Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS389: Actuarial Science Problem Lab II
Major 2, Term 251
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DATE: Nov. 20, 2025
DURATION: 100 MINUTES

Name: ______________________ **ID:** ______________________

Instructions

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary.
6. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
7. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only.

The test is 100 minutes, GOOD LUCK, and you may begin now!

Work Space

Questions

1. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{1}{30}x(1+3x) & \text{for } 1 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find $E[\frac{1}{X}]$.

A) $\frac{1}{12}$ **B)** $\frac{7}{15}$ **C)** $\frac{45}{103}$ **D)** $\frac{11}{20}$ **E)** $\frac{14}{15}$

2. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

A) $\frac{1}{5}$ **B)** $\frac{3}{5}$ **C) 1** **D)** $\frac{28}{15}$ **E)** $\frac{12}{5}$

3. An insurer's annual weather related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \frac{2.5(200)^{2.5}}{x^{3.5}} & \text{for } x \geq 200 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

A) 35 **B) 93** **C) 124** **D) 131** **E) 298**

4. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

If the median of this distribution is $\frac{1}{3}$, then

$$\lambda =$$

A) $\frac{1}{3}\ln\frac{1}{2}$ **B)** $\frac{1}{3}\ln 2$ **C)** $2\ln\frac{3}{2}$ **D)** $3\ln 2$ **E)** **3**

5. X has a discrete uniform distribution on the integers $0, 1, 2, \ldots, n$ and Y has a discrete uniform distribution on the integers $1, 2, 3, \ldots, n$. Find $Var[X] - Var[Y]$.

A) $\frac{2n+1}{12}$ **B)** $\frac{1}{12}$ **C)** **0** **D)** $-\frac{1}{12}$ **E)** $-\frac{2n+1}{12}$

6. The probability that a particular machine breaks down in any day is .20 and is independent of the breakdowns on any other day. The machine can break down only once per day. Calculate the probability that the machine breaks down two or more times in ten days.

A) .1075 **B) .0400** **C) .2684** **D) .6242** **E) .9596**

7. A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.

(ii) In any calendar year, the probability of a hurricane is 0.05.

(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

A) 0.06 B) 0.19 C) 0.38 D) 0.62 E) 0.92

8. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

A) 0.096 B) 0.192 C) 0.235 D) 0.376 E) 0.469

9. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation 0.0056h. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation 0.0044h. Assuming the two measurements are independent random variables, what is the probability that their average value is within 0.005h of the height of the tower?

A) 0.38 B) 0.47 C) 0.68 D) 0.84 E) 0.90

10. A new car battery is sold for \$100 with a 3-year limited warranty. If the battery fails at time t $(0 < t < 3)$, the battery manufacturer will refund $\$100(1 - \frac{t}{3})$. After analyzing battery performance, the battery manufacturer uses the (continuous) uniform distribution on the interval $(0, n)$ as the model for time until failure for the battery (n in years). The battery manufacturer determines that the expected cost of the warranty is \$10. Find n.

A) 3 B) 5 C) 10 D) 15 E) 30

11. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

A) 6,321 B) 7,358 C) 7,869 D) 10,256 E) 12,642

12. A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000?

A) 3858 B) 4449 C) 5382 D) 5644 E) 7235

13. A wheel is spun with the numbers 1, 2 and 3 appearing with equal probability of $\frac{1}{3}$ each. If the number 1 appears, the player gets a score of 1.0; if the number 2 appears, the player gets a score of 2.0; if the number 3 appears, the player gets a score of X, where X is a normal random variable with mean 3 and standard deviation 1. IF W represents the player's score on 1 spin of the wheel, then what is $P[W \leq 1.5]$?

A) .13 B) .33 C) .36 D) .40 E) .64

14. Let X and Y be discrete loss random variables with joint probability function

$$f(x,y) = \begin{cases} \frac{y}{24x} & \text{for } x = 1,2,4; y = 2,4,8; x \leq y \\ 0, & \text{otherwise} \end{cases}$$

An insurance policy pays the full amount of loss X and half of loss Y. Find the probability that the total paid by the insurer is no more than 5.

A) $\frac{1}{8}$ **B)** $\frac{7}{24}$ **C)** $\frac{3}{8}$ **D)** $\frac{5}{8}$ **E)** $\frac{17}{24}$

15. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 0.75x & \text{for } 0 < x < 2 \text{ and } 0 < y \leq 2 - x \\ 0, & \text{otherwise} \end{cases}$$

What is $P[X > 1]$?

A) $\frac{1}{8}$ **B)** $\frac{1}{4}$ **C)** $\frac{3}{8}$ **D)** $\frac{1}{2}$ **E)** $\frac{3}{4}$

16. Let X and Y be independent random variables with $\mu_X = 1$, $\mu_Y = -1$, $\sigma_X^2 = \frac{1}{2}$, and $\sigma_Y^2 = 2$. Calculate $E[(X+1)^2(Y-1)^2]$.

A) 1 **B)** $\frac{9}{2}$ **C) 16** **D) 17** **E) 27**

17. Let X and Y be discrete random variables with joint probability function

$$p(x,y) = \begin{cases} 0.250, & \text{for } x=0, y=0 \\ 0.250, & \text{for } x=1, y=0 \\ 0.125, & \text{for } x=0, y=1 \\ 0.375, & \text{for } x=1, y=1. \end{cases}$$

Calculate Corr (X,Y), the correlation coefficient of X and Y.

A) 0.06 **B) 0.23** **C) 0.26** **D) 0.38** **E) 0.63**

18. The independent random variables X and Y have the same mean. The coefficients of variation of X and Y are 3 and 4, respectively. Calculate the coefficient of variation of $\frac{1}{2}(X+Y)$.

A) 5/4 **B) 7/4** **C) 5/2** **D) 7/2** **E) 7**

19. An actuary is studying hurricane models. A year is classified as a high, medium, or low hurricane year with probabilities 0.1, 0.3, and 0.6, respectively. The numbers of hurricanes in high, medium, and low years follow Poisson distributions with means 20, 15, and 10, respectively. Calculate the variance of the number of hurricanes in a randomly selected year.

A) 11.25 B) 12.50 C) 12.94 D) 13.42 E) 23.75

20. The lifetime of a certain electronic device has an exponential distribution with mean 0.50. Calculate the probability that the lifetime of the device is greater than 0.70, given that it is greater than 0.40.

A) 0.203 B) 0.247 C) 0.449 D) 0.549 E) 0.861

Solutions

1. **Solution:** We need to compute $E\left[\frac{1}{X}\right] = \int_1^3 \frac{1}{x} \cdot \frac{1}{30} x(1+3x)dx$

$$E\left[\frac{1}{X}\right] = \frac{1}{30}\int_1^3 (1+3x)dx = \frac{1}{30}\left[x + \frac{3x^2}{2}\right]_1^3$$

$$= \frac{1}{30}\left[\left(3+\frac{27}{2}\right) - \left(1+\frac{3}{2}\right)\right] = \frac{1}{30}\left[\frac{33}{2} - \frac{5}{2}\right] = \frac{1}{30}\cdot\frac{28}{2} = \frac{28}{60} = \frac{7}{15}$$

Answer: **B)** $\frac{7}{15}$

2. **Solution:** For the expected value of X:

$$E[X] = \int_{-2}^4 x \cdot \frac{|x|}{10}dx = \frac{1}{10}\int_{-2}^4 |x| \cdot x dx$$

Split the integral at $x = 0$:

$$E[X] = \frac{1}{10}\left[\int_{-2}^0 (-x)\cdot x dx + \int_0^4 x \cdot x dx\right] = \frac{1}{10}\left[\int_{-2}^0 -x^2 dx + \int_0^4 x^2 dx\right]$$

$$= \frac{1}{10}\left[-\left[\frac{x^3}{3}\right]_{-2}^0 + \left[\frac{x^3}{3}\right]_0^4\right] = \frac{1}{10}\left[-\left(0-(-\frac{8}{3})\right) + \left(\frac{64}{3} - 0\right)\right]$$

$$= \frac{1}{10}\left[-\frac{8}{3} + \frac{64}{3}\right] = \frac{1}{10}\cdot\frac{56}{3} = \frac{56}{30} = \frac{28}{15}$$

Answer: **D)** $\frac{28}{15}$

3. **Solution:** This is a Pareto distribution with $\alpha = 2.5$ and $\theta = 200$. The CDF is:

$$F(x) = 1 - \left(\frac{\theta}{x}\right)^\alpha = 1 - \left(\frac{200}{x}\right)^{2.5}$$

For the 30th percentile $p_{0.3}$:

$$0.3 = 1 - \left(\frac{200}{p_{0.3}}\right)^{2.5} \Rightarrow \left(\frac{200}{p_{0.3}}\right)^{2.5} = 0.7 \Rightarrow p_{0.3} = \frac{200}{(0.7)^{1/2.5}}$$

For the 70th percentile $p_{0.7}$:

$$0.7 = 1 - \left(\frac{200}{p_{0.7}}\right)^{2.5} \Rightarrow \left(\frac{200}{p_{0.7}}\right)^{2.5} = 0.3 \Rightarrow p_{0.7} = \frac{200}{(0.3)^{1/2.5}}$$

The difference is:

$$p_{0.7} - p_{0.3} = 200\left(\frac{1}{(0.3)^{0.4}} - \frac{1}{(0.7)^{0.4}}\right) \approx 200\left(\frac{1}{0.617} - \frac{1}{0.867}\right) \approx 200(1.621 - 1.153) \approx 200(0.468) \approx 93.6$$

Answer: **B) 93**

4. **Solution:** This is an exponential distribution with parameter λ. The median m satisfies:

$$\int_0^m \lambda e^{-\lambda x} dx = 0.5$$

$$\left[-e^{-\lambda x}\right]_0^m = 0.5 \Rightarrow 1 - e^{-\lambda m} = 0.5 \Rightarrow e^{-\lambda m} = 0.5$$

Given $m = \frac{1}{3}$:

$$e^{-\lambda/3} = 0.5 \Rightarrow -\frac{\lambda}{3} = \ln(0.5) \Rightarrow \lambda = -3\ln(0.5) = 3\ln(2)$$

Answer: **D)** $3\ln 2$

5. **Solution:** For discrete uniform distribution on integers $0, 1, \ldots, n$:

$$Var[X] = \frac{(n+1)^2 - 1}{12} = \frac{n(n+2)}{12}$$

For discrete uniform distribution on integers $1, 2, \ldots, n$:

$$Var[Y] = \frac{n^2 - 1}{12}$$

Thus:

$$Var[X] - Var[Y] = \frac{n(n+2)}{12} - \frac{n^2 - 1}{12} = \frac{n^2 + 2n - n^2 + 1}{12} = \frac{2n+1}{12}$$

Answer: **A)** $\frac{2n+1}{12}$

6. **Solution:** This is a binomial distribution with $n = 10$, $p = 0.20$.

$$P(2 \text{ or more}) = 1 - P(0) - P(1)$$

$$P(0) = (0.8)^{10} = 0.1074$$

$$P(1) = \binom{10}{1}(0.2)^1(0.8)^9 = 10 \times 0.2 \times 0.1342 = 0.2684$$

$$P(2 \text{ or more}) = 1 - 0.1074 - 0.2684 = 0.6242$$

Answer: **D) .6242**

7. **Solution:** This follows a binomial distribution with $n = 20$, $p = 0.05$.

$$P(\text{fewer than } 3) = P(0) + P(1) + P(2)$$

$$P(0) = (0.95)^{20} = 0.3585$$

$$P(1) = \binom{20}{1}(0.05)^1(0.95)^{19} = 20 \times 0.05 \times 0.3774 = 0.3774$$

$$P(2) = \binom{20}{2}(0.05)^2(0.95)^{18} = 190 \times 0.0025 \times 0.3972 = 0.1886$$

Sum = 0.3585 + 0.3774 + 0.1886 = 0.9245 0.92 Answer: **E) 0.92**

8. **Solution:** For one group of 10, probability that at least 9 complete:

$$P(\geq 9) = P(9) + P(10)$$

$$P(9) = \binom{10}{9}(0.8)^9(0.2)^1 = 10 \times 0.1342 \times 0.2 = 0.2684$$

$$P(10) = (0.8)^{10} = 0.1074$$

$$P(\geq 9) = 0.2684 + 0.1074 = 0.3758$$

Probability that exactly one group has at least 9 completers:

$$2 \times 0.3758 \times (1 - 0.3758) = 2 \times 0.3758 \times 0.6242 = 0.469$$

Answer: **E) 0.469**

9. **Solution:** Let E_1 and E_2 be the errors. Then:

$$E_1 \sim N(0, (0.0056h)^2), \quad E_2 \sim N(0, (0.0044h)^2)$$

The average error: $\bar{E} = \frac{E_1+E_2}{2}$

$$E[\bar{E}] = 0, \quad Var[\bar{E}] = \frac{1}{4}[(0.0056h)^2 + (0.0044h)^2] = \frac{h^2}{4}[0.00003136 + 0.00001936] = \frac{h^2}{4}(0.00005072)$$

$$\sigma_{\bar{E}} = h\sqrt{0.00001268} = 0.00356h$$

We want $P(|\bar{E}| \leq 0.005h)$:

$$Z = \frac{0.005h}{0.00356h} = 1.404$$

$$P(|Z| \leq 1.404) = 2\Phi(1.404) - 1 \approx 2(0.92) - 1 = 0.84$$

Answer: **D) 0.84**

10. **Solution:** The refund amount is $R(t) = 100(1 - t/3)$ for $0 < t < 3$, and 0 for $t \geq 3$. Expected refund:

$$E[R] = \int_0^3 100(1 - t/3) \cdot \frac{1}{n} dt = \frac{100}{n} \int_0^3 (1 - t/3) dt$$

$$= \frac{100}{n}\left[t - \frac{t^2}{6}\right]_0^3 = \frac{100}{n}\left[3 - \frac{9}{6}\right] = \frac{100}{n} \times 1.5 = \frac{150}{n}$$

Set equal to 10: $\frac{150}{n} = 10 \Rightarrow n = 15$ Answer: **D) 15**

11. **Solution:** For exponential with mean 2, failure rate $\lambda = 0.5$. Full refund probability: $P(T \leq 1) = 1 - e^{-0.5} = 0.3935$ Half refund probability: $P(1 < T \leq 2) = e^{-0.5} - e^{-1} = 0.6065 - 0.3679 = 0.2386$ Expected refund per printer: $200 \times 0.3935 + 100 \times 0.2386 = 78.70 + 23.86 = 102.56$ For 100 printers: $100 \times 102.56 = 10,256$ Answer: **D) 10,256**

12. Solution: For exponential with mean 10, failure rate $\lambda = 0.1$. Payment in first year: x with probability $1 - e^{-0.1} = 0.09516$ Payment in second/third year: $0.5x$ with probability $e^{-0.1} - e^{-0.3} = 0.9048 - 0.7408 = 0.1640$ Expected payment: $x \times 0.09516 + 0.5x \times 0.1640 = 0.09516x + 0.0820x = 0.17716x$ Set equal to 1000: $0.17716x = 1000 \Rightarrow x = 5644$ Answer: **D) 5644**

13. Solution: Using law of total probability:

$$P(W \leq 1.5) = P(W \leq 1.5|1)P(1) + P(W \leq 1.5|2)P(2) + P(W \leq 1.5|3)P(3)$$
$$= 1 \times \frac{1}{3} + 0 \times \frac{1}{3} + P(X \leq 1.5) \times \frac{1}{3}$$

For $X \sim N(3,1)$, $P(X \leq 1.5) = \Phi\left(\frac{1.5-3}{1}\right) = \Phi(-1.5) = 0.0668$ Thus: $P(W \leq 1.5) = \frac{1}{3} + 0 + \frac{1}{3} \times 0.0668 = 0.3333 + 0.0223 = 0.3556 \approx 0.36$ Answer: **C) .36**

14. Solution: First find the constant: Sum over all valid (x,y): . . . Answer: **E)**

15. Solution: First find the constant by integrating:

$$\int_0^2 \int_0^{2-x} 0.75xdydx = \int_0^2 0.75x(2-x)dx = 0.75 \int_0^2 (2x - x^2)dx$$
$$= 0.75 \left[x^2 - \frac{x^3}{3}\right]_0^2 = 0.75 \left[4 - \frac{8}{3}\right] = 0.75 \times \frac{4}{3} = 1$$

So constant is correct.

$$P(X > 1) = \int_1^2 \int_0^{2-x} 0.75xdydx = \int_1^2 0.75x(2-x)dx$$
$$= 0.75 \int_1^2 (2x - x^2)dx = 0.75 \left[x^2 - \frac{x^3}{3}\right]_1^2$$
$$= 0.75 \left[(4 - \frac{8}{3}) - (1 - \frac{1}{3})\right] = 0.75 \left[\frac{4}{3} - \frac{2}{3}\right] = 0.75 \times \frac{2}{3} = 0.5$$

Answer: **D)** $\frac{1}{2}$

16. Solution: Since X and Y are independent:

$$E[(X+1)^2(Y-1)^2] = E[(X+1)^2] \cdot E[(Y-1)^2]$$
$$E[(X+1)^2] = E[X^2 + 2X + 1] = E[X^2] + 2E[X] + 1$$
$$E[X^2] = Var[X] + (E[X])^2 = \frac{1}{2} + 1^2 = 1.5$$

So $E[(X+1)^2] = 1.5 + 2(1) + 1 = 4.5$

$$E[(Y-1)^2] = E[Y^2 - 2Y + 1] = E[Y^2] - 2E[Y] + 1$$
$$E[Y^2] = Var[Y] + (E[Y])^2 = 2 + (-1)^2 = 3$$

So $E[(Y-1)^2] = 3 - 2(-1) + 1 = 3 + 2 + 1 = 6$

Thus: $E[(X+1)^2(Y-1)^2] = 4.5 \times 6 = 27$ Answer: **E) 27**

17. **Solution:** First find marginal distributions: $P(X = 0) = 0.250 + 0.125 = 0.375$, $P(X = 1) = 0.250+0.375 = 0.625$ $P(Y = 0) = 0.250+0.250 = 0.500$, $P(Y = 1) = 0.125+0.375 = 0.500$

    $E[X] = 0 \times 0.375 + 1 \times 0.625 = 0.625$ $E[Y] = 0 \times 0.500 + 1 \times 0.500 = 0.500$

    $E[X^2] = 0^2 \times 0.375 + 1^2 \times 0.625 = 0.625$ $E[Y^2] = 0^2 \times 0.500 + 1^2 \times 0.500 = 0.500$

    $Var[X] = 0.625 - (0.625)^2 = 0.625 - 0.390625 = 0.234375$ $Var[Y] = 0.500 - (0.500)^2 = 0.500 - 0.250 = 0.250$

    $E[XY] = (0)(0)(0.250) + (1)(0)(0.250) + (0)(1)(0.125) + (1)(1)(0.375) = 0.375$

    $Cov(X, Y) = E[XY] - E[X]E[Y] = 0.375 - (0.625)(0.500) = 0.375 - 0.3125 = 0.0625$

    $\rho = \frac{Cov(X,Y)}{\sqrt{Var[X]Var[Y]}} = \frac{0.0625}{\sqrt{0.234375 \times 0.250}} = \frac{0.0625}{\sqrt{0.05859375}} = \frac{0.0625}{0.24206} \approx 0.258$ Answer: **C) 0.26**

18. **Solution:** σ/μ. Given: $\text{CV}_X = 3 \Rightarrow \sigma_X/\mu = 3 \Rightarrow \sigma_X = 3\mu$ $\text{CV}_Y = 4 \Rightarrow \sigma_Y/\mu = 4 \Rightarrow \sigma_Y = 4\mu$

    For $Z = \frac{1}{2}(X+Y)$: $E[Z] = \frac{1}{2}(\mu+\mu) = \mu$ $Var[Z] = \frac{1}{4}(Var[X]+Var[Y]) = \frac{1}{4}(9\mu^2+16\mu^2) = \frac{1}{4}(25\mu^2) = 6.25\mu^2$ $\sigma_Z = 2.5\mu$

    $\text{CV}_Z = \sigma_Z/E[Z] = 2.5\mu/\mu = 2.5 = 5/2$

    Answer: **C) 5/2**

19. **Solution:** Let N be number of hurricanes. Using law of total variance: Var[N] = E[Var[N—type]] + Var[E[N—type]]

    E[N—type]: High:20, Medium:15, Low:10 E[E[N—type]] = 0.1×20 + 0.3×15 + 0.6×10 = 2 + 4.5 + 6 = 12.5 Var[E[N—type]] = 0.1×(20-12.5)² + 0.3×(15-12.5)² + 0.6×(10-12.5)² = 0.1×56.25 + 0.3×6.25 + 0.6×6.25 = 5.625 + 1.875 + 3.75 = 11.25

    E[Var[N—type]] = 0.1×20 + 0.3×15 + 0.6×10 = 2 + 4.5 + 6 = 12.5

    Total variance = 12.5 + 11.25 = 23.75 Answer: **E) 23.75**

20. **Solution:** For exponential distribution with mean 0.5, failure rate $\lambda = 2$. Memoryless property: $P(T > 0.70|T > 0.40) = P(T > 0.30) = e^{-2\times 0.30} = e^{-0.6} = 0.5488 \approx 0.549$ Answer: **D) 0.549**.