King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Fractional Differential Equations
Comprehensive Exam (202)

Duration: 120 minutes

---

(1) (a) Solve the initial-value problem

$$\begin{cases} u''(t) + {}^{RL}D^{\alpha}u(t) = 0, \quad t > 0, \quad 0 < \alpha < 1, \\ u(0) = 1, \quad u'(0) = 1. \end{cases}$$

(b) Compare the solution in (a) with the solution of

$$\begin{cases} u''(t) + {}^{C}D^{\alpha}u(t) = 0, \quad t > 0, \quad 0 < \alpha < 1, \\ u(0) = 1, \quad u'(0) = 1. \end{cases}$$

(2) Consider the function $f(x) = \sqrt{x}$ and $\alpha = 1/2$, $\beta = 3/2$. Compute

$$^{RL}D^{\alpha}\left[^{RL}D^{\beta} f(x)\right], \qquad ^{RL}D^{\beta}\left[^{RL}D^{\alpha} f(x)\right], \qquad \text{and} \qquad ^{RL}D^{\alpha+\beta} f(x).$$

(3) (a) Find the first derivative of $E_\alpha(t^\alpha)$, $\alpha > 0$, $t > 0$ in terms of a Mittag-Leffler function and discuss the case $\alpha = 1$.

(b) Show that $E_{1,2}(t) = 1 + tE_{1,3}(t)$.

(4) Solve using Laplace transform.

$$\begin{cases} D^{\alpha}u(t) - \lambda u(t) = 0, \qquad t > 0, \quad n-1 < \alpha < n, \quad \lambda > 0, \\ u^{(k)}(0) = b_k \in R, \qquad k = 0, ..., n-1. \end{cases}$$

(5) Use successive approximation to compute $u_1(t)$ and $u_2(t)$ (only)

$$\begin{cases} D^{\alpha}u(t) = 1 + tu(t) + u^2(t), \qquad 0 < \alpha < 1, \qquad t > 0, \\ u(0) = 0. \end{cases}$$

Formulas

$$L(^{C}D^{\alpha} f(t)) = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0)$$

$$L(^{RL}D^{\alpha} f(t)) = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^{k}\left[D^{\alpha-k-1} f(t)\right]_{t=0}$$

Q	1	2	3	4	5	Total
Max	20	20	20	20	20	100
Points						