Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS481 Actuarial Contingencies 2
Dr. Mohammad H. Omar
Major 2 Exam Term 231 FORM CODE 001

Name_____ ID#:_____ Serial #:___

Instructions.

1. Mobile calculators, I-pad, smart watches, or communicable devices are disallowed. Please do not bring your cell phones, smart watches, or other electronic devices in the exam. Any student caught with these devices switched on during the exam will be considered under the cheating rules of the University.
2. If you finish the test earlier and want to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
5. Do not spend too much time on any one question. If a question seems too di¢ cult, leave it and go on.
6. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
7. Only answers supported by work will be considered. Unsupported guesses will not be graded.
8. Submit the physical OMR page to your proctor
9. Use regular scientific calculators or financial calculators only.
10. Record your final answers to the multiple-choice questions on the OMR sheet. And write important steps to arrive at the solution of the last two problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Sunday Nov 5 6pm-7.20pm bldg 63 137

Q1. At all ages greater than 45, the force of failure for smokers is **double** that for non-smokers. Suppose the age-at-failure random variable for nonsmokers has a **uniform** distribution with $\omega = 70$. If (60) and (50) have independent lifetimes, where (60) is a *nonsmoker* and (50) is a *smoker*, find the value of $\mathring{e}_{60:50}$.

A) 0.32968
B) 3.54167
C) 5
D) 7.08324
E) 10

Q2. Let T_{80} and T_{85} be independent random variables with uniform distributions with $\omega = 100$. Find the probability that the **second failure** occurs **within six** years.

A) 1/12
B) 2/5
C) 3/25
D) 21/50
E) 7/10

Q3. A continuous two-life annuity has actuarial present value of 1380. The annuity pays at annual rate of

i) 100 while ***both*** (*x*) and (*y*) survive,

ii) 70 while (*x*) survives ***after*** the failure of (*y*), and

iii) 50 while (*y*) survives ***after*** the failure of (*x*).

Given that $\bar{a}_x = 14$ and $\bar{a}_y = 10$, find the value of $\bar{a}_{\overline{xy}}$.

A) 24 B) 23 C) 21.5 D) 19 E) 12

Q4. Oil drilling machine A follows a survival model defined by $\mu_x^A = \frac{1.90}{9-x}$, for $0 < x < 9$, and Oil drilling machine B follows a survival model defined by $\mu_x^B = \frac{1.60}{9-x}$, for $0 < x < 9$.

The two Oil drilling machines have **independent** lifetimes.

Find the probability that Oil drilling machine A **fails before** Oil drilling machine B.

A) 0.2385
B) 0.3741
C) 0.4545
D) 0.5429
E) 0.5454

Q5. The APV for a **last-survivor** whole life insurance on $(\overline{xy})$ with future lifetimes for (x) and (y) is calculated

1) with **constant hazard** rate 0.06 for each
2) with **unit benefit** paid at the instant of failure of the status,
3) under the force of interest, $\delta = 0.04$.
4) under the assumption of **independence**.

However, it is now discovered that

1) although the **total hazard rate** of 0.06 each is correct, the two lifetimes are **not independent**
2) each lifetime includes a **common shock** hazard factor with constant force of 0.02.

Calculate the **APV increase** that results from recognizing the common shock element.

A) 0.035714 B) 0.03921 C) 0.42424 D) 0.45 E) 0.485714

Q6. Annual yields for coupon-bearing bonds are reported in the table below.

Maturity (in years)	1	2	3	4
Annual Yield for Coupon-bearing Bonds	2.0%	4.0%	6.0%	8.0%

An actuary uses the above to calculate the corresponding zero-coupon yields of the same maturities. Assuming coupons are paid annually, which of the following is **correct**?

A) $z_1 = 0.04$ B) $z_2 = 0.05$ C) $z_1 + z_2 = 0.07014$ D) $z_3 = 0.06530$ E) $z_4 = 0.08447$

Q7. A company sells insurance in a country where only one-year bonds are available as investments to back its business. Our task is to compare the interest sensitivity of the following three products in this environment.

(i) Product A: A 5-year immediate annuity-certain, where payments are made regardless of survival status.

(ii) Product B: A 5-year immediate life annuity.

(iii) Product C: A single premium 5-year term insurance contract.

The applicable failure rates are

$q_x = 0.10,\ q_{x+1} = 0.15,\ q_{x+2} = 0.20,\ q_{x+3} = 0.25,$ and $q_{x+4} = 0.30.$

Assuming today's interest rate is 7%, the actuarial present value for each of the three products are calculated using each of the following two interest rate scenarios:

(1) *Increasing*: rates rise by 1% each year, but do not exceed 11% in any year.

(2) *Decreasing*: rates fall by 1% each year, but do not fall below 3% in any year.

Which of the products is **least interest sensitive** in this environment?

A) All products B) Product A C) Product B D) Product C E) None

Q8. A five-year pure endowment contract issued to a person age 60 is funded with level annual premiums and has a maturity benefit of $15 000. Premiums are payable at the beginning of each year, and the benefit is payable at the end of the fifth year. Table below shows mortality rates for a 60-year-old and forward rates that are currently available. Note that $f_{0,5} = z_5$.

n	0	1	2	3	4
$f_{n,5-n}$	4.0%	5.0%	6.0%	7.0%	8.0%
x	60	61	62	63	64
q_x	0.03	0.04	0.05	0.06	0.07

Use this information to calculate the net level annual premium for the pure endowment.

A) 1200 B) 1427.99 C) 1476.02 D) 1500 E) 2141.98

Q9. Annual coupon-bearing bond yield rates with various maturities are reported below.

Maturity (in years)	1	2	3	4
Annual Yield Rates for Coupon-bearing Bonds	2.0%	4.0%	6.0%	8.0%

Which of the following is correct about the ***forward rates*** that can be obtained from the above table?

A) Total number of possible forward rates = 6 and $f_{1,3} = 0.10685$
B) Total number of possible forward rates = 10 and $f_{1,2} = 0.06169$
C) Total number of possible forward rates = 4 and $f_{1,1} = 0.04041$
D) Total number of possible forward rates = 5 and $f_{2,2} = 0.13040$
E) Total number of possible forward rates = 5 and $f_{3,1} = 0.15578$

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS
AS 481 Actuarial Contingencies 2
Major 2 Exam Sunday Nov 5, 2023 (6:00 pm - 7:20 pm)
Your instructor's name: Dr. Mohammad H. Omar
Name: _____ ID #:_____ Serial#:_____

Part 1 (2 marks each). Please mark the correct answer to each of the questions by completely darkening the oval of your choice with a dark pen or pencil.

MULTIPLE CHOICE:	A	B	C	D	E
Q1	O	O	O	O	O
Q2	O	O	O	O	O
Q3	O	O	O	O	O
Q4	O	O	O	O	O
Q5	O	O	O	O	O

MULTIPLE CHOICE:	A	B	C	D	E
Q6	O	O	O	O	O
Q7	O	O	O	O	O
Q8	O	O	O	O	O
Q9	O	O	O	O	O

Score: $\frac{\quad}{25}$

Code: 001

Q10. (3pts) A contingent contract pays a benefit of amount b at the end of the year of the *second failure* of independent lives (x) and (y). The net annual premium is 100 paid at the beginning of each year while both (x) and (y) survive and 40 per year after the first failure. Find the value of b, given the following values:

$A_{xy} = 0.85 \quad \ddot{a}_x = 8 \quad \ddot{a}_y = 7 \quad d = 0.05$

Q11. (1+3=4 marks) Using the n-year forward one-year rates in the following table, answer the following questions:

n	0	1	2	3	4
$f_{n,1}$	4.1%	5	6	7	8

a) How many total spot rates can be determined? _____.

b) Find the spot rate z_3.