



College of Computing and Mathematics
Department of Mathematics

Syllabus of the Comprehensive Exam
MATH 531: Real Analysis

Topics:

1. **Lebesgue measure:** The Lebesgue outer measure; The sigma-algebra of Lebesgue measurable sets; Outer and inner approximations of Lebesgue measurable sets; Properties of the Lebesgue measure; The Borel-Cantelli lemma.
2. **Lebesgue measurable functions:** Sums, products, and compositions; Simple approximation; Littlewood's three principles; Egoroff's theorem; Lusin's theorem.
3. **Integration Theory:** The Riemann integral; The Lebesgue integral (any type of functions); Properties of the Lebesgue integral; Convergence theorems (Fatou's lemma, the monotone convergence theorem, the Lebesgue's dominated convergence theorem); Uniform integrability and the Vitali convergence theorem; Convergence in measure; Characterizations of Riemann and Lebesgue integrability.
4. **Differentiation and integration:** Continuity and differentiability of monotone functions; Functions of bounded variations; Absolutely continuous functions; Integration derivative; Convex functions.
5. **The Classical L_p Spaces:** Inequalities in L_p (Hölder's and Minkowski's inequalities); Completeness of L_p and the Riesz-Fisher theorem; Convergence in L_p ; The Riesz representation for the dual of \mathbb{R}^n ; Weak sequential convergence in \mathbb{R}^n .
6. **General measure spaces:** Measures and measurable sets; Signed measures (the Hahn and Jordan decompositions); Measurable functions; Integration of general measurable functions; the Carathéodory measure induced by an outer measure; the construction of an outer measure

References

Real Analysis by H.L. Royden and P.M. Fitzpatrick, Fourth Edition.

The material

Chapter 2: Lebesgue Measure (sections 1-5)

Chapter 3: Lebesgue Measurable Functions (sections 1-3)

Chapter 4: Lebesgue Integration (sections 1-6)

Chapter 5: Lebesgue Integration, Further Topics (sections 1-3)

Chapter 6: Differentiation and Integration (sections 1-6)

Chapter 7: The L_p -spaces: Completeness and Approximation (sections 1-3)

Chapter 8: The L_p -spaces: Duality and Weak Convergence (sections 1-2)

Chapter 17: General Measure Spaces, Their Properties and Construction (sections 1-4)

Chapter 18: Integration Over General Measure Spaces (sections 1-2)