



College of Computing and Mathematics  
Department of Mathematics

Syllabus of the Comprehensive Exam  
MATH 531: Real Analysis

**Topics:**

- 1. Lebesgue measure:** The Lebesgue outer measure; The sigma-algebra of Lebesgue measurable sets; Outer and inner approximations of Lebesgue measurable sets; Properties of the Lebesgue measure; The Borel-Cantelli lemma.
- 2. Lebesgue measurable functions:** Sums, products, and compositions; Simple approximation; Littlewood's three principles; Egoroff's theorem; Lusin's theorem.
- 3. Integration Theory:** The Riemann integral; The Lebesgue integral (any type of functions); Properties of the Lebesgue integral; Convergence theorems (Fatou's lemma, the monotone convergence theorem, the Lebesgue's dominated convergence theorem); Uniform integrability and the Vitali convergence theorem; Convergence in measure; Characterizations of Riemann and Lebesgue integrability.
- 4. Differentiation and integration:** Continuity and differentiability of monotone functions; Functions of bounded variations; Absolutely continuous functions; Integration derivative; Convex functions.
- 5. The Classical  $L_p$  Spaces:** Inequalities in  $L_p$  (Hölder's and Minkowski's inequalities); Completeness of  $L_p$  and the Riesz-Fisher theorem; Convergence in  $L_p$ ; The Riesz representation for the dual of  $\mathbb{R}^n$ ; Weak sequential convergence in  $\mathbb{R}^n$ .
- 6. General measure spaces:** Measures and measurable sets; Signed measures (the Hahn and Jordan decompositions); Measurable functions; Integration of general measurable functions; the Carathéodory measure induced by an outer measure; the construction of an outer measure

## References

Real Analysis by H.L. Royden and P.M. Fitzpatrick, Fourth Edition.

## The material

Chapter 2: Lebesgue Measure (sections 1-5)

Chapter 3: Lebesgue Measurable Functions (sections 1-3)

Chapter 4: Lebesgue Integration (sections 1-6)

Chapter 5: Lebesgue Integration, Further Topics (sections 1-3)

Chapter 6: Differentiation and Integration (sections 1-6)

Chapter 7: The  $L_p$ -spaces: Completeness and Approximation (sections 1-3)

Chapter 8: The  $L_p$ -spaces: Duality and Weak Convergence (sections 1-2)

Chapter 17: General Measure Spaces, Their Properties and Construction (sections 1-4)

Chapter 18: Integration Over General Measure Spaces (sections 1-2)