King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 514 Comprehensive Exam-2

Time Allowed: 180 Minutes

Name: ______________________ ID#: ______________

- Mobiles and calculators are not allowed in this exam.
- Write neatly and clearly. You may lose points for messy work.
- Show all your work. No points for answers without justification.

Question #	Marks	Maximum Marks
1		10
2		10
3		10
4		10
5		13
6		12
7		10
8		10
9		15
Total		100

Q.1 (10 points) Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin(\pi x)}{(x^2+4)(x^2+9)}dx$

Q.2 (10 points) Use Laplace transform to solve the integral equation

$$f(t) = t\sin(t) + 2\int_0^t f'(\tau)\sin(t-\tau)d\tau, \quad f(0) = 0$$

Q.3 (10 points) Use Laplace transform to solve the wave equation

$$a^2 u_{xx} = u_{tt} + b\sin(2t),\ \ x > 0,\ \ t > 0$$

under the following conditions

$$u(x,0) = 0, \quad u_t(x,0) = 0, \quad x > 0$$

and

$$u(x,0) = 0 \qquad \lim_{x\to\infty} |u(x,t)| < \infty$$

Q.4 (10 points) Solve the integral equation for $f(x)$ using the Fourier transform

$$\int_{-\infty}^{\infty} f(t)f(x-t)dt = \frac{1}{x^2+a}.$$

Q.5 (13 points) Use appropriate Fourier transform to solve the Laplace equation

$$u_{xx} + u_{yy} = 0, \;\; x > 0, \;\; y > 0$$

under the following conditions $u(0, y) = k, \;\; y > 0$ and $u(x, 0) = 0, \;\; x > 0.$ Solution is bounded as $x \to \infty$.

Q.6 (6+6 points) Use Mellin transform to show the following:

(a) $\mathcal{M}\{x^m e^{-nx}\} = \dfrac{\Gamma(m+p)}{n^{m+p-1}}$

(b) $\mathcal{M}\left\{\dfrac{1}{x^2+1}\right\} = \dfrac{\pi}{2}\csc\left(\dfrac{p\pi}{2}\right)$

Q.7 (10 points) Show the Hankel transform

$$\mathcal{H}_o\{(a^2 - r^2)H(a - r)\} = \frac{4a}{\alpha^3} J_1(a\alpha) - \frac{2a^2}{\alpha^2} J_0(a\alpha)$$

Q.8 (10 points) Find a solution $\Phi(x, y)$ of the Laplace equation

$$u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty, \quad y > 0$$

under the condition $\lim_{y \to 0^+} = \begin{cases} T_o & x < -1 \\ T_1 & |x| < 1 \\ T_2 & x > 1 \end{cases}$. Hint: Use Poisson's formula.

Q.9 (15 points) Solve the integral equation using Wiener-Hopf technique

$$\int_0^\infty e^{-|x-\xi|}u(\xi)d\xi = -\frac{1}{4}u(x) + 1, \quad 0 < x < \infty.$$

$u(x)$ is bounded as $x \to \infty$.

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