King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

AS 491
Exam I– Term 222
Sunday, March 5, 2023

Allowed Time: 90 minutes

Instructor: **Dr. Boubaker Smii**

Name: ______________________________

ID #: ____________________

Section #: ___________ Serial Number: ___________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification !
3. **Calculators and Mobiles are not allowed.**

Question #	**Grade**	**Maximum Points**
1		**9**
2		**15**
3		**10**
4		**15**
5		**8**
6		**9**
7		**14**
Total:		**80**

Exercise 1:(9)
Let X and Y be random variables with joint density:

$$f(x,y) = \frac{3}{2}, \;\; 0 \leq x \leq 1, \;\; x^2 \leq y \leq 1. \tag{a}$$

(a) Find the probability density $f_Y(y)$ of Y.

(b) Find $\mathbf{E}(X \mid Y = \frac{1}{2})$, the conditional expectation of X given that $Y = \frac{1}{2}$.

Exercise 2: (15)

Claim amounts on a home insurance policy have density function

$$f(x) = \begin{cases} \dfrac{3}{x^4} & if \quad x > 1, \\ \\ 0 & otherwise. \end{cases} \tag{b}$$

Suppose two such claims are made, and assume the claim amounts are independent.

(a) Find the probability density function of the **larger** of the two claims.

(b) Find the probability that the amount of the second claim is at least **twice** that of the first claim.

Exercise 3:(10)

A- Let X be the number of times that a fair coin, flipped 40 times, lands heads.
Find the probability that $X = 20$.
(Express the result as $\Phi(a)$, where Φ is the standard Normal distribution function and a a real number).

B- Let X and Y be two independent random variables.
Assume that X and Y follows Poisson distributions with parameters λ_1 and λ_2 respectively. Use moment generating functions to show that $X + Y$ has a Poisson distribution and find the corresponding paramter.

Exercise 4:(15)
Consider the Markov chain $\{X_n, n \geq 0\}$ with three states, $S = 1, 2, 3$, that has the following transition matrix

$$P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.6 & 0.1 \end{pmatrix}$$

1. Draw the state transition diagram for this chain.

2. Is there any absorbing state ? Explain.

3. Find P^2 and $P(X_5 = 3 \mid X_3 = 1,\ X_2 = 1)$.

4. Determine $P(X_8 = 3, X_7 = 1, X_5 = 2 \mid X_3 = 2, X_2 = 1)$.

Exercise 5: (8)
A company offers collision insurance and liability insurance. The claim amount under collision insurance is normally distributed with mean $10,000$ and standard deviation $2,000$; the claim amount under liability insurance is normally distributed with mean $9,000$ and standard deviation $2,000$. Assuming independence, find the probability that the claim amount under liability insurance exceeds the claim amount under collision insurance.

(Express the result as $\Phi(x)$, where Φ is the standard Normal distribution function and x a real number. **Note:** $\sqrt{8000000} = 2828$.)

Exercise 6:(9)

An insurance company pays out claims at times of a Poisson process with rate 4 per week. Suppose that the mean payment is 400 and the standard deviation is 200. Find the mean and standard deviation of the total payments for 6 weeks.

Exercise 7:(14)
The probability density function (pdf) of the duration of the (independent) interarrival times between successive cars on the Dammam-Riyadh Highway is given by

$$f_T(t) = \begin{cases} \frac{1}{12}\, e^{-\frac{t}{12}}, & t \geq 0, \\ 0, & t < 0, \end{cases} \tag{c}$$

where these durations are measured in seconds.
A- An old fennec fox requires 12 seconds to cross the highway, and he starts out immediately after a car goes by. What is the probability that he will survive?

B- Another old fennec, slower but tougher, requires 24 seconds to cross the road, but it takes two cars to kill him. If he starts out at an arbitrary time, determine the probability that he survives.

C- If both these fennec foxes start out at the same time, immediately after a car goes by, what is the probability that exactly one of them survives?
(**Hint:** Consider a random variables N_1 = the number of cars in the first 12 seconds and N_2 = the number of cars in the second 12 seconds.)