Department of Mathematics and Statistics, KFUPM

Comprehensive Exam, Math 571, 20 Jan, 2021, Duration: 150 mins

Instructions

- There are 5 problems for total of 70 points. Credit awarded will be based on the correctness and clarity of the answers.
- Write your name and your ID number on the top of the first page.
- Write each problem in separate page.

Problem 1 (20 points) : Consider the initial value problem

$$y' = x\sin(y) \quad \text{for } x \in [0,2], \quad y(0) = \pi/2 \tag{1}$$

a) Show that (1) has a unique solution $y \in C^1[0,b]$ for some $b > 0$.
b) Define the one-step explicit Euler scheme and show that the global error is $O(h)$.
c) Define the one-step implicit Euler scheme and show that the truncation error T_n is $O(h)$.

Problem 2 (16 points) : Given that δ is a positive real number, consider the linear two-step method

$$y_{n+2} - \delta y_{n+1} = \frac{h}{2}(3f_{n+1} - f_n)$$

on the mesh $\{x_n : x_n = x_0 + nh,\ n = 0, \cdots, N\}$ of spacing $h, h > 0$.
a) For which values of δ the method is zero-stable?
b) Is the method convergent for $\delta = 1$? If **No**, justify your answer. If **yes** do the following:

1. Determine the order of accuracy and the error constant.
2. Give a bound for the truncation error T_n.

Problem 3 (12 points) : Consider the following two-point BVP:

$$-y''(x) + y'(x) + y(x) = x^2 \quad \text{for } x \in (0,1) \quad \text{with} \quad y(0) = y(1) = 0, \tag{2}$$

a) Develop a second order accurate finite difference scheme for the above BVP.
b) Show (briefly) that the truncation error of the numerical scheme in part a is of order two.

Problem 4 (10 points) : Give an example of a consistent $O(h^3)$ accurate three-stage RK method (Justify your answer).

Problem 5 (12 points) : A predictor P and a corrector C are defined by their characteristic polynomials:

$$P: \ \rho^*(z) = z^2 - z, \quad \sigma^*(z) = \frac{1}{2}(3z - 1)$$
$$C: \ \rho(z) = z^2 - z, \quad \sigma(z) = \frac{1}{2}(z^2 + z)$$

a) Find the stability polynomial $\pi_{P(EC)^m E}$ of this method.
b) Assuming that $m = 1$, use Schur's criterion to calculate the associated intervals of absolute stability. Is this method $A-$ stable? (Justify your answer).

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Good luck