Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS481: Actuarial Contingencies 2
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Major Exam 2 Term 241
Sunday, November 3, 2024
6.00 PM - 8.00 PM

Name……. ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. ***Only answers supported by work will be considered. Unsupported guesses will not be graded***.
9. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***

The test is 120 minutes, GOOD LUCK, and you may begin now!

Question	Total Mark	Mark Obtained	Comments
1-6	0.5*6 = 3		
7-11	1*5 = 5		
12	3		
13	3		
14	3		
15	3		
Total	20		

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PART A

1. Which of the following statements best describes the role of interest rates in calculating the actuarial present value (APV) in a multi-decrement model?

A) Higher interest rates always reduce the APV of future benefits, regardless of the timing of the decrements.

B) Higher interest rates reduce the APV of future benefits because future payments are discounted more heavily.

C) Higher interest rates increase the APV of future benefits because the rate of return on investments increases.

D) Interest rates do not affect the APV in a multi-decrement model because the decrement probabilities are more important than interest assumptions.

2. A policyholder owns a life insurance policy with a reduced paid-up non-forfeiture option. How does the reduced paid-up insurance compare to extended term insurance in terms of actuarial present value?

A) The APV of reduced paid-up insurance is generally higher because it provides lifetime coverage with no further premiums.

B) The APV of reduced paid-up insurance is generally lower because the death benefit is reduced compared to extended term insurance.

C) The APV of reduced paid-up insurance is the same as extended term insurance because both are based on the policy's cash surrender value.

D) The APV of reduced paid-up insurance varies depending on the insurer's assumptions about mortality rates for different decrements.

3. In a multi-decrement model, the Actuarial Present Value (APV) of a benefit is most affected by which of the following factors?

A) The choice of decrement probabilities, interest rates, and the timing of benefit payments.

B) The future value of benefit payments without regard to decrement probabilities or interest rates.

C) The mortality improvement assumptions applied only to one of the decrements.

D) The assumption that decrements are independent, even if the decrement intensities are constant over time.

4. In the context of non-forfeiture options, which option provides the highest guaranteed death benefit but for a shorter period of coverage?

A) Cash surrender value

B) Reduced paid-up insurance

C) Extended term insurance

D) Continuation of the original policy at a lower premium

5. Which of the following best describes the Actuarial Present Value (APV) of a benefit in a multi-decrement model?

A) The expected value of future benefit payments, discounted to the present using a fixed interest rate.

B) The future value of benefit payments adjusted for different types of decrements, without discounting.

C) The expected present value of future payments where multiple causes of decrement are considered, and the payment is discounted at an appropriate rate.

D) The sum of all future benefit payments without taking into account decrements or interest.

6. Which of the following describes the extended term insurance non-forfeiture option?

A) The policyholder exchanges the cash surrender value for a reduced paid-up whole life insurance policy.

B) The policyholder exchanges the cash surrender value for a new policy that provides coverage for a fixed term with the same death benefit, but no further premiums are required.

C) The policyholder can withdraw part of the policy's face value as a lump sum.

D) The policyholder continues paying premiums at a reduced rate to maintain the original death benefit.

7.

If T_x and T_y are independent, calculate the value of ${}_2|q_{xy}$ given the following values:

$q_x = 0.08$ $q_{x+1} = 0.09$ $q_{x+2} = 0.11$
$q_y = 0.10$ $q_{y+1} = 0.15$ $q_{y+2} = 0.20$

A) 0.000002
B) 0.000108
C) 0.179328
D) 0.184452
E) 0.456006

8.

You are given the following:

$a_{xy} = 3,$
$a_{x|y} = 2$, and
$a_{y|x} = 4.$

Calculate $a_{\overline{xy}}$

A) 3
B) 5
C) 7
D) 9
E) 11

9.

A common shock model is assumed where both T_x^* and T_y^* have exponential distributions, and therefore constant hazard rates $\lambda_x = 0.3$ and $\lambda_y = 0.4$, respectively. Under this assumption, the common shock hazard rate $\lambda = 0.2$. When $\delta = 0.06$, calculate the APV of a last-survivor whole life insurance, with benefit paid at the moment of failure of the status.

A) 0.02841
B) 0.85367
C) 0.86448
D) 0.89286
E) 0.90909

10.

A continuous two-life annuity has actuarial present value of 1380. The annuity pays at annual rate of

i) 100 while ***both*** (x) and (y) survive,
ii) 70 while (x) survives ***after*** the failure of (y), and
iii) 50 while (y) survives ***after*** the failure of (x).

Given that $\bar{a}_x = 14$ and $\bar{a}_y = 10$, find the value of $\bar{a}_{\overline{xy}}$.

A) 24 B) 23 C) 21.5 D) 19 E) 12

11.

Oil drilling machine A follows a survival model defined by $\mu_x^A = \frac{1.90}{9-x}$, for $0 < x < 9$, and Oil drilling machine B follows a survival model defined by $\mu_x^B = \frac{1.60}{9-x}$, for $0 < x < 9$.

The two Oil drilling machines have **independent** lifetimes.

Find the probability that Oil drilling machine A **fails before** Oil drilling machine B.

A) 0.2385
B) 0.3741
C) 0.4545
D) 0.5429
E) 0.5454

PART B

12.

Consider the joint density function of T_x and T_y given by

$$f_{x,y}(t_x,t_y) = \frac{4}{(1+t_x+2t_y)^3},$$

for $t_x > 0$ and $t_y > 0$.

I) Show that T_x and T_y are not independent.

II) find an expression for ${}_n q_{\overline{xy}}$.

13.

For two persons alive at ages x and y at time 0, show that the Kolmogorov differential equation for ${}_t p_{xy}^{03}$ solves for

$$ {}_n p_{xy}^{03} = {}_n q_{\overline{xy}} + \lambda \cdot \overset{\circ}{e}_{xy}, $$

where λ is the constant common shock hazard.

14.

A company hires all new employees at age 25. An employee can leave the company via death while employed (Decrement 1), resignation prior to age 65 (Decrement 2), or retirement at age 65. The company provides the following benefits for its employees:

(a) Employees who retire at age 65 receive continuous retirement income at an annual rate of 500 for each year of employment with the company.

(b) Employees who die while employed receive a one-time death benefit of 200,000 at the precise time of death.

(c) Employees who resign prior to age 65, but survive on to age 65, receive continuous retirement income at an annual rate of 400 for each year of employment with the company (including partial years).

Write expressions involving continuous annuities and/or integrals for the APV at time of hire for each of the three benefits.

15.

A health insurer divides its insured population into the three risk classes of low, moderate, and high. The following table shows the percentage of insureds in each risk class in Year Z, and the reallocation of them among the three risk classes in Year $Z+1$.

Risk Class	Year Z Distribution	Year $Z+1$ Distribution		
		Low	Moderate	High
Low	69.5%	57.4%	11.7%	0.4%
Moderate	28.7%	9.9%	17.7%	1.1%
High	1.8%	0.2%	0.9%	0.7%
Totals	100%	67.5%	30.3%	2.2%

(a) From the data in the table, develop a matrix of transition probabilities for transition among risk classes.

(b) The average claim costs in Year Z for the three risk classes are 500 for low, 10,000 for moderate, and 50,000 for high. Find the average claim cost in Year Z for the entire insured population.

(c) Assuming a homogeneous discrete time Markov process, find the percentage distribution of the insured population in Year $Z+2$ and in Year $Z+3$.