King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Numerical Analysis of Ordinary Differential Equations
Comprehensive Exam

Name:	
ID :	

Q		Points
1		**20**
2		**20**
3		**20**
4		**10**
5		**20**
6		**20**
Total		**100**

Q1 Consider the Runge-Kutta method with tableau given below

$1/4$	$7/24$	$-1/24$
$3/4$	$13/24$	$5/24$
	$1/2$	$1/2$

a) Find the stability function of the method.

b) Is the method A-stable?

Q2 Let f be a continuous function and satisfies a Lipschitz condition for $[a,b]\times R^1$.

Consider the linear multistep method $y_n = y_{n-1} + 2hf(x_{n-1}, y_{n-1}) - hf(x_{n-2}, y_{n-2})$

Determine if the method is consistent and stable. If it is consistent then find the order.

Q3 Prove the following Theorem:

Theorem: Let f be a continuous function and satisfies a Lipschitz condition for $[a,b]\times R^1$. Let $y \in C^2[a,b]$ be the solution of $\begin{cases} y' = f(x,y), \ x \in [a,b], \ y \in R^1 \\ y(a) = y_0 \end{cases}$

Then, $\exists$ a constant K > 0 such that e_k , the global error in Euler's method, satisfies

$$|e_k| \le Kh, \quad k = 0,1,2,\ldots,N$$

Euler's Method: $y_{k+1} = y_k + hf(x_k, y_k) \quad k = 0,1,\cdots$

Q4 Prove that: Explicit Runge–Kutta methods can never be A-stable

Q5 Solve the given boundary value problem using the finite difference method

$$u'' + (x^2+1)u = \frac{x}{x^2+5} \quad \text{with boundary conditions} \quad u(0)=0, u(4)=1.$$

Suppose the finite difference approximation for the second-order derivative $u'' \approx (u_{i+1} - 2u_i + u_{i-1})/h^2$ and the given interval is divided into four equal subintervals. (Just write the linear system. Don't solve the system)

Q6 Consider the following three problems

(BVP)

$u'' - 3x^2u' + 3xu = 5$

$u(0)=1, \qquad u(1)=2.5$

(IVP-1)

$u_1'' - 3x^2u_1' + 3xu_1 = 5$

$u_1(0)=1, \qquad u_1'(0)=0$

(IVP-2)

$u_2'' - 3x^2u_2' + 3xu_2 = 5$

$u_2(0)=1, \qquad u_2'(0)=1$

The numerical solutions of the (IVP-1) and (IVP-2) are given in column(1) and column(2), respectively. Use shooting method to find the numerical solution of the (BVP) then fill column(3).

x	$u_1(x)$	$u_2(x)$	$u(x)$
0	1	1	
0.2	1.095654	1.294856	
0.4	1.357517	1.745005	
0.6	1.719056	2.25898	
0.8	2.074545	2.702588	
1	2.315669	2.955164	