Department of Mathematics and Statistics

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AS389: Actuarial Science Problem Lab II

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FINAL EXAM, Term 241

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Questions

1. A survey of 1000 people determines that 80% like walking and 60% like biking, and all like at least one of the two activities. What is the probability that a randomly chosen person in this survey likes biking but not walking?

A) 0 B) .1 C) .2 D) .3 E) .4

2. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

A) 0.26 B) 0.38 C) 0.40 D) 0.48 E) 0.62

3. (SOA) An insurance policy reimburses a loss up to a benefit limit of 10 . The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \frac{2}{y^3} & \text{for } y > 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

A) 1.0 B) 1.3 C) 1.8 D) 1.9 E) 2.0

4. (SOA) A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E[X]$.

A) $2 + \frac{1}{3}e^{-6}$ B) $2 - 2e^{-2/3} + 5e^{-4/3}$ C) 3 D) $2 + 3e^{-2/3}$ E) 5

5. (SOA) The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine's age at failure, X has density function

$$f(x) = \begin{cases} \frac{1}{5} & \text{for } 0 < x < 5 \\ 0 & \text{otherwise.} \end{cases}$$

Let Y be the age of the machine at the time of replacement. Determine the variance of Y.

A) 1.3 B) 1.4 C) 1.7 D) 2.1 E) 7.5

6. (SOA) An investment account earns an annual rate R that follows a uniform distribution on the interval $(0.04, 0.08)$. The value of a 10,000 initial investment in this account after one year is given by $V = 10,000e^R$. Determine the cumulative distribution function, $F(v)$, of V for values of v that satisfy $0 < F(v) < 1$.

A) $\frac{10,000e^{v/10,000} - 10,408}{425}$ B) $25e^{v/10,000} - 0.04$ C) $\frac{v - 10,408}{10,833 - 10,408}$

D) $\frac{25}{v}$ E) $25\left[ln(\frac{v}{10,000}) - 0.04\right]$

7. Three boxes are numbered 1, 2 and 3. For k = 1, 2, 3, box k contains k blue marbles and 5 - k red marbles. In a two-step experiment, a box is selected and 2 marbles are drawn from it without replacement. If the probability of selecting box k is proportional to k, what is the probability that the two marbles drawn have different colors?

A) 17/60 B) 34/75 C) 1/2 D) 8/15 E) 17/30

8. In Canada's national 6-49 lottery, a ticket has 6 numbers each from 1 to 49, with no repeats. Find the probability of matching exactly 4 of the 6 winning numbers if the winning numbers are all randomly chosen.

A).00095 B) .00097 C).00099 D) .00101 E) .00103

9. A model for world population assumes a population of 6 billion at reference time 0, with population increasing to a limiting population of 30 billion. The model assumes that the rate of population growth at time $t \geq 0$ is

$$\frac{Ae^t}{(.02A+e^t)^2}$$

billion per year, where t is regarded as a continuous variable. According to this model, at what time will the population reach 10 billion (nearest .1)?

A) .3 B) 4 C) 5 D) .6 E) .8

10. Calculate the area of the closed region in the xy-plane bounded by $y = x - 5$ and $y^2 = 2x + 5$.

A) 8 B) 74/3 C) 98/3 D) 122/3 E) 128/3

11. Let $F(x) = \int_0^{x^{1/3}} \sqrt{1+t^4}dt$. F'(0) =

A) 0 B) 1/3 C) 2/3 D) 1 E) Does not exist

12. If a loss occurs, the amount of loss will be uniform between \$1000 and \$2,000. The probability of the loss occurring is .2. An insurance policy pays the total loss, if a loss occurs. Find the standard deviation of the amount paid by the insurer.

A) 584 B) 614 C) 634 D) 654 E) 674

13. X and Y are random losses with joint density function $f(x,y) = \frac{x}{500{,}000}$ for $0 < x < 100$ and $0 < y < 100$.
An insurance policy on the losses pays the total of the two losses to a maximum payment of 100. Find the expected payment the insurer will make on this policy (nearest 1).

A) 90 B) 92 C) 94 D) 96 E) 98

14. (SOA) A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000 ?

A) 3858 B) 4449 C) 5382 D) 5644 E) 7235

15. (SOA) A company offers earthquake insurance. Annual premiums are modeled by an exponential random variable with mean 2. Annual claims are modeled by an exponential random variable with a mean of 1. Premiums and claims are independent. Let X denote the ratio of claims to premiums. What is the density function of X?

A) $\frac{1}{2x+1}$ B) $\frac{2}{(2x+1)^2}$ C) e^{-x} D) $2e^{-2x}$ E) xe^{-x}

16. (SOA) The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f, where $f(x)$ is proportional to $(10+x)^{-2}$.
Calculate the probability that the lifetime of the machine part is less than 6.

A) 0.04 B) 0.15 C) 0.47 D) 0.53 E) 0.94

17. X is a continuous random variable with density function $f(x) = ce^{-x}$, $x > 1$.
Find $P[X < 3|X > 2]$.

A) $1-e^{-1}$ B) e^{-1} C) $1-e^{-2}$ D) $e^{-1}-e^{-2}$ E) $e^{-2}-e^{-3}$

18. The time until the occurrence of a major hurricane is exponentially distributed. It is found that it is 1.5 times as likely that a major hurricane will occur in the next ten years as it is that the next major hurricane will occur in the next five years. Find the expected time until the next major hurricane.

A) 5 B) $5\,ln\,2$ C) $\frac{5}{ln\,2}$ D) $10\,ln\,2$ E) $\frac{10}{ln\,2}$

19. The number of claims received each day by a claims center has a Poisson distribution. On Mondays, the center expects to receive 2 claims but on other days of the week, the claims center expects to receive 1 claim per day. The numbers of claims received on separate days are mutually independent of one another. Find the probability that the claims center receives at least 3 claims in a 5 day week (Monday to Friday).

A) .90 B) .92 C) .94 D) .96 E) .98

20. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x,y) = \begin{cases} 2x & \text{for } 0 < x < 1,\ x < y < x+1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the conditional variance of Y given that $X = x$?

A) $\frac{1}{12}$ B) $\frac{7}{6}$ C) $x + \frac{1}{2}$ D) $x^2 - \frac{1}{6}$ E) $x^2 + x + \frac{1}{3}$

21. Let X and Y be continuous random variables having a bivariate normal distribution with means μ_X and μ_Y, common variance σ^2, and correlation coefficient ρ_{XY}. Let Fx and Fy be the cumulative distribution functions of X and Y respectively. Determine which of the following is a necessary and sufficient condition for Fx(t) ≥Fy (t) for all t.

A) $\mu_X \geq \mu_Y$ B) $\mu_X \leq \mu_Y$ C) $\mu_X \geq \rho_{XY}\mu_Y$ D) $\mu_X \leq \rho_{XY}\mu_Y$ E) $\rho_{XY} \geq 0$

22. As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5.

Calculate the probability that the sixth person tested is the first one with high blood pressure.

A) 0.000 B) 0.053 C) 0.080 D) 0.316 E) 0.394

23. Let X be a random variable with moment generating function

$$M(t) = \left(\frac{2+e^t}{3}\right)^9, \quad -\infty < t < \infty .$$

Calculate the variance of X.

A) 2 B) 3 C) 8 D) 9 E) 11

24. Smith is offered the following gamble: he is to choose a coin at random from a large collection of coins and toss it randomly. $\frac{3}{4}$ of the coins in the collection are loaded towards a head (LH) and $\frac{1}{4}$ are loaded towards a tail. If a coin is loaded towards a head, then when the coin is tossed randomly, there is a $\frac{3}{4}$ probability that a head will turn up and a $\frac{1}{4}$ probability that a tail will turn up. Similarly, if the coin is loaded towards tails, then there is a $\frac{3}{4}$ chance of tossing a tail on any given toss. If Smith tosses a head, he loses \$100, and if he tosses a tail, he wins \$200. Smith is allowed to obtain "sample information" about the gamble. When he chooses the coin at random, he is allowed to toss it once before deciding to accept the gamble with that same coin. Suppose Smith tosses a head on the sample toss. Find Smith's expected gain/loss on the gamble if it is accepted.

A) loss of 20 B) loss of 10 C) loss of 0 D) gain of 10 E) gain of 20

25. A survey of 1000 Saudi Arabia sports fans who indicated they were either hockey fans or football fans or both, had the following result.

- 800 indicated that they were hockey fans
- 600 indicated that they were football fans

Based on the sample, find the probability that a Saudi Arabia sports fan is not a hockey fan given that she/he is a football fan.

A) 1/5 B) ¼ C) 1/3 D) ½ E) 1

26. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both ball: are the same color is 0.44. Calculate the number of blue balls in the second urn.

A) 4 B) 20 C) 24 D) 44 E) 64

27. The random variable X has density function $f(x) = ce^{-|x|}$ for $-\infty < x < \infty$. Find the variance of X.

A) 0 B) .5 C) 1.0 D) 1.5 E) 2.0

28. The joint density function of two random losses X and Y is

$$f(x,y) = \begin{cases} x+y, & \text{for } 0<x<1 \text{ and } 0<y<1 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the probability that loss X is less than double the loss Y.

A) $\frac{7}{32}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{19}{24}$ E) $\frac{7}{8}$

29. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails. Let X and Y be the times at which the first and second circuits fail, respectively. X and Y have joint probability density function

$$f(x,y) = \begin{cases} 6e^{-x}e^{-2y} & \text{for } 0 < x < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected time at which the device fails?

A) 0.33 B) 0.50 C) 0.67 D) 0.83 E) 1.50

30. An insurance company designates 10% of its customers as high risk and 90% as low risk. The number of claims made by a customer in a calendar year is Poisson distributed with mean θ and is independent of the number of claims made by that customer in the previous calendar year. For high risk customers $\theta = 0.6$, while for low risk customers $\theta = 0.1$. Calculate the probability that a customer of unknown risk profile who made exactly one claim in 1997 will make exactly one claim in 1998.

A) 0.08 B) 0.12 C) 0.16 D) 0.20 E) 0.24

SOLUTIONS

1. C
2. D
3. D
4. D
5. C
6. E
7. E
8. B
9. A
10. E
11. E
12. B
13. B
14. D
15. B
16. C
17. A
18. C
19. C
20. A
21. B
22. B
23. A
24. B
25. C
26. A
27. E
28. D
29. D
30. C