Department of Mathematics and Statistics

King Fahd University of Petroleum & Minerals

AS389: Actuarial Science Problem Lab II

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FINAL EXAM, Term 251

Date: Dec. 11, 2025

Duration: 100 Minutes

Instructions:

1. Only materials provided by the instructor can be present on the table during the exam.

2. No student will be allowed to go out once the Exam starts.

3. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

4. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

5. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only.

Answers:

1. C	6. D	11. D	16. B
2. E	7. C	12. B	17. A
3. B	8. C	13. D	18. D
4. A	9. E	14. C	19. D
5. E	10. C	15. B	20. D

1. If E and F are events for which $P[E \cup F] = 1$, then $P[E' \cup F'] =$

A) 0
B) $P[E'] + P[F'] - P[E'] \cdot P[F']$
C) $P[E'] + P[F']$
D) $P[E'] + P[F'] - 1$
E) 1

2. Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have a probability of .08 of having an accident, but new drivers with driver education have only a .05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accidents the first year?

A) $\frac{5}{6}$

B) $\frac{(.92)(.4)}{(.95)(.6)+(.92)(.4)}$

C) $\frac{(.95)(.4)}{(.95)(.6)+(.92)(.4)}$

D) $\frac{(.95)(.4)}{(.95)(.4)+(.92)(.6)}$

E) $\frac{(.95)(.6)}{(.95)(.6)+(.92)(.4)}$

3. A loss distribution random variable X has a pdf of $f(x) = ae^{-x} + be^{-2x}$ for $x > 0$. If the mean of X is 1 , find the probability $P[X < 1]$.

A) .52 B) .63 C) .74 D) .85 E) .96

4. If $f_X(x) = xe^{-x^2/2}$ for $x > 0$, and $Y = ln\, X$, find the density function for Y.

A) $e^{2y-\frac{1}{2}e^{2y}}$ B) $(ln\, y)e^{-(ln\, y)^2/2}$ C) $e^{y-\frac{1}{2}e^{2y}}$ D) $ye^{-y^2/2}$ E) $e^{-\frac{1}{2}e^{2y}}$

5. An insurer estimates that Smith's time until death is uniformly distributed on the interval $[0, 5]$ and Jones' time until death is uniform on the interval $[0, 10]$. The insurer assumes that the two times of death are independent of one another. Find the probability that Smith is the first of the two to die.

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

6. If X has a normal distribution with mean 1 and variance 4, then $P[X^2 - 2X \leq 8] = ?$

A) .13 B) .43 C) .75 D) .86 E) .93

7. The pdf of X is $f(x) = \begin{cases} 2x \text{ for } 0<x<1 \\ 0, \text{ elsewhere} \end{cases}$. The mean of X is μ .
Find $\frac{E[|X-\mu|]}{Var[X]}$.

A) $\frac{20}{9}$ B) $\frac{26}{9}$ C) $\frac{32}{9}$ D) $\frac{19}{81}$ E) $\frac{22}{81}$

8. Two players put one dollar into a pot. They decide to throw a pair of dice alternately. The first one who throws a total of 5 on both dice wins the pot. How much should the player who starts add to the pot to make this a fair game?

A) $\frac{9}{17}$ B) $\frac{8}{17}$ C) $\frac{1}{8}$ D) $\frac{2}{9}$ E) $\frac{8}{9}$

9. An analysis of economic data shows that the annual income of a randomly chosen individual from country A has a mean of \$18,000 and a standard deviation of \$6000, and the annual income of a randomly chosen individual from country B has a mean of \$31,000 and a standard deviation of \$8000. 100 individuals are chosen at random from Country A and 100 from Country B. Find the approximate probability that the average annual income from the group chosen from Country B is at least \$15,000 larger than the average annual income from the group chosen from Country A (all amounts are in US\$).

A) .9972 B) .8413 C) .5000 D) .1587 E) .0228

10. Three individuals are running a one kilometer race. The completion time for each individual is a random variable. X_i is the completion time, in minutes, for person i.

X_1 : uniform distribution on the interval $[2.9, 3.1]$
X_2 : uniform distribution on the interval $[2.7, 3.1]$
X_3 : uniform distribution on the interval $[2.9, 3.3]$

The three completion times are independent of one another.
Find the expected latest completion time (nearest .1).

A) 2.9 B) 3.0 C) 3.1 D) 3.2 E) 3.3

11.

An insurer will pay the amount of a loss in excess of a deductible amount α. Suppose that the loss amount has a continuous uniform distribution between 0 and $C > \alpha$. When a loss occurs, let the expected payout on the policy be $f(\alpha)$. Find $f'(\alpha)$.

A) $\frac{\alpha}{C}$ B) $-\frac{\alpha}{C}$ C) $\frac{\alpha}{C}+1$ D) $\frac{\alpha}{C}-1$ E) $1-\frac{\alpha}{C}$

12.

Coins K and L are weighted so the probabilities of heads are .3 and .1, respectively. Coin K is tossed 5 times and coin L is tossed 10 times. If all the tosses are independent, what is the probability that coin K will result in heads 3 times and coin L will result in heads 6 times?

A) $\binom{5}{3}(.3)^3(.7)^2 + \binom{10}{6}(.1)^3(.9)^2$

B) $\binom{5}{3}(.3)^3(.7)^2\binom{10}{6}(.1)^6(.9)^4$

C) $\binom{15}{9}(.4)^9(.6)^6$

D) $\frac{\binom{5}{3}\binom{10}{6}}{\binom{15}{9}}$

E) $(.6)(.9)$

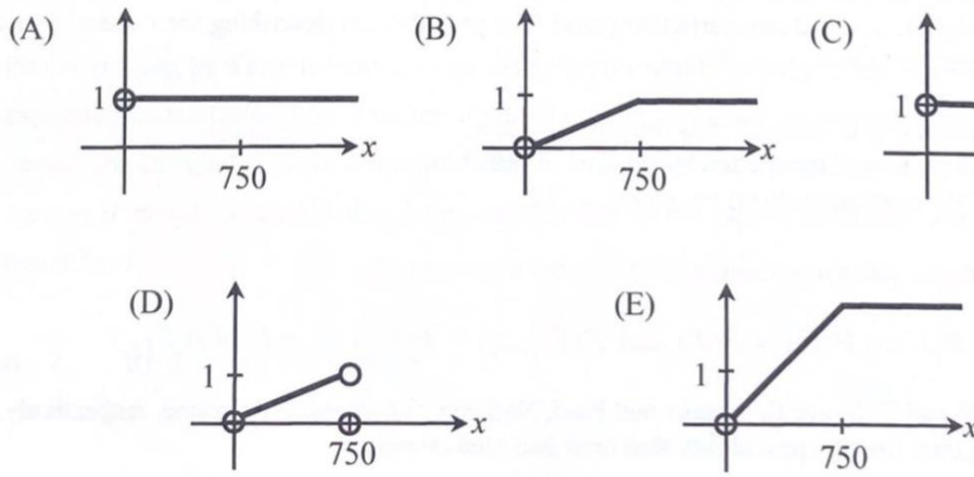
13.

The value, v, of an appliance is based on the number of years since purchase, t, as follows: $v(t) = e^{(7-.2t)}$. If the appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance. After seven years the warranty pays nothing. The time until failure of the appliance has an exponential distribution with a mean of 10. Calculate the expected payment from the warranty.

A) 98.70 B) 109.66 C) 270.43 D) 320.78 E) 352.16

14.

An insurance policy is written that reimburses the policyholder for all losses incurred up to a benefit limit of 750 . Let $f(x)$ be the benefit paid on a loss of x . Which of the following most closely resembles the graph of the derivative of f ?



15.

A test for a disease correctly diagnoses a diseased person as having the disease with probability .85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of .10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

A) .0085 B) .0791 C) .1075 D) .1500 E) .9000

16.

Let X and Y be discrete random variables with joint probability function $f(x,y)$ given by the following table:

		x			
		2	3	4	5
	0	.05	.05	.15	.05
y	1	.40	0	0	0
	2	.05	.15	.10	0

For this joint distribution, $E[X]=2.85$ and $E[Y]=1$. Calculate $Cov[X,Y]$.

A) $-.20$ B) $-.15$ C) .95 D) 2.70 E) 2.85

17. For a Poisson random variable X with mean λ it is found that it is twice as likely for X to be less than 3 as it is for X to be greater than or equal to 3. Find λ (nearest .1).

A) 2.0 B) 2.2 C) 2.4 D) 2.6 E) 2.8

18. Let X and Y be discrete random variables with joint probability function

$$f(x,y) = \begin{cases} \frac{2^{x+1-y}}{9} & \text{for } x=1,2 \text{ and } y=1,2 \\ 0, \text{otherwise} \end{cases}.$$

Calculate $E[\frac{X}{Y}]$.

A) $\frac{8}{9}$ B) $\frac{5}{4}$ C) $\frac{4}{3}$ D) $\frac{25}{18}$ E) $\frac{5}{3}$

19. People passing by a city intersection are asked for the month in which they were born. It is assumed that the population is uniformly divided by birth month, so that any randomly passing person has an equally likely chance of being born in any particular month. Find the minimum number of people needed so that the probability that no two people have the same birth month is less than .5 .

A) 2 B) 3 C) 4 D) 5 E) 6

20.

One of the questions asked by an insurer on an application to purchase a life insurance policy is whether or not the applicant is a smoker. The insurer knows that the proportion of smokers in the general population is .30, and assumes that this represents the proportion of applicants who are smokers. The insurer has also obtained information regarding the honesty of applicants:

- 40% of applicants that are smokers say that they are non-smokers on their applications,
- none of the applicants who are non-smokers lie on their applications.

What proportion of applicants who say they are non-smokers are actually non-smokers?

A) 0 B) $\frac{6}{41}$ C) $\frac{12}{41}$ D) $\frac{35}{41}$ E) 1