Department of Mathematics and Statistics

King Fahd University of Petroleum & Minerals

AS389: Actuarial Science Problem Lab II

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Major 2, Term 241

Date: Nov. 20, 2024

Duration: 80 Minutes

Questions

1. X has a discrete uniform distribution on the integers $0, 1, 2, \ldots, n$ and Y has a discrete uniform distribution on the integers $1, 2, 3, \ldots, n$. Find $Var[X] - Var[Y]$.

A) $\frac{2n+1}{12}$ B) $\frac{1}{12}$ C) 0 D) $-\frac{1}{12}$ E) $-\frac{2n+1}{12}$

2. The probability that a particular machine breaks down in any day is .20 and is independent of the breakdowns on any other day. The machine can break down only once per day. Calculate the probability that the machine breaks down two or more times in ten days.

A) .1075 B) .0400 C) .2684 D) .6242 E) .9596

3. If X has a continuous uniform distribution on the interval from 0 to 10, then what is $P[X + \frac{10}{X} > 7]$?

A) $\frac{3}{10}$ B) $\frac{31}{70}$ C) $\frac{1}{2}$ D) $\frac{39}{70}$ E) $\frac{7}{10}$

Problems 4 and 5 relate to the following information. Three individuals are running a one kilometer race. The completion time for each individual is a random variable. X_i is the completion time, in minutes, for person i.

X_1 : uniform distribution on the interval $[2.9, 3.1]$

X_2 : uniform distribution on the interval $[2.7, 3.1]$

X_3 : uniform distribution on the interval $[2.9, 3.3]$

The three completion times are independent of one another.

4. Find the probability that the earliest completion time is less than 3 minutes.

A) .89 B) .91 C) .94 D) .96 E) .98

5. Find the probability that the latest completion time is less than 3 minutes (nearest .01).

A) .03 B) .06 C) .09 D) .12 E) .15

6.

The time until the occurrence of a major hurricane is exponentially distributed. It is found that it is 1.5 times as likely that a major hurricane will occur in the next ten years as it is that the next major hurricane will occur in the next five years. Find the expected time until the next major hurricane.

A) 5 B) $5\,ln\,2$ C) $\frac{5}{ln\,2}$ D) $10\,ln\,2$ E) $\frac{10}{ln\,2}$

7. Let X be a Poisson random variable with $E[X] = ln\,2$. Calculate $E[cos(\pi X)]$

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) $2\,ln\,2$

8.

A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000 ?

A) 3858 B) 4449 C) 5382 D) 5644 E) 7235

9.

Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation 0.0056h . The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation 0.0044h . Assuming the two measurements are independent random variables, what is the probability that their average value is within 0.005h of the height of the tower?

A) 0.38 B) 0.47 C) 0.68 D) 0.84 E) 0.90

10.

An insurer uses the exponential distribution with mean μ as the model for the total annual claim occurring from a particular insurance policy in the current one year period. The insurer assumes an inflation factor of 10% for the one year period following the current one year period. Using the insurer's assumption, find the coefficient of variation ($\frac{standard\ deviation}{expected\ value}$) for the annual claim paid on the policy for the one year period following the current one year period.

A) 1.21 B) 1.1 C) 1 D) $\frac{1}{1.1}$ E) $\frac{1}{1.21}$

11.

Average loss size per policy on a portfolio of policies is 100. Actuary 1 assumes that the distribution of loss size has an exponential distribution with a mean of 100, and Actuary 2 assumes that the distribution of loss size has a pdf of $f_2(x) = \frac{2\theta^2}{(x+\theta)^3}$, $x > 0$. If m_1 and m_2 represent the median loss sizes for the two distributions, find $\frac{m_1}{m_2}$.

A) .6 B) 1.0 C) 1.3 D) 1.7 E) 2.0

12. A fair die is tossed until a 2 is obtained. If X is the number of trials required to obtain the first 2, what is the smallest value of x for which $P[X \leq x] \geq \frac{1}{2}$?

A) 2 B) 3 C) 4 D) 5 E) 6

13. (SOA) The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

A) 6,321 B) 7,358 C) 7,869 D) 10,256 E) 12,642

14. A box contains 10 white and 15 black marbles. Let X denote the number of white marbles in a selection of 10 marbles selected at random and without replacement. Find $\frac{Var[X]}{E[X]}$.

A) $\frac{1}{8}$ B) $\frac{3}{16}$ C) $\frac{2}{8}$ D) $\frac{5}{16}$ E) $\frac{3}{8}$

15. (SOA) The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

A) 0.07 B) 0.29 C) 0.38 D) 0.42 E) 0.57

16. An analysis of auto accidents shows that one in four accidents results in an insurance claim. In a series of independent accidents, find the probability that the first accident resulting in an insurance claim is one of the first 3 accidents.

A) .50 B) .52 C) .54 D) .56 E) .58

17. An insurer has 5 independent one-year term life insurance policies. The face amount on each policy is 100,000. The probability of a claim occurring in the year for any given policy is .2. Find the probability the insurer will have to pay more than the total expected claim for the year.

A) .06 B) .11 C) .16 D) .21 E) .26

18. The number of claims per year from a particular auto insurance policy has a Poisson distribution with a mean of 1, and probability function p_k. Based on a number of years of experience, the insurer decides to change the distribution, so that the new probability of 0 claims is $p_0^* = .5$, and the new probabilities p_k^* for $k \geq 1$ are proportional to the old (Poisson) probabilities according to the relationship $p_k^* = c \cdot p_k$ for $k \geq 1$. Find the mean of the new claim number distribution.

A) .79 B) .63 C) .5 E) .37 E) .21

19. The probability generating function of a discrete non-negative integer valued random variable N is a function of the real variable t: $P(t) = \sum_{k=0}^{\infty} t^k \cdot P[N = k] = E[t^N]$.

Which of the following is the correct expression for the probability generating function of the Poisson random variable with mean 2?

A) e^{-2t} B) e^{1-2t} C) e^{2t} D) e^{2t-1} E) $e^{2(t-1)}$

20. An insurer issues two independent policies to individuals of the same age.
The insurer models the distribution of the completed number of years until death for each individual, and uses the geometric distribution $P[N = k] = (.99)^k(.01)$, where $k = 0, 1, 2, ...$ and N is the completed number of years until death for each individual.
Find the probability that the two individuals die in the same year.

A) .001 B) .003 C) .005 D) .007 E) .009