Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS484: Actuarial Risk Theory and Credibility
Dr. Mohammad H. Omar
MidTerm Exam Term 231   FORM 001

Name________________________________ ID#:_________ Serial #:___

Instructions.

1. Mobile calculators, I-pad, smart watches, or communicable devices are disallowed. Please do not bring your cell phones, smart watches, or other electronic devices in the exam. Any student caught with these devices switched on during the exam will be considered under the cheating rules of the University.
2. If you finish the test earlier and want to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
5. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
6. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
7. Only answers supported by work will be considered. Unsupported guesses will not be graded.
8. Submit the physical OMR page to your proctor
9. Use regular scientific calculators or financial calculators only.
10. Record your final answers to the multiple-choice questions on the OMR sheet. And write important steps to arrive at the solution of the last two problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

1. The severities of individual claims have the gamma distribution with parameters $\alpha = 5$ and $\theta = 1000$. Use the central limit theorem to approximate the probability that the sum of 100 independent claims exceeds 525 000.
   (A) 1.11800
   (B) 0.25328
   (C) 0.13178
   (D) 0.10275
   (E) 0.05000

2. Consider three loss distributions for an insurance company. Losses for the next year are estimated to be 100 million with variance of 50 000 million^2 (or standard deviation 223.607 million). You are interested in finding high quantiles of the distribution of losses. Using the normal, Pareto and Weilbull distributions, obtain the *VaR* at the 90%, 99.0%, and 99.9% security levels.
   Which of the following is **true**?
   (A) At 99.9%, $VaR_{normal} > VaR_{Pareto} > VaR_{Weibull}$
   (B) At 99%, $VaR_{normal} = 396.56$
   (C) At 99.0%, $VaR_{normal} > VaR_{Pareto} > VaR_{Weibull}$
   (D) At 90%, $VaR_{normal} > VaR_{Weibull} > VaR_{Pareto}$
   (E) At 99.0%, $VaR_{Weibull} = 3258.85$

3. Claims have a Pareto distribution with $\alpha = 2$ and unknown θ. Claims the following year experience 5% uniform inflation.
   Let *r* be the **ratio** of the **proportion** of claims that will **exceed *d* next** year to the **proportion** of claims that **exceed** *d* **this year**.

   Determine the limit of *r* as *d* goes to infinity.
   (A) 1.1025
   (B) 1.1236
   (C) 1.5601
   (D) 1.9501
   (E) 2.2050

4. Seventy – five percent of claims have a normal distribution with a mean of 3500 and a variance of 1 000 000. The remaining 25% have a normal distribution with mean of 4500 and a variance of 1 000 000.
   Determine the **probability** that a randomly selected claim **exceeds** 5000.
   (A) 0.0568
   (B) 0.06681
   (C) 0.12724
   (D) 0.25951
   (E) 0.30854

5. For models involving general liability insurance, actuaries at the *Insurance Services Office* once considered a mixture of two Pareto distributions. This model contains a total of five parameters. They decided that five parameters were not necessary and selected the more parsimonious mixture distribution with following distribution function.

$$F(x) = 1 - a\left(\frac{\theta_1}{\theta_1 + x}\right)^{\alpha} - (1-a)\left(\frac{\theta_2}{\theta_2 + x}\right)^{\alpha+2}$$

where $\theta_1 = 2000, \quad \theta_2 = 4000, \quad \alpha = 3, \quad a = 0.25$.

Which of the following provides the **correct description** of some moments of this two-point mixture distribution?

A) $E[X]$ = 1 500

B) $E[X]$ = 2 000

C) $E[X]$ = 3 000 000

D) $E[X^2]$ = 2 000 000

E) $E[X^2]$ = 3 000 000

6. Let X have a Pareto distribution. Which of the following provides the **correct** cumulative distribution function (cdf) of specific transformation (the inverse, transformed, or inverse transformed) of X?

(A) *Transformed*: $F_Y(y) = \left(\frac{\theta}{\theta + y^{\tau}}\right)^{\alpha}$

(B) *Inverse transformed*: $F_Y(y) = 1 - \left[\frac{y^{\tau}}{y^{\tau} + (\theta^{-1/\tau})^{\tau}}\right]^{\alpha}$

(C) *Inverse Transformed*: $F_Y(y) = 1 - \left(\frac{\theta}{\theta + y^{\tau}}\right)^{\alpha}$

(D) *Inverse*: $F_Y(y) = \left(\frac{\theta}{\theta + y^{\tau}}\right)^{\alpha}$

(E) *Inverse*: $F_Y(y) = \left(\frac{y}{y + \theta^{-1}}\right)^{\alpha}$

7. Losses in 1993 follow the density function $f(x) = 3x^{-4}$, $x \geq 1$, where x is the loss *in millions of dollars*. Inflation of 8% impacts all claims uniformly from 1993 to 1994. Using the cdf of losses for 1994, determine the probability that a 1994 loss exceeds 2.2 million dollars.

   (A) 0.1250 (B) 0.1183 (C) 0.0939 (D) 0.0800 (E) 0.0671

8. A discrete distribution has a probability generating function of the form $P_N(z) = e^{\lambda(z-1)}$. Which of the following is **correct**?

   (A) $M_N(z) = e^{\lambda(z-1)}$
   (B) $P'(1) = 0$
   (C) $P''(1) = \lambda^2$
   (D) $P(N = 1) = P''(0)/2$
   (E) $P''(1) - P'(1) = \lambda$

9. Consider an ETNB distribution with r = 0.5 and = $\beta = 1$. With $p_0^M = 0.6$ set arbitrarily, which of the following is correct probability for its truncated version or for its modified version?
(A) $p_1^M = 0.341421$
(B) $p_1^T = 0.583335$
(C) $p_2^T = 0.853553$
(D) $p_3^M = 0.042678$
(E) $p_3^T = 0.106694$

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS DEPT OF MATHEMATICS

AS484: Actuarial Risk Theory and Credibility - Semester 231,

MidTerm Exam Thursday Oct 12, 2023 (6:00 pm - 7:30 pm)

Your instructor's name: Dr. Mohammad H. Omar

Name: ________________________________ ID #:____________________ Serial#:_______________

Part 1 (2 marks each). Please mark the correct answer to each of the questions by completely darkening the oval of your choice with a dark pen or pencil.

MULT CHOICE:	A	B	C	D	E
Q1	O	O	O	O	O
Q2	O	O	O	O	O
Q3	O	O	O	O	O
Q4	O	O	O	O	O
Q5	O	O	O	O	O

MULT CHOICE:	A	B	C	D	E
Q6	O	O	O	O	O
Q7	O	O	O	O	O
Q8	O	O	O	O	O
Q9	O	O	O	O	O

Code: 001

Score: $\frac{\quad}{28}$

Part 2. Please show important steps to the next two questions below.

Q10. **(5 marks)** Consider a negative binomial random variable with parameters $\beta = 0.6$ and $r = 2.6$. The zero-modified probabilities has the probability at zero of $P_0^M = 0.6$. Complete the missing probabilities below where

a) $p_k = P(N = k)$ from the negative binomial distribution

b) $p_k^T = P(N^T = k)$ from the zero-truncated negative binomial distribution

c) $p_k^M = P(N^M = k)$ from the zero-modified negative binomial distribution

k	p_k	p_k^T	p_k^M
0	0.294638	A	0.6
1	0.287272	C	0.162907
2	B	0.274906	0.109962
3	0.111497	0.158071	D

A =	B =	C =	D =

Q11. **(1+2+2=5 marks)** A continuous distribution is a member of the linear exponential distribution family with the following specifications:

I. $p(x) = x^{3-1}/\Gamma(3) = x^2/\Gamma(3)$

II. Normalizing constant $q(\theta) = \theta^3 = 2^3$

III. Canonical parameter $r(\theta) = -\frac{1}{\theta} = -\frac{1}{2}$

a) Provide the **probability density function** for this continuous random variable

.

b) Compute E[X]

.

c) Compute Var(X)

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