# Dept of Mathematics and Statistics
# King Fahd University of Petroleum & Minerals
# AS481: Actuarial Contingencies 2
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# Final Exam Term 241
# Wednesday, December 18, 2024
# 8.00 AM - 11.00 AM

Name……. ID#:_____ Serial #:___

**Instructions.**

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra exam time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. ***Only answers supported by work will be considered. Unsupported guesses will not be graded***.
9. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. ***Write important steps to arrive at the solution of the exam problems.***

The test is 180 minutes, GOOD LUCK, and you may begin now!

| Question | Total Mark | Mark Obtained | Comments |
|---|---|---|---|
| 1-7 | 2*7 = 14 | | |
| 8-13 | 1*6 = 6 | | |
| 14 | 3 | | |
| 15 | 3 | | |
| 16 | 3 | | |
| 17 | 3 | | |
| 18 | 3 | | |
| Total | 35 | | |

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1. Consider a simple two-state model with transition probabilities given by

$$\mathbf{P}^{(0)} = \begin{vmatrix} .60 & .40 \\ .70 & .30 \end{vmatrix}$$

over the first interval of the process, and

$$\mathbf{P}^{(1)} = \begin{vmatrix} .50 & .50 \\ .80 & .20 \end{vmatrix}$$

for the second interval of the process. If the process is known to begin in State 1 at time 0, what is the probability that the process will be in State 0 at time 2?

A. 0.59 B. 0.41 C. 0.70 D. 0.30 E. 0.50

2. A four-state Markov process, with states denoted as States 0,1,2,3, begins in State 0 at time 0 for a person age x at time 0. The process can transition only from State 0 to one of States 1, 2, or 3. The forces of transition, all for $t \geq 0$, are

$$\mu_{x+t}^{01} = .30,\ \mu_{x+t}^{02} = .50, \text{ and } \mu_{x+t}^{03} = .70,$$

Calculate the value of $Pr\left[X(1) = 2 \mid X(0) = 0\right]$.

A. 0.23 B. 0.24 C. 0.25 D. 0.26

3.

An electronic device relies on two essential components, denoted $(x)$ and $(y)$, for its operation. The joint density function of the lifetimes of $(x)$ and $(y)$, measured in months, is

$$f_{x,y}(t_x,t_y) = \frac{t_x+t_y}{27},$$

for $0<t_x<3$ and $0<t_y<3$.

Find the correlation coefficient of $T_x$ and $T_y$.

A. 0.90 B. 0.09 C. -0.90 D. -0.09

4.

If $\mu_{x+t}^{(1)} = .10$ and $\mu_{x+t}^{(2)} = .20$ for all $t$, find ${}_tp_x^{(\tau)}$

A. Exp(-0.30t) B. 1 - Exp(-0.10t) C. (1- Exp(-0.30t))/3 D. 1/3

5.

A five-year bond, issued at time 0, faces the decrements of (1) Default, (2) Call (i.e., prepayment), and (3) Maturity.

The probabilities of decrement by curtate duration and cause are shown in the following table:

| Curtate Duration $k$ | Default $q_k^{(1)}$ | Call $q_k^{(2)}$ | Maturity $q_k^{(3)}$ |
|---|---|---|---|
| 0 | .02 | .03 | .00 |
| 1 | .02 | .04 | .00 |
| 2 | .02 | .05 | .00 |
| 3 | .02 | .06 | .00 |
| 4 | .02 | .00 | .98 |

A guarantor has contracted to pay 1000 at the end of the year of default if default occurs, and nothing otherwise. Find the APV of this contingent payment contract using an annual interest rate of 6%.

A. 75.33 B. 76.33 C. 77.33 D. 78.33

6.

A whole life insurance with immediate payment of claims pays 1000 at the instant of death by natural causes (NC) or 2000 at the instant of death by accidental causes (AC). Find the APV of the insurance using a force of interest of .05 and constant hazard rates (forces of mortality) $\mu_y^{NC} = .01$ for natural causes and $\mu_y^{AC} = .002$ for accidental causes, for all $y$.

A. 205.81 B. 215.81 C. 225.81 D. 235.81 E. 245.81

7. A five-year pure endowment contract issued to a person age 60 is funded with level annual premiums and has a maturity benefit of $10,000. Premiums are payable at the beginning of each year, and the benefit is payable at the end of the fifth year. The Table below shows mortality rates for a 60-year-old and forward rates that are currently available. Use this information to calculate the net level annual premium for the pure endowment. Note that $f_{0,5} = z_5$.

| $y$ | $f_{y,5-y}$ | $x$ | $q_x$ |
|---|---|---|---|
| 0 | 4.0% | 60 | .02 |
| 1 | 5.0 | 61 | .03 |
| 2 | 6.0 | 62 | .04 |
| 3 | 7.0 | 63 | .05 |
| 4 | 8.0 | 64 | .06 |

A. 1476.02 B. 1376.02 C. 1276.02 D. 1176.02 E. 1076.02

8. How does the corridor factor behave as the policyholder ages?
   a) It increases to account for higher mortality risk.
   b) It decreases as cash values grow larger.
   c) It remains constant throughout the policy term.
   d) It eliminates the need for mortality assumptions.

9. How do non-forfeiture options relate to surrender benefits?
   a) Non-forfeiture options ensure the policyholder receives value if they lapse or surrender the policy.
   b) Non-forfeiture options only apply to policies with no cash value.
   c) Non-forfeiture options eliminate all surrender charges on early termination.
   d) Non-forfeiture options apply to riders, not the base policy.

10. The internal rate of return (IRR) in profit analysis is defined as:
    a) The rate at which the net present value (NPV) of cash flows equals zero.
    b) The rate that maximizes the total profit vector.
    c) The discount rate applied to pre-contract expenses.
    d) The rate used to determine premium reserves.

11. What is the discounted payback period?
    a) The time taken for cumulative discounted cash flows to turn positive.
    b) The time when gross premiums equal total benefits and expenses.
    c) The time when reserves are fully funded for a policy.
    d) The time needed for the total cash flows to match initial pre-contract expenses.

12. Which of the following best describes the profit signature?
    a) A sequence of annual profits and losses over a policy's lifetime.
    b) The projected cumulative profit at maturity of the insurance policy.
    c) A graphical representation of the relationship between premium reserves and cash flows.
    d) The total profits distributed to policyholders over time.

13. In participating insurance, how are distributable profits shared with policyholders?
    a) Through cash, premium reductions, bonuses, or additional insurance purchases.
    b) By refunding pre-contract expenses to participating policyholders.
    c) By adjusting mortality rates in the premium calculation.
    d) By directly reducing future reserves required under the policy.

**PART B**

14.

Find the value of $1000q_x'^{(1)}$, given $q_x^{(1)} = .02$, $q_x^{(2)} = .06$, and each decrement is uniformly distributed over $(x, x+1)$ in its associated single-decrement table.

15.

A block of 1000 fully discrete insurances, issued at age 70, are in force at age 79. The gross premium is $G = 16$, the ninth gross premium reserve is 115.00, the tenth gross premium reserve is 128.83, the tenth year death benefit is 1000, the tenth year withdrawal benefit is 110, and the assumed interest rate is .06. Expenses are 3 per policy, incurred at the beginning of the year, and there are no claim settlement expenses. Withdrawals can occur only at the end of the contract year. The assumed decrement rates are $q'^{(d)}_{79} = .01$ and $q'^{(w)}_{79} = .10$. During the tenth contract year there are 15 deaths and 100 withdrawals. Calculate, in order, (a) the gain from mortality and (b) the gain from withdrawal on this block of policies.

16.

A company sells insurance in a country where only one-year bonds are available as investments to back its business. Our task is to compare the interest sensitivity of the following three products in this environment.

(i) A 5-year immediate annuity-certain, where payments are made regardless of survival status.
(ii) A 5-year immediate life annuity.
(iii) A single premium 5-year term insurance contract.

The applicable failure rates are $q_x = .10, q_{x+1} = .15, q_{x+2} = .20, q_{x+3} = .25,$ and $q_{x+4} = .30.$

(a) Assuming today's interest rate is 7%, calculate the actuarial present value for each of the three products using each of the following two interest rate scenarios:

(1) Increasing: rates rise by 1% each year, but do not exceed 11% in any year.
(2) Decreasing: rates fall by 1% each year, but do not fall below 3% in any year.

(b) Which of the products is least interest sensitive in this environment? Explain.

17.

A policy with a gross premium of 1000 and a survival model given by

$$q_{x+t} = .05 + .01t,$$

for $t = 0,1,2,3,$ produces the profit signature

$$\mathbf{\Pi} = (-300, 100, 90, 80, 70).$$

The hurdle rate is $r = .10$.

(a) Calculate the net present value of the profits at issue.

(b) Show that the internal rate of return is 5.54%.

(c) Find the discounted payback period.

(d) Calculate the profit margin.

18. A discrete three-year term policy of 1,000,000 face amount has annual gross premium of 17,500.00, pre-contract expenses of 3,000.00, per policy expenses after issue of 100.00, reserves of ${}_1V^G = {}_2V^G = 800.00$, and no settlement expenses. Assume mortality rates of $q_x = .015$, $q_{x+1} = .017$, and $q_{x+2} = .019$, a cash flow accumulation rate of $i = .05$, and a risk discount rate of $r = .10$.
(a) Determine the profit vector.
(b) Determine the profit signature.
(c) Calculate the net present value.
(d) Determine the internal rate of return.
(e) What is the policy's breakeven term?
(f) What is the policy's profit margin?