AS484 Term 222

Midterm

Duration: 150 minutes

Name:	ID:

1. Only SOA approved calculators are allowed.
2. This exam has 15 questions.
3. Show all of your work. Points will be deducted for results without work.
4. Write clearly. Justify every step in the calculations. You may lose points just writing the equation or results.
5. No credits will be given to wrong steps.

Questions

1. [5 points] The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution: $F(x) = 1 - 0.8\, e^{-0.02x} - 0.2\, e^{-0.001x}$, $x > 0$. Calculate expected payment for one claim.
2. [5 points] The random variable X has pdf $f(x) = 0.02x$, if $0 < x < 10$.
   Calculate mean and variance of $(X-5)_+$
3. [5 points] A random loss is uniformly distributed on (0,100). The premium of an ordinary insurance with deductible 10 is calculated by expected claim plus 15. The premium of a complete insurance is calculated by expected claim times k. If two premiums are equal. Determine k.
4. [5 points] Suppose X has uniform distribution on [0,1000]. Calculate x satisfying $e(600) = 2\, e(x)$,where $e(x)$ is the mean excess loss function.
5. [5 points] Losses X follow Weibull distribution with $\tau = 2$, $\theta = 1000$. Calculate $VaR_{0.90}(X)$.
6. [5 points] X follows a Beta distribution with $\theta = 100$, $a = 2$ and $b = 1$. Calculate $TVaR_{0.90}(X)$.
7. [10 points] X follows normal distribution. $TVaR_{0.5}(X) = 67.55$, $TVaR_{0.8}(X) = 80.79$. Find $TVaR_{0.9}(X)$.
8. [5 points] The distribution of X is a two point mixture:
   a. With probability 0.6, X has two parameter Pareto distribution $\alpha = 2$, $\theta = 100$.
   b. With probability 0.4, X has two parameter Pareto distribution $\alpha = 4$, $\theta = 3000$.

   Calculate S(200).
9. [10 points] An insurance company sells hospitalization reimbursement insurance. You are given:
   a. Benefit payment for a standard hospital stay follows a lognormal distribution with $\mu = 7$, $\sigma = 2$.
   b. Benefit payment for a hospital stay due to an accident is twice as much as the standard benefit.
   c. 25% of all hospitalizations are for accidental causes.

   Calculate the probability that benefit payment exceeds 10,000.
10. [5 points] Examine the tail of the Gamma distribution by looking at the hazard rate function. ($\alpha < 1$).

11. [10 points] You are given:
    a. A portfolio consists of 75 liability risks and 25 property risks.
    b. The risks have identical claim count distribution.
    c. Loss sizes for liability risks follow a Pareto distribution with parameters $\theta = 300$, $\alpha=4$.
    d. Loss sizes for property risks follow a Pareto distribution with parameters $\theta = 1000$, $\alpha=3$.

    Determine the variance of the claim size distribution for this portfolio for a single claim.
12. [5 points] A loss distribution is a two component spliced model using Weibull distribution with $\theta = 1500$ and $\tau = 1$ for losses up to 4000, and a Pareto distribution with $\theta = 12000$ and $\alpha = 2$ for losses 4000 and greater. The probability that losses are less than 4000 is 0.6. Calculate the probability that losses are less than 25000.
13. [5 points] You are given:
    a. In 1998, claim sizes follow a Pareto distribution with parameters θ (unknown), $\alpha=2$.
    b. Inflation of 6% affects all claims uniformly from 1998 and 1999.
    c. r is the ratio of the proportion of claims that exceed d in 1999 to the proportion of claims that exceed d in 1998.

    Determine the limit of r as d goes to infinity.
14. [10 points] Let N have Poisson distribution with mean Λ. Let Λ have a gamma distribution with mean 1 and variance 2. Determine the unconditional probability that $N = 1$.
15. [10 points] Given a value of $\Theta=\theta$, the random variable X has an exponential distribution with hazard rate function $h(x) = \theta$, a constant. The random variable Θ has uniform distribution on the interval (1,11). Determine $S_X(0.5)$ for the unconditional distribution.