(1)

$E[X_i] = \alpha\theta = 250$

$Var[X_i] = \alpha\theta^2 = \sigma_i^2 \quad \sigma_i = 250$

$E[X] = 16 \times 250 = 4000$ (1)

$\sigma = \sqrt{16 \times 250^2} = 4 \times 250 = 1000$ (1)

$P(X > 6000) = P\left(\frac{X - 4000}{1000} > \frac{6000-4000}{1000}\right)$ (6)

$= P(Z > 2)$ (2)

$= 1 - N(2) = 0.02275.$

(2) $TVaR_{0.95}(X) = VaR_{0.95} + e_{VaR_{0.95}}(X)$. 6

For an exponential, mean excess loss $e(x) = \theta$ regardless of x. So, the difference is $e_{VaR(X)} = \theta = 1000$. 2

2

~~Problem set~~

(3) Mean excess loss with deductible of d is

$$\frac{E[X] - E[X \wedge d]}{1 - F(d)}.$$

We are given that $F(1000) = 1$, so that all losses are < 1000.

2

$E[X] = E[X \wedge 1000] = 331$. Then $\frac{E[X] - E[X \wedge d]}{1 - Fd}$

5

$$= \frac{331 - 91}{1 - 0.2} = 300$$

2

4. $F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha}$ $E[X] = \frac{\theta}{\alpha-1}$

and $E[X \wedge x] = \frac{\theta}{\alpha-1}\left(1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right)$

Expected cost per loss is

$$E[X] - E[X \wedge d] = \left(\frac{\theta}{\alpha-1}\right)\left(\frac{\theta}{d+\theta}\right)^{\alpha-1} = 1105$$

Expected cost per payment is

$$\frac{E[X] - E[X \wedge d]}{1 - F(d)} = \frac{d+\theta}{\alpha-1} = 1778$$

Loss elimination ratio 1

$$\frac{E[X \wedge d]}{E[X]} = 1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1} = 0.2633$$

$$E[X \wedge d] = 0.2633\,E[X] \quad E[X](1 - 0.2633)$$

$$= 1105$$

$$\Rightarrow E[X] = 1500$$

1

$$\frac{\theta}{d+\theta} = \frac{\theta}{\alpha-1} \Big/ \frac{d+\theta}{\alpha-1} = \frac{1500}{1778} = 0.8436$$

$$1105 = \frac{\theta}{\alpha-1} \times \left(\frac{\theta}{d+\theta}\right)^{\alpha-1} = 1500 \; 0.8436^{\alpha-1}$$

$$\Rightarrow \alpha = 2.8 \quad 2$$

$$1778 = \frac{d+\theta}{\alpha-1} = \frac{d+\theta}{1.8} \qquad d+\theta = 3200$$

$$\frac{\theta}{d+\theta} = 0.8436$$

$$\Rightarrow \begin{array}{l} \theta = 2700 \quad 2 \\ d = 500 \quad 2 \end{array}$$

With deductible $2d = 1000$

$$ECP = \frac{E[X] - E[X \wedge 2d]}{1 - F(2d)} = \frac{2d+\theta}{\alpha-1} = 2056$$

2

5. $a(x) = 1 \quad A(x) = x$ 2

$$M_Q(z) = E[e^{\theta z}] = \int_1^{11} e^{\theta z} \frac{1}{10} \, d\theta$$

$$= \frac{e^{\theta z}}{10z}\Big|_1^{11} = \frac{e^{11z}}{10z} - \frac{e^{z}}{10z}$$ 3

$$S_x(x) = M_x(-x) = \frac{e^{-x}}{10x} - \frac{e^{-11x}}{10x}$$ 3 $S_x(0.5) = 0.12$ 2

6. Let p be the probability, and a the multiple of the exponential distribution. Then $F(200) = p$, and that plus $Pr(X > 200)$ must equal 1, so

$$p + a e^{-200/400} = 1 \quad 4$$

Since the density of the uniform distribution is $\frac{p}{200}$ and equals the exponential at 200

$$\frac{p}{200} = \frac{a e^{-200/400}}{400}$$

$$a = 2p e^{1/2} \quad 4$$

$$\Rightarrow p + 2p = 1 \qquad p = 1/3 \quad 2$$

7 $1 - Pr(N=0) = 1 - \frac{1}{(1+\beta)^4}$. Integrate over mixing distribution which has density $1/2$. 2

$$1 - Pr(N=0) = \frac{1}{2}\int_0^2 1 - \frac{1}{(1+\beta)^4}\,d\beta$$ 6

$= 0.839506$. 2

8 The expected present value of the claim is $0.5(10/1.04^3)$ and the expected present value of the legal fees is $5/1.04^3$ for a total of $10/1.04^3 = 8.89$.

Let I be indicator variable for whether the payment is required.

$$Var(X) = Var(E[X|I]) + E[Var(X|I)]$$

2

The expected value of the claim is 0 with probability 50% and $\frac{10}{1.04^3}$ with probability 50%.

Bernoulli shortcut says..

2

$$Var(E[X|I]) = 0.5 \cdot 0.5 \left(\frac{10}{1.04^3}\right)^2 = 19.7579$$

The variance of the claim is 0 with probability 50% and $\left(\frac{20}{1.04^3}\right)^2$ with probability 50%.

2

$$E[Var(X|I)] = 0.5 \left(\frac{20}{1.04^3}\right)^2 = 158.0629$$

$$Var(X) = 19.7579 + 158.0629 = 177.8208$$ 2

$$8.89 + 0.02(177.8208) = 12.4464$$

2

9. $E[X^{-1}] = \dfrac{\theta^{-1}\Gamma(\alpha-1)}{\Gamma(\alpha)} = \dfrac{1}{\theta(\alpha-1)}$ 4

$E[X^{-2}] = \dfrac{1}{\theta^2(\alpha-1)(\alpha-2)}$ 4

2

$Var(Y) = E[X^{-2}] - E[X^{-1}]^2 = 0.00888889$

(10). $\Pr(N=1) = e^{-\lambda}\lambda$

$\alpha\theta = 1 \quad \alpha\theta^2 = 2 \quad \alpha = \frac{1}{2} \quad \theta = 2$ (2)

$$\Pr(N=1) = \int_0^\infty e^{-\lambda}\lambda \frac{(\lambda/\theta)^\alpha e^{-\lambda/\theta}}{\lambda\,\Gamma(\alpha)}\,d\lambda$$

$$= \int_0^\infty \frac{\lambda^{1/2} e^{-3\lambda/2}}{2^{1/2}\,\Gamma(\alpha)}\,d\lambda \qquad u = \frac{3\lambda}{2}$$

$$= \frac{1}{\sqrt{2}}\int_0^\infty \frac{\left(\frac{2}{3}u\right)^{1/2} e^{-u}}{\Gamma(\alpha)}\,\frac{2}{3}\,du$$

$$= \frac{2}{3\sqrt{3}\,\Gamma(\alpha)}\int_0^\infty u^{1/2} e^{-u}\,du$$

$$= \frac{2}{3\sqrt{3}\,\Gamma(\frac{1}{2})}\Gamma\left(\frac{3}{2}\right) = \frac{2 \cdot \frac{1}{2}\,\Gamma(\frac{1}{2})}{3\sqrt{3}\,\Gamma(\frac{1}{2})}$$ (6)

$$= \frac{1}{3\sqrt{3}}$$ (2)