King Fahd University of Petroleum and Minerals
Department of Mathematics
SYLLABUS COMPREHENSIVE EXAM
Semester I: 2025-2026 **(251)**

Course #: MATH 565
Title: Advanced Ordinary Differential Equations I

Textbooks: 1.Nonlinear Differential Equations and Dynamical Systems by F. Verhulst (Second Edition, 1996. Revised 2006)
2.The qualitative theory of ordinary diffential equations: an introduction. By F. Brauer and J. A. Nohel, Dover Publications, Inc. NY (1969).

Topics to be covered
Existence and uniqueness of solution to an initial value problem
Continuation of the interval of definition of the solution
Gronwall inequalities
Phase space & orbits of autonomous equations
Critical points and linearization of autonomous equations
Periodic solutions of autonomous equations
First integrals and integral manifolds of autonomous equations
Two-dimensional linear systems
Critical points of nonlinear equations
Existence of stable and unstable manifolds
Bendixson's criterion
Geometric auxiliaries, preparation for the Poincaré-Bendixson theorem (positive & negative orbits, alpha & omega limit sets, invariant sets, minimal sets)
The Poincaré-Bendixson theorem
Applications of the Poincaré-Bendixson theorem
Stability of equilibrium solutions of non-autonomous equations
Stability of periodic solutions of non-autonomous equations
Linearization of non-autonomous equation at a periodic solution
Stability of linear equations with constants coefficients
Stability of linear equations with coefficients which have a limit
Stability of linear equations with periodic coefficients (The Floquet theory)
Asymptotic stability of the trivial solution
Instability of the trivial solution
Stability of periodic solutions of autonomous equations
Lyapunov functions for autonomous equations
Hamiltonian systems and systems with first integrals
Stability by the direct method also known as Lyapunov's second method
Applications of Stability analysis by the direct method