Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS481 Actuarial Contingencies 2
Dr. Mohammad H. Omar
Major 1 Exam Term 231 FORM CODE 001

Name________________________________ ID#:_________ Serial #:___

Instructions.

1. Mobile calculators, I-pad, smart watches, or communicable devices are disallowed. Please do not bring your cell phones, smart watches, or other electronic devices in the exam. Any student caught with these devices switched on during the exam will be considered under the cheating rules of the University.
2. If you finish the test earlier and want to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
5. Do not spend too much time on any one question. If a question seems too di¢ cult, leave it and go on.
6. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
7. Only answers supported by work will be considered. Unsupported guesses will not be graded.
8. Submit the physical OMR page to your proctor
9. Use regular scientific calculators or financial calculators only.
10. Record your final answers to the multiple-choice questions on the OMR sheet. And write important steps to arrive at the solution of the last two problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Sunday Oct 1 6pm-7.20pm bldg 24 151

Q1. A four state Markov process, with states denoted as States 0, 1, 2, 3, begins in State 0 at time 0 for a person age x at time 0. The process can transition only from State 0 to one of States 1, 2, or 3. (This is the standard multiple decrement model presented in section 14.4.1) The forces of transition are $\mu_{x+t}^{01} = 0.30$, $\mu_{x+t}^{02} = 0.50$ and $\mu_{x+t}^{03} = 0.70$ all for $t \geq 0$.

Which of the following is **correct**?

A) This Markov process is non-homogeneous
B) ${}_rp_x^{00} = -1.50r$ is the solution of the Kolmogorov Forward Equation
C) $\Pr[X(1) = 2|X(0) = 0] = 0.25896$
D) $\Pr[X(1) = 3|X(0) = 0] = 0.74104$
E) $\Pr[X(1) = 1|X(0) = 0] = 0.30$

Q2. A 3-state Markov process begins in State 0 at time 0. The process is defined by the following transition probability matrices

$$P^{(0)} = P^{(1)} = \begin{bmatrix} 0.60 & 0.30 & 0.10 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and for } k = 2, 3, \ldots\ P^{(k)} = \begin{bmatrix} 0 & 0.30 & 0.70 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

I. A payment of 1 is made at discrete times $t = 0, 1, 2, \ldots$ provided the process is in either State 0 or State 1 at time t.
II. A payment of 4 is made at discrete times $t = 1, 2, 3, \ldots$, provided the process is in State 1 at time t.
III. Neither the payment in (I) nor (II) is invested (i.e., investment rate $i = 0$)

Which of the following is **correct**?

A) Expected value of the payments in (I) above is infinite
B) Expected value of the payments in (II) above is infinite
C) Expected value of the payments in (I) above is 8.452
D) Expected value of the payments in (II) above is 2.352
E) Total expected value of payments in (I) and (II) is 6.300

Q3. A 2-state homogeneous Markov survival model has a constant force of transition function $\mu_{x+t}^{10} = 0$ and $\mu_{x+t}^{01} = \mu_{x+t}^{0} = \mu = 0.4$.

Using the Kolmogorov's forward equation, which of the following is the **correct** solution for ${}_3p_x^{01}$?

A) 0.2
B) 0.4
C) 0.32968
D) 0.45071
E) 0.69881

Q4. Given the following extract from a double-decrement table,

$\boldsymbol{x}$	$\boldsymbol{l_x^{(\tau)}}$	$\boldsymbol{q_x^{(1)}}$	$\boldsymbol{q_x^{(2)}}$	$\boldsymbol{q'^{(1)}_x}$	$\boldsymbol{q'^{(2)}_x}$
40	2000	0.24	0.10	0.25	y
41	--	--	--	0.20	$2y$

find the value of $l_{42}^{(\tau)}$.

A) 1210.13
B) 802.56
C) 750.14
D) 660.02
E) 608.00

Q5. Find the value of $1000q'^{(1)}_x$, given $q^{(1)}_x = 0.025$, but $q^{(2)}_x = 0.05$ and each decrement is uniformly distributed over (x, x+1) in its **associated single decrement** table.

A) 25.6496 B) 50.34 C) 125.67 D) 136.53 E) 198.25

Q6. A triple decrement model allows for mortality (Decrement 1), disability (Decrement 2), and withdrawal (Decrement 3). Mortality and disability are uniformly distributed over each year of age in their associated **single decrement** tables, but withdrawals can occur **only at the end** of a year of age. Given the values $q'^{(1)}_x = 0.01$, $q'^{(2)}_x = 0.04$ and $q'^{(3)}_x = 0.10$, find the value of $q^{(3)}_x$.

A) 0.04960 B) 0.09504 C) 0.10000 D) 0.15012 E) 0.9504

Q7. Consider the general double-decrement model with $b^{(1)}_{t+1} = 1000$ and $b^{(2)}_{t+1} = 100$, $q^{(1)}_{x+t} = 0.01$ and $q^{(2)}_{x+t} = 0.025$, $G_{t+1} = 100$, $r_{t+1} = 0.05$, $e_{t+1} = 20$, and $i_{t+1} = 0.04$ for all t = 0, 1, 2,⋯ . Given ${}_2AS = 75$, find **asset share** value at time k = 3.

A) 100 B) 104 C) 130.5408 D) 148.7047 E) 150

Q8. A person is currently employed at age x at time 0, which we call State 0. Let State 1 denote *unemployment* and State 2 denote *deceased*. The transition forces between states are as follows:

(i) $\mu_{x+t}^{01} = 0.20 + 0.002t^2$

(ii) $\mu_{x+t}^{02} = \mu_{x+t}^{12} = 0.05$

(iii) $\mu_{x+t}^{10} = 0.80 - 0.04t$

(iv) $\mu^{20} = \mu^{20} = 0$

Using half-year time steps to approximate the solutions of the Kolmogorov differential equations, the estimates of ${}_{10}p_x^{00}$ and ${}_{10}p_x^{01}$ below are obtained for t = 0.5, 1.0, …, 10.0.

t	${}_tp_x^{00}$	${}_tp_x^{01}$		t	${}_tp_x^{00}$	${}_tp_x^{01}$
0.5	0.875	0.100		5.5	0.571	0.186
1.0	0.805	0.146		6.0	0.552	0.186
1.5	0.759	0.167		6.5	0.533	0.187
2.0	0.726	0.177		7.0	0.514	0.187
2.5	0.699	0.182		7.5	0.496	0.188
3.0	0.675	0.184		8.0	0.478	0.189
3.5	0.653	0.185		8.5	0.461	0.189
4.0	0.631	0.185		9.0	0.443	b
4.5	0.611	0.186		9.5	0.426	0.192
5.0	0.591	0.186		10.0	a	0.193

Specifically, which of the following **correctly describe** the missing values in the above table?

A) a = 0.409 and b = 0.191

B) a = 0.409 and b = 0.1905

C) a = 0.410 and b = 0.190

D) a = 0.415 and b = 0.191

E) a = 0.420 and b = 0.191

Q9. Consider the general double-decrement model with $b_{t+1}^{(1)} = 1000$ and $b_{t+1}^{(2)} = 100$, $s_{t+1}^{(1)} = 10$ and $s_{t+1}^{(2)} = 5$, $q_{x+t}^{(1)} = 0.01$ and $q_{x+t}^{(2)} = 0.025$, ${}_{t}V^{G} = 70$ and ${}_{t+1}V^{G} = 80$, $G_{t+1} = 100$, $r_{t+1} = 0.05$, $e_{t+1} = 20$, and $i_{t+1} = 0.05$. Let the actual earned interest rate in the $(t+1)^{st}$ year be denoted by $i_{t+1}^{*} = 0.06$, and the actual Cause 2 decrement probability be denoted by $q_{x+t}^{*(2)} = 0.02$. For N =1000 policies, if the gain from interest is calculated first and the gain from the Cause 2 decrement is calculated second, which of the following is the **additional gain** from the **Cause 2** decrement?

A) -120 B) 0 C) 125 D) 145 E) 180

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DEPARTMENT OF MATHEMATICS AND STATISTICS
AS 481 Actuarial Contingencies 2
Major 1 Exam Sunday Oct 1, 2023 (6:00 pm - 7:20 pm)
Your instructor's name: Dr. Mohammad H. Omar
Name: ______________________________ ID #:___________________ Serial#:______________

Part 1 (2 marks each). Please mark the correct answer to each of the questions by completely darkening the oval of your choice with a dark pen or pencil.

MULTIPLE CHOICE:	A	B	C	D	E
Q1	O	O	O	O	O
Q2	O	O	O	O	O
Q3	O	O	O	O	O
Q4	O	O	O	O	O
Q5	O	O	O	O	O

MULTIPLE CHOICE:	A	B	C	D	E
Q6	O	O	O	O	O
Q7	O	O	O	O	O
Q8	O	O	O	O	O
Q9	O	O	O	O	O

Score: $\frac{\quad}{30}$

Code: 001

Q10. (3+2=5 pts) Given the following probabilities of decrement, for each of two decrements at certain ages, complete the construction of the full multiple-decrement (double-decrement) table. Assume an initial group (or radix) of size 1000.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(\tau)}$	$p_x^{(\tau)}$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	0.010	0.10	0.110	0.890	1000.00	10.00	100.00
41	0.011	0.10			890.00	9.79	89.00
42	0.012	0.10	0.112	0.888	a	9.49	79.12
43	0.013	0.10	0.113	0.887	702.59	b	70.26
44	0.014	0.10	0.114	0.886	623.20	8.72	c
45	0.015	0.10	0.115	0.885	552.16	8.28	55.22

a) _a=__________ _b=__________ _c=__________

b) calculate ${}_{3|}q_{40}^{(2)}$

Q11. (6+1=7 marks) Consider a three-year endowment insurance, with gross annual premium and annual expenses paid at the beginning of each year and benefits paid at the end of the year. The contingent benefit is 1000 for death (Decrement 1) within the three-year period, or at time t = 3 if death has not previously occurred. A withdrawal benefit (Decrement 2) will be paid in the event of withdrawal from the plan at the end of any of the first two years. All parameter values for the insurance are shown in the following table:

Curtate Duration k	Percent of premium expense	Constant Contract Expense	Failure Benefit Amount	Withdrawal benefit amount	Endowment Benefit Amount	$q_{x+k}^{(1)}$	$q_{x+k}^{(2)}$	$q_{x+k}^{(\tau)}$	$p_{x+k}^{(\tau)}$
0	0.05	25	1000	50	0	0.02	0.30	0.32	0.68
1	0.05	25	1000	100	0	0.03	0.20	0.23	0.77
2	0.05	25	1000	0	1000	0.04	0	0.04	0.96

All cash flows are discounted at annual interest rate 6%.

(a) Find the gross annual premium using the equivalence principle.

(b) Find the gross premium reserve at time t =3, ${}_3V^G$.