Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS481: Actuarial Contingencies 2
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Major Exam 1 Term 241
Sunday, September 29, 2024
6.30 PM - 8.30 PM

Name………………………………………………………. ID#:_________ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. ***Only answers supported by work will be considered. Unsupported guesses will not be graded***.
9. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***

The test is 120 minutes, GOOD LUCK, and you may begin now!

Question	Total Mark	Mark Obtained	Comments
1	1		
2	1		
3	1		
4	1		
5	2		
6	3		
7	3		
8	2		
9	2		
10	2		
Bonus	2		
Total	18		

Extra blank page

1. How can you distinguish between a transient state and a recurrent state in a Markov chain?

A) A transient state is always followed by an absorbing state

B) A recurrent state will eventually be visited again, while a transient state may not be revisited

C) A transient state is always visited before a recurrent state

D) A recurrent state cannot have any transitions to other states.

2. In which of the following cases can a Markov chain be considered to have a mix of recurrent and transient states?

A) If there are some states that the process can return to and others that might not be revisited

B) If every state is absorbing

C) If all states are either transient or absorbing

D) If all states are recurrent

3. When modeling a real-life process where transition probabilities change with seasons, such as animal migration patterns, which type of Markov process would be most appropriate?

A) Homogeneous process

B) Non-homogeneous process

C) Absorbing process

D) Recurrent process

4. Which of the following is an example of a competing risks scenario in actuarial science?

A) A pension plan where a member may retire, die, or leave for another job, but only one of these events will occur

B) A policyholder simultaneously holding multiple insurance policies

C) A life insurance policy that covers accidental death and natural death, paying out for both events

D) A medical plan that provides coverage for two different treatments.

5.

A certain animal species can be classified as thriving (State 0), endangered (State 1), or extinct (State 2). Movement among states is governed by a non-homogeneous Markov process defined by the following transition probability matrices:

$$\mathbf{P}^{(0)} = \begin{bmatrix} .85 & .15 & 0 \\ 0 & .70 & .30 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{P}^{(1)} = \begin{bmatrix} .90 & .10 & 0 \\ .10 & .70 & .20 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{P}^{(2)} = \begin{bmatrix} .95 & .05 & 0 \\ .20 & .70 & .10 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{P}^{(k)} = \begin{bmatrix} .95 & .05 & 0 \\ .50 & .50 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $k = 3, 4, 5, \cdots$.

If the species is thriving at time n, is it possible for it to be extinct at time $n+1$? Why?

A. Yes
B. No.

6.

A discrete-time Markov process has only two states, denoted as State 0 and State 1. A person is age x at time 0, and is located in State 0. We are given the age-specific one-year transition probability values

$$p_{x+k}^{00} = .70 + \frac{.10}{k+1}$$

and

$$p_{x+k}^{11} = .60 + \frac{.20}{k+1}.$$

Find the value of $Pr[X_3 = 0 \mid X_1 = 0]$.

A. 0.6333
B. 0.2500
C. 0.3333
D. 0.3145

7. Given the following probabilities of decrement, for each of two decrements at certain ages.

x	$q_x^{(1)}$	$q_x^{(2)}$
45	.011	.100
46	.012	.100
47	.013	.100
48	.014	.100
49	.015	.100
50	.016	.100

Assume an initial group (called the radix of the table) of size 1000, calculate

$_{3}p_{46}^{(\tau)}$

A. 0.3021
B. 0.885
C. 0.6979
D. 0.1321

8. From Question 6, calculate

$_{2|2}q_{45}^{(1)}$

A. 0.02006
B. 0.00980
C. 0.02354
D. 0.01324

9.

If $\mu_{x+t}^{(1)} = .10$ and $\mu_{x+t}^{(2)} = .20$ for all t, find

${}_{\infty}q_x^{(1)}$

A. 0.3333
B. 0.6667
C. 0.2000
D. 0.8000

10.

If $q_x^{(1)} = .20$ and $q_x^{(2)} = .10$, and both decrements are uniformly distributed over the interval $(x, x+1]$ in the multiple-decrement context, find $q_x'^{(2)}$.

A. 0.1121
B. 0.8879
C. 0.1326
D. 0.8674

Bonus Question.

Given the following extract from a double-decrement table, find the value of $l_{42}^{(\tau)}$.

x	$l_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x'^{(1)}$	$q_x'^{(2)}$
40	2000	.24	.10	.25	y
41	--	--	--	.20	$2y$

A. 1320
B. 608.56
C. 802.56
D. 2000