King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 642
Comprehensive exam- Linear control theory- 2021-2022
Net Time Allowed: 120 minutes

Name:____

ID:____________________Section:__________Serial:______________

Q#	Marks	Maximum Marks
1		20
2		20
3		20
4		20
5		20
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. The state-space representation of a dynamical system is given as follows:

$$x'(t) = \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t), \quad x(0) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} x(t)$$

(a) Find the unit step response for the given system

(b) Compute $x(t)$ as $t \to \infty$. Which state blows up? Also find $y(\infty)$.

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2. Consider $A = \begin{bmatrix} \lambda & \lambda T & \lambda T^2/2 \\ 0 & \lambda & \lambda T \\ 0 & 0 & \lambda \end{bmatrix}$ with $\lambda \neq 0$ and $T > 0$.

(a) Find a matrix Q, such that

$$Q^{-1} A Q = J$$

where J is the Jordan-form representation of A.

(b) Find the minmal polynomial of the matrix A.

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3. Find a force control $u(t)$ that we can apply to bring the the following system to equilibrium in 2 seconds

$$x'(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad x(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

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4. You are given the following SISO system:

$$
\begin{aligned}
x'(t) &= \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)
\end{aligned}
$$

Design an observer-based controller (i.e., $u(t) = K\hat{x}(t)$) for the above system such that the desired eigenvalues for the closed loop system are all at $\lambda_{cl} = \{-10, -10\}$ for both the controller and the observer.

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5. Consider the continuous time system

$$\begin{aligned} x'(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned}$$

Use LQR with $Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 1$ to find the optimal state feedback gain K.

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