

Syllabus

Math 533 Complex Analysis- Comprehensive Exam

1. Topics

- 1) **Complex numbers:** the algebra of complex numbers, geometric representation of complex numbers, spherical representation.
- 2) **Complex functions:** limits, continuity, analyticity, rational functions, exponential functions, trigonometric functions, branches of logarithm.
- 3) **Analytic functions:** harmonic functions, harmonic conjugates, Cauchy-Riemann equations, power series representation.
- 4) **Conformal mappings:** mapping properties of elementary functions, linear (fractional) transformations.
- 5) **Complex integration:** line integral, index or winding number, Cauchy's theorem, Cauchy's integral formula, Cauchy's estimate, Morera's theorem, Goursat's theorem.
- 6) **Zeros of analytic functions:** entire functions, Liouville's theorem, fundamental theorem of algebra, open mapping theorem.
- 7) **Singularities:** removable and essential singularities, poles, Laurent series, residues, residue theorem, calculus of residues and evaluation of real integrals, meromorphic functions, argument principle, Rouché's theorem.
- 8) **The maximum modulus theorem:** The maximum modulus principle, Schwarz's lemma.

2. References

- 1) Ahlfors, *Complex Analysis*, 2nd ed., McGraw-Hill, 1979, **Ch. 1-5**.
- 2) Conway, *Functions of One Complex Variable*, 2nd ed., Springer-Verlag, 1978, **Ch. 1-6**.