Term 241

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DURATION: 60 MINUTES

Questions

1.

$$\lim_{N\to\infty} \frac{5^N}{N!} =$$

A) 0 B) $\frac{1}{2}$ C) $5\,ln\,5$ D) $+\infty$ E) None of A,B,C,D

2.

Let f be a continuous function on $\mathbb{R}^2$ and let $I = \int_0^2 \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y)\,dy\,dx$.

Which of the following expressions is equal to I with the order of integration reversed?

A) $\int_{-2}^{2}\int_{y^2}^{2} f(x,y)\,dx\,dy$

D) $\int_{-\sqrt{2}}^{\sqrt{2}}\int_{y^2}^{2} f(x,y)\,dx\,dy$

B) $\int_{-2}^{2}\int_{y}^{\sqrt{2}} f(x,y)\,dx\,dy$

E) $\int_{0}^{2}\int_{2}^{y^2} f(x,y)\,dx\,dy$

C) $\int_{0}^{2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy$

3.

(SOA) An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage.

A) 0 B) $\frac{47}{144}$ C) $\frac{1}{2}$ D) $\frac{97}{144}$ E) $\frac{7}{9}$

4.

(SOA) A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

A) 24 B) 36 C) 41 D) 52 E) 60

5.

You are given that $P[A] = .5$ and $P[A \cup B] = .7$.
Actuary 1 assumes that A and B are independent and calculates $P[B]$ based on that assumption.
Actuary 2 assumes that A and B mutually exclusive and calculates $P[B]$ based on that assumption.
Find the absolute difference between the two calculations.

A) 0 B) .05 C) .10 D) .15 E) .20

6.

A survey of 1000 Canadian sports fans who indicated they were either hockey fans or lacrosse fans or both, had the following result.

- 800 indicated that they were hockey fans
- 600 indicated that they were lacrosse fans

Based on the sample, find the probability that a Canadian sports fan is not a hockey fan given that she/he is a lacrosse fan.

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

7.

Three boxes are numbered 1, 2 and 3. For $k = 1, 2, 3$, box k contains k blue marbles and $5 - k$ red marbles. In a two-step experiment, a box is selected and 2 marbles are drawn from it without replacement. If the probability of selecting box k is proportional to k, what is the probability that the two marbles drawn have different colors?

A) $\frac{17}{60}$ B) $\frac{34}{75}$ C) $\frac{1}{2}$ D) $\frac{8}{15}$ E) $\frac{17}{30}$

8.

In Canada's national 6-49 lottery, a ticket has 6 numbers each from 1 to 49, with no repeats. Find the probability of matching exactly 4 of the 6 winning numbers if the winning numbers are all randomly chosen.

A) .00095 B) .00097 C) .00099 D) .00101 E) .00103

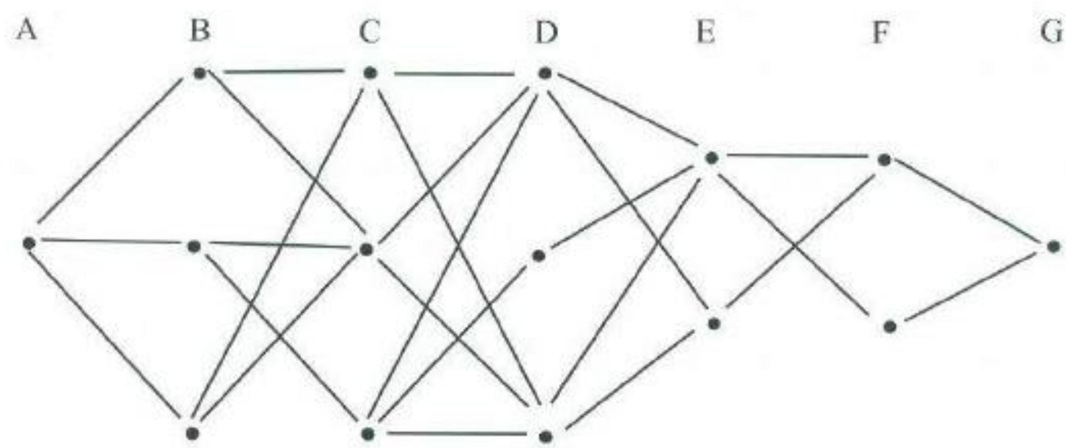
9.

A number X is chosen at random from the series 2,5,8, ... and another number Y is chosen at random from the series 3,7,11, ... Each series has 100 terms. Find $P[X = Y]$.

A) .0025 B) .0023 C) .0030 D) .0021 E) .0033

10.

In the following diagram, A, B,... refer to successive states through which a traveler must pass in order to get from A to G, moving from left to right. A path consists of a sequence of line segments from one state to the next. A path must always move to the next state until reaching state G. Determine the number of possible paths from A to G.



A) 30 B) 32 C) 34 D) 36 E) 38

11.

Let X be a discrete random variable with probability function

$P[X = x] = \frac{2}{3^x}$ for $x = 1, 2, 3, \ldots$ What is the probability that X is even?

A) $\frac{1}{4}$ B) $\frac{2}{7}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

12.

In a small metropolitan area, annual losses due to storm, fire, and theft are independently distributed random variables. The pdf's are:

	Storm	Fire	Theft
$f(x)$	e^{-x}	$\frac{2e^{-2x/3}}{3}$	$\frac{5e^{-5x/12}}{12}$

Determine the probability that the maximum of these losses exceeds 3 .

A) 0.002 B) 0.050 C) 0.159 D) 0.287 E) 0.414

13.

(SOA) An insurance company insures a large number of homes. The insured value, X of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4} & \text{for } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?

A) 0.578 B) 0.684 C) 0.704 D) 0.829 E) 0.875

14.

The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & \text{for } 0 < x < 20 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, what is the probability that it exceeds 16?

A) $\frac{1}{25}$ B) $\frac{1}{9}$ C) $\frac{1}{8}$ D) $\frac{1}{3}$ E) $\frac{3}{7}$

15.

The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f, where $f(x)$ is proportional to $(10+x)^{-2}$. Calculate the probability that the lifetime of the machine part is less than 6.

A) 0.04 B) 0.15 C) 0.47 D) 0.53 E) 0.94

16.

A recent study indicates that the annual cost of maintaining and repairing a car in a town in Ontario averages 200 with a variance of 260. If a tax of 20% is introduced on all items associated with the maintenance and repair of cars (i.e., everything is made 20% more expensive), what will be the variance of the annual cost of maintaining and repairing a car?

A) 208 B) 260 C) 270 D) 312 E) 374

17.

An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function $M_X(t) = \frac{1}{(1-2500t)^4}$.

Determine the standard deviation of the claim size for this class of accidents.

A) 1,340 B) 5,000 C) 8,660 D) 10,000 E) 11,180

18.

(SOA) Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} 1/3 & \text{for } x = 0 \\ 2/3 & \text{for } x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the moment generating function, $M(t)$, of $Y = X_1X_2X_3$.

A) $\frac{19}{27} + \frac{8}{27}e^t$ B) $1 + 2e^t$ C) $(\frac{1}{3} + \frac{2}{3}e^t)^3$ D) $\frac{1}{27} + \frac{8}{27}e^{3t}$ E) $\frac{1}{3} + \frac{2}{3}e^{3t}$

19.

A random variable has the cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{x^2-2x+2}{2} & \text{for } 1 \le x < 2 \\ 1 & \text{for } x \ge 2 \end{cases}$$

Calculate the variance of X

A) $\frac{7}{72}$ B) $\frac{1}{8}$ C) $\frac{5}{36}$ D) $\frac{4}{32}$ E) $\frac{23}{12}$

20.

Smith is offered the following gamble: he is to choose a coin at random from a large collection of coins and toss it randomly. The proportion of the coins in the collection that are loaded towards a head is p. If a coin is loaded towards a head, then when the coin is tossed randomly, there is a $\frac{3}{4}$ probability that a head will turn up and a $\frac{1}{4}$ probability that a tail will turn up. Similarly, if the coin is loaded towards tails, then there is a $\frac{3}{4}$ probability that a tail will turn up and a $\frac{1}{4}$ probability that a head will turn up. If Smith tosses a head, he loses \$100, and if he tosses a tail, he wins \$200. Find the proportion p for which Smith's expected gain is 0 when taking the gamble.

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{5}{6}$